# Aggregate Productivity Shocks, Zeros and Gravity

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**Abstract**

We modify the Helpman-Melitz-Rubinstein (HMR) model to account for firm-level productivity shocks with aggregate effects. Our model assumes that firms' productivities are subject to idiosyncratic firm-level shocks as well as to aggregate country-level shocks. This leads to two nested specifications of the gravity equation, of which one permits to segregate the effect of productivity shocks on the extensive margin from that of trading costs shocks. The empirical results obtained from the latter specification suggests that over time, the selection effect is increasingly driven by aggregate productivity shocks. This corroborates the recent finding of Gabaix (2011) that the 100 largest US firms account for one-third of variations in aggregate output.

**Keywords:** Aggregate Productivity Shocks – Gravity Model – Heterogeneity – Random Trade Costs – Sample Selection – Trade Flows.

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# 1 Introduction

Helpman, Melitz and Rubinstein (2008) (henceforth HMR) developed an insightful theoretical model capable of explaining three important stylized facts about international trade. First, as in Melitz's (2003) classic model, firms are heterogenous when it comes to their productivity and capacity to export. This allows for the possibility of no trade occurring between a given country $i$ and another country $j$ and it permits to identify the determinants of the selection of countries into trading relationship. Second, the model allows for asymmetric trade flows between country pairs. In particular, the model has the potential to explain why country $i$ exports to, but does not import from country $j$. Third, the model generates a gravity equation in which distance and GNPs along with other variables condition positive trade flows. The resulting empirical gravity equation features a correction for unobserved firm heterogeneity and a sample selection bias-correction term involving the inverse Mills ratio. Interestingly, the former correction is found to be more important than the latter (HMR, p.471).

In the HMR model, each country $j$ in the world is endowed with a continuum of firms of measure $N_j$. Firms are heterogenous in productivity, and their distribution across different productivity levels is described by a truncated Pareto measure. At equilibrium, the model implies that a fraction $\pi_{ij}$ of the firms of country $j$ export to country $i$ while allowing for the possibility that $\pi_{ij} = 0$. This is an important feature of trade data that previous models with symmetric firms could not explain. Still, it also means that the number of exporting firms is infinite when $\pi_{ij} > 0$, in contrast with the fact that the total number of firms is not very large for the majority of industries within a given country. Several industries are characterized by natural entry barriers such as large sunk investments that limit the number of players. For instance, a large proportion of international trade in cars is dominated by a few companies and the same can be said about trade in aircraft, processed agricultural products and electronics. Two firms, Samsung and Hyundai, jointly account for 35% of South Korea's exports (Gabaix 2011, p.784). As pointed out by Eaton, Kortum and Sotelo (2012) "*it is hard to reconcile the small (sometimes zero) number of firms engaged in selling from one country to another with a continuum.*"

The HMR model posits a spatial distribution for firms' productivity while being mute regarding their temporal behaviour. At first glance, this spatial distribution is consistent with two radically different temporal behaviour. In the first case, each firm is endowed with a constant productivity level so that when it exports, it does so consistently over time (everything else equal). In the second case, firms switch across different productivity levels randomly over time in a manner that keeps their spatial distribution unaltered. However, the latter interpretation may not allow for the occurrence of zero aggregate trade flows when the number of candidates exporting firms is infinite. Thus, we are left with the first interpretation which implies a chain of competitive advantage such that if aggregate exports from country $j$ to country $i$ are consistently positive over time, it must be the case that the most productive firm in country $j$ is exporting consistently. Failure to export on the part of this firm entails that no other firm from country $j$ can export to country $i$. However, there is empirical evidence that firms' productivity vary over time and that there is much "ins and outs" at the firm level as well. Using establishments and enterprises data for Canada, Sabuhoro, Larue and Gervais (2006) found that for one-third of all establishments the length of an export episode does not exceed one month and that firms learn from past failures as the number of past exits increases export survival. They also found that the hazard of exiting foreign markets varies negatively and significantly with the relative size of the exporting establishment and the number of exported products. The last point is also documented by Bernard, Redding and Schott (2011). Still, even very large exporting firms with a history of high productivity are not immune to negative productivity shocks and may be forced to exit.[1]

If the number of candidate exporting firms is infinite, introducing some randomness in firms productivity is irrelevant for determining aggregate trade flows as long as firms behave independently. This is probably what Eaton, Kortum and Sotelo (2012) meant by saying that "*shocks to individual firms can never have an aggregate effect.*" For the shocks on individual firms' productivity to have observable implications on aggregate trade flows, one must either allow the firms to be in finite number and subject to random productivity shocks or introduce a common shock that affects the productivity of all firms.[2] Either of these assumptions leads to an aggregate productivity shock affecting trade flows.[3] This paper examines the theoretical and empirical implications of the presence of such aggregate productivity shocks in the HMR model.

We assume that each country in the world is endowed with $N_j$ firms that are homogenous ex-ante with respect to the distribution of productivity, but heterogenous ex-post regarding realized productivity. At the firm level, productivity shocks may be justified by unforeseen equipment plants failures and human errors. At the country level, productivity shocks may originate from the volatility of certain inputs' prices. We assume that the ex-ante distribution of the productivity index is described by the truncated Pareto measure postulated by HMR for the spatial distribution of firms. Consequently, realized trade flows ($\widetilde{M}_{ij}$) between country pairs are random but expected trade flows ($E(\widetilde{M}_{ij}) = M_{ij}$) are equal to the expressions that one obtains when the number of firms per country is infinite. Thus, the realized export from country $j$ to country $i$ can be represented as $\widetilde{M}_{ij} = M_{ij}\widetilde{R}_{ij}$, where $M_{ij}$ is the expected trade flow and $\widetilde{R}_{ij}$ is an error with unit mean. The size of $M_{ij}$ depends on the respective GNPs of $i$ and $j$, the distance between $i$ and $j$, the fixed and variable costs for exporting from $j$ to $i$ as well as some other country-specific fixed effects. The error $\widetilde{R}_{ij}$ is an aggregate shock induced by firm-level productivity shocks and it vanishes (i.e., converges to one) as the economy of country $j$ improves so that $N_j$ increases to infinity. To obtain non-vanishing aggregate shocks $\widetilde{R}_{ij}$, we assume that firms' productivity are exposed to a shock common to all firms of the same country (as in Eaton and Kortum, 2002) as well as to idiosyncratic shocks. Further specifying export costs as random conditional on country-pair specific regressors (as in HMR, p.457) implies that $M_{ij} = E(M_{ij})\, R_{ij}$, where $R_{ij}$ is another error term with unit mean. In total, we obtain an expression of the form $\widetilde{M}_{ij} = E\left(\widetilde{M}_{ij}\right) R_{ij}\widetilde{R}_{ij}$ for realized trade flows where $R_{ij}$ is due to the randomness of trade costs (and other demand side shocks) and $\widetilde{R}_{ij}$ is due to the finiteness of the number of firms (and other supply side shocks).

When the number of (candidate exporting) firms is infinite, the aggregate productivity shock $\widetilde{R}_{ij}$ is necessarily positive under the assumptions of the model. This happens because the most productive of an infinite number of firms virtually attains the maximum efficiency level allowed by the model. That is, if country $j$ has an infinite number or firms and its most productive firm is unable to export to destination $i$, it must be that trade costs are prohibitive and $\widetilde{M}_{ij} = 0$. Hence, our extension of the HMR model has no observable implication when the number of firms is infinite. When the number of firms is finite however, both $R_{ij}$ and $\widetilde{R}_{ij}$ have point-masses at zero (i.e., non-zero probability of being equal to zero), which implies that aggregate productivity shocks ($\widetilde{R}_{ij}$) contribute to the selection of countries into trading relationship in synergy with trade costs shocks ($R_{ij}$). Thus, our framework can

[1] For instance, the US import refusal on September 4 of 2012 of a shipment of beef contaminated with E. coli from Canada's XL Foods led to the largest meat recall in Canada's history, the temporary closure of the plant, a highly publicized break in its exports and the sale of the plant.

[2] Gabaix (2011) contends that firm-specific shocks do not cancel each other and that volatility affecting the largest 100 US firms account for 33% of variations in US output. This suggests that firm-specific shocks can have an impact on aggregate trade flows.

[3] In its issue of June 22, 2013, "The Economists" published an article entitled "The Goliaths" which elegantly makes the point that "the fate of large firms help explain economic volatility."

be used to assess how important is the contribution of productivity shocks to the overall probability of trade. Two different gravity equations arises depending on whether the number of firms is finite or not. We estimate these two models separately for the years 1980 and 1989 using the world trade data available on Elhanan Helpman's website and find supportive evidence for the specification that assumes a finite number of firms. The results suggest that on average, aggregate productivity shocks contributed up to 51% of the sample selection effect in 1980 and up to 57% in 1989.

The remainder of the paper is organized as follows. Section 2 revisits the original HMR model. Our version of the HMR model with a finite number of firms is presented in Section 3. In Section 4, we present an alternative approach to introduce aggregate productivity shocks in trade flows without having to assume that the number of firms is finite. We present the two gravity equations that can be deduced from our theoretical framework in Section 5 and derive their feasible versions in Section 6. Section 7 presents the empirical application and Section 8 concludes.

## 2 Revisiting the Helpman-Melitz-Rubinstein Model

HMR assumes that there are $J$ countries in the world, and each country produces and consumes a continuum of products. The demand of country $j$ for product $l$ is given by:

$$x_j(l) = \frac{p_j(l)^{-\varepsilon} Y_j}{P_j^{1-\varepsilon}}, \tag{1}$$

where $p_j(l)$ is the price at which product $l$ sells in country $j$, $P_j$ and $Y_j$ are respectively the ideal price index and the income of country $j$, and $\varepsilon > 1$ is the elasticity of substitution across products. The price index $P_j$ is defined as:

$$P_j = \left[\int_{l\in B_j} p_j(l)^{1-\varepsilon}\, dl\right]^{1/(1-\varepsilon)}, \tag{2}$$

where $B_j$ is the set of goods available for consumption in country $j$.

Each firm of the world economy produces a distinct product, and each country $j$ has a continuum of firms of measure $N_j$. A country-$j$ firm produces one unit of output with a combination of inputs whose value is $c_j a$, where $a$ is the number of inputs used and $c_j$ is the price of that input. The productivity, $\frac{1}{a}$, is firm-specific while the factor price, $c_j$, is country-specific. According to these assumptions, a country-$j$ firm with productivity $\frac{1}{a}$ maximizes its profits by setting a domestic price equal to $q_{jj}(a) = \frac{c_j a}{\alpha}$, where $\alpha \equiv \frac{\varepsilon-1}{\varepsilon}$. When the same firm exports from country $j$ to country $i$, its price is $q_{ij}(a) = \tau_{ij}\frac{c_j a}{\alpha}$, where $\tau_{ij} > 1$ reflects all variable costs necessary to deliver the firm's product to the importing country. This variable cost may include transport cost as well as other costs induced by trade resistance factors. At the price $q_{ij}$, the demand of the remote market for a product from country $j$ is given by:

$$x_{ij}(a) = \left(\tau_{ij}\frac{c_j a}{\alpha}\right)^{-\varepsilon} \frac{Y_i}{P_i^{1-\varepsilon}}. \tag{3}$$

There are also fixed costs $c_j f_{ij}$ incurred by an exporting firm for serving country $i$. Thus the profit from exporting a product from $j$ to $i$ is:

$$\pi_{ij}(a) = (1-\alpha)\left(\frac{\tau_{ij} c_j a}{\alpha P_i}\right)^{1-\varepsilon} Y_i - c_j f_{ij},$$

where the fixed export costs are defined by $c_j f_{ij}$. The minimum productivity required for a country-$j$ firm to be able to export to country $i$ is implicitly defined by $\pi_{ij}(a_{ij}) = 0$. Rearranging, we obtain:

$$a_{ij} = \frac{\alpha P_i}{c_j \tau_{ij}} \left( \frac{c_j f_{ij}}{(1-\alpha) Y_i} \right)^{1/(1-\varepsilon)}. \tag{4}$$

In order to determine the volume of importation of country $i$ from country $j$, one needs to know the relative proportion of each type of firms within the exporting country. HMR assume that these proportions are the same for all countries and are described by a truncated Pareto distribution with support $[a_L, a_H]$. The cumulative distribution function of the heterogeneity index $a$ is thus given by:

$$\Pr(a \leq x) \equiv G(x) = \frac{x^k - a_L^k}{a_H^k - a_L^k}, a_L \leq x \leq a_H, \tag{5}$$

where $k > \varepsilon - 1$. However, these proportions must be multiplied by $N_j$ in order to reflect the relative size of the different economies. Accordingly, the value of the imports of country $i$ from country $j$ is:

$$\begin{aligned} M_{ij} &\equiv N_j \int_{a_L}^{a_{ij}} x_{ij}(a)\, q_{ij}(a)\, dG(a) \\ &= N_j \left( \frac{\tau_{ij} c_j}{\alpha P_i} \right)^{1-\varepsilon} Y_i V_{ij}, \end{aligned} \tag{6}$$

where

$$V_{ij} = \begin{cases} \int_{a_L}^{a_{ij}} a^{1-\varepsilon} dG(a) & \text{for } a_{ij} > a_L \\ 0 & \text{otherwise.} \end{cases}.$$

An explicit calculation of $V_{ij}$ yields $V_{ij} = \frac{k a_L^{k-\varepsilon+1}}{(k-\varepsilon+1)\left(a_H^k - a_L^k\right)} W_{ij}$, where

$$W_{ij} = \max\left\{ \left( \frac{a_{ij}}{a_L} \right)^{k-\varepsilon+1} - 1, 0 \right\}. \tag{7}$$

Overall, the HMR model generalizes Anderson and van Wincoop's (2003) model in two respects. First, it highlights the extensive margin of trade flows and its relationship to firm heterogeneity and fixed trade costs. Second, it accounts for the fact that the volume of exports from $j$ to $i$ is potentially different from the volume of exports from $i$ to $j$. However, a maintained assumption in the HMR model is that each country $j$ in the world has a continuum of firms of measure $N_j$. A direct consequence of this assumption is that the number of country-$j$ firms exporting to country $i$ is either zero (when no firm is qualified to export to $i$) or infinite (when a proportion $G(a) > 0$ of firms are qualified to export to $i$). However, there are factors in the real world that cause most exporting industries not to involve a large number of players. First, large sunk investment playing as entry barriers often limits the number of candidates exporting firms. Second, a firm that is qualified to export to a remote market still faces a high probability of exit within a relatively short length of time (Sabuhoro, Larue and Gervais, 2006). In the next section, we accommodate these empirical facts by re-formulating the HMR model for a finite number of firms facing random productivity shocks.

# 3 Sources of Randomness in Aggregate Trade Flows

The aim of this section is to show that trade flows are likely to be described by an equation of type $\widetilde{M}_{ij} = E(M_{ij}) R_{ij} \widetilde{R}_{ij}$, where $R_{ij}$ is a random error caused by trade costs and the macroeconomic volatility of the importing country and $\widetilde{R}_{ij}$ is caused by the finiteness of the number of firms and the macroeconomic volatility of the exporting country. An approach is proposed to measure the relative importance of each of these two sources of randomness in determining the selection of countries into trading relationship.

### 3.1 Finiteness of the Number of Firms

The measure $G(a)$ that gives the spatial distribution of the productivity index in (5) puts nonzero weights on firms that do no export at all. An alternative approach to compute trade flows might account for the a priori knowledge that only firms for which $a \leq a_{ij}$ are able to export. In this case, one would compute $V_{ij}$ using the distribution of the index $a$ within the subset of exporting firms, which is given by:

$$G_{ij}(x) \equiv \Pr\left(a \leq x | a_L \leq a \leq a_{ij}\right) = \frac{x^k - a_L^k}{a_{ij}^k - a_L^k}.$$

While $G(a)$ is the unconditional distribution of $a$ within each country (common to all countries), $G_{ij}(x)$ is the distribution of $a$ in country $j$ conditional on the firm actually exporting to country $i$. Hence, countries are homogenous ex ante regarding the distribution $G(a)$ but heterogenous ex post with respect to the conditional distribution $G_{ij}(x)$. Using $G_{ij}(x)$ to compute trade flows yields:

$$M_{ij} = N_{ij} \frac{1}{G\left(a_{ij}\right)} \left(\frac{\tau_{ij} c_j}{\alpha P_i}\right)^{1-\varepsilon} Y_i V_{ij}, \tag{8}$$

where $N_{ij}$ is the measure of firms that export from $j$ to $i$.

Equation (8) states that the aggregate trade flow $M_{ij}$ is equal to the measure of the firms that export, $N_{ij}$, times the average value of exports of a representative exporting firm $\frac{1}{G(a_{ij})}\left(\frac{\tau_{ij}c_j}{\alpha P_i}\right)^{1-\varepsilon} Y_i V_{ij}$. In turn, this average value has two components. The first component, the ratio $\frac{1}{G(a_{ij})}$, is interpreted as the extensive margin component because it depends solely on the proportion of exporting firms $G\left(a_{ij}\right)$. The second component, $\left(\frac{\tau_{ij}c_j}{\alpha P_i}\right)^{1-\varepsilon} Y_i V_{ij}$, may be attributed to the intensive margin as it relates to the volume of trade of firms already involved in trade. The latter component is increasing in the proportion of firms that export according to the following equality:

$$V_{ij} = \frac{k a_L^{k-\varepsilon+1}}{(k-\varepsilon+1)\left(a_H^k - a_L^k\right)} \max\left\{\left(\frac{a_H^k - a_L^k}{a_L^k} G\left(a_{ij}\right) + 1\right)^{\frac{k-\varepsilon+1}{k}} - 1, 0\right\}. \tag{9}$$

The expressions of $M_{ij}$ in (6) and in (8) are reconciled if and only if $N_{ij} = N_j G\left(a_{ij}\right)$. The latter equality necessarily holds in the HMR model because firms are in infinite number while each firm is deterministically identified by its productivity level. According to a Law of Large Numbers, this equality remains valid if one chooses to interpret $a_{ij}$ as a random variable draw from a Pareto distribution with support $[a_H, a_L]$. As an implication from Equation (6), keeping $\left(\frac{\tau_{ij}c_j}{\alpha P_i}\right)^{1-\varepsilon}$ and $Y_i$ constant, the only way for country $j$ to increase its export to country $i$ is by increasing $V_{ij}$ through the extensive margin $G\left(a_{ij}\right)$. This would be the case for example if the productivity of country-$j$ firms had improved exogenously. In this case, an increase in trade volume arises only from the emergence of new trade relations at the firm level. By contrast, if $\left(\frac{\tau_{ij}c_j}{\alpha P_i}\right)^{1-\varepsilon}$ and $G\left(a_{ij}\right)$ are kept constant, an increase in trade volume may arise following an increase of the GNP $Y_i$ of the importing country. In this case, a fixed number of country $j$ exporting firms must produce more intensively in order to respond to a higher level of demand in country $i$.

Interestingly, the equation $N_{ij} = N_j G\left(a_{ij}\right)$ does not hold exactly in a context where the number of firms is finite and a firm's productivity is random. To see this, let each country $j$ have a finite number $N_j$ of firms. At the beginning of each period $t$, a firm's productivity is a random variable $1/a$, where $a$ follows the truncated Pareto distribution given by (5). In mature industries, the technology is known to all and often readily available. However, even a mastered technology may give rise to random productivity because of uncertainties about the marginal product of certain inputs (e.g., absenteeism, equipment failures or human error). Hence, we shall assume that each firm knows whether it is able to export only after observing the realization of its productivity $a$ and the threshold $a_{ij}$ at the end of the period. Under these assumptions, the observed aggregate trade flow from country $j$ to country $i$ is given by:

$$\widetilde{M}_{ij} = \sum_{k=1}^{N_j} x_{ij}\left(a_{(kj)}\right) q_{ij}\left(a_{(kj)}\right) 1\left(a_{(kj)} \in [a_L, a_{ij}]\right), \tag{10}$$

where quantities and prices $x_{ij}(a_k)$ and $q_{ij}(a_k)$ are defined as in Section 2, $1\left(a_{(kj)} \in [a_L, a_{ij}]\right) = 1$ if $a_{(kj)} \in [a_L, a_{ij}]$ and $1\left(a_{(kj)} \in [a_L, a_{ij}]\right) = 0$ otherwise.

The current modified HMR model has three main implications. First, the realized trade flow (10) is random and it satisfies

$$\begin{aligned} \widetilde{M}_{ij} &= Y_i \left(\frac{\tau_{ij} c_j}{\alpha P_i^{1-\varepsilon}}\right)^{1-\varepsilon} \sum_{k=1}^{N_j} a_{(kj)}^{1-\varepsilon} 1\left(a_{(kj)} \in [a_L, a_{ij}]\right) \text{ and} \\ E\left(\widetilde{M}_{ij}\right) &= N_j Y_i \left(\frac{\tau_{ij} c_j}{\alpha P_i}\right)^{1-\varepsilon} V_{ij} \equiv M_{ij}, \end{aligned}$$

where the expression of $M_{ij}$ is the same as in Equation (6). Second, the number of exporting firms, $N_{ij} = \sum_{k=1}^{N_j} 1\left(a_{(kj)} \in [a_L, a_{ij}]\right)$, is also random such that $E(N_{ij}) = N_j G(a_{ij})$. Third, $G(a_{ij})$ is no longer the proportion of firms that trade and now denotes the probability of trade at the firm level. This means that the ex-post number of firms involved in trade (i.e., the realization of $N_{ij}$) can be zero even though $G(a_{ij})$ is strictly positive. Firms that are expected to export may not do so because of adverse productivity shocks. Indeed, $N_{ij}$ follows the Binomial distribution $B(N_j, G(a_{ij}))$ and the probability that country $i$ imports from $j$ is given by:

$$\Pr(N_{ij} > 0) = 1 - (1 - G(a_{ij}))^{N_j}. \tag{11}$$

The latter implication relates our framework to the balls-and-bins model of Armenter and Koren (2012). These authors propose a disaggregated trade model where shipments are treated as balls that are randomly assigned to bins. The balls are identified by the HS classification in the origin country while the bins are labelled by the classification in the destination country. Each ball has probability $s_{ij}$ of originating from category $i$ and landing into the category $j$. Similarly to our model, a bin may end up being empty even though its ex-ante probability of receiving balls is strictly positive. Another model considered by Eaton, Kortum and Sotelo (2012) assumes that the number of country $j$ firms achieving at least a given productivity level $a$ is generated by a Poisson distribution with intensity $\mu_j(a)$, where $\mu_j(.)$ is a continuous measure on the support of $a$. Here, we follow a different route that is rather closer in spirit to the original HMR (2008) model.

Keeping in mind that $\widetilde{M}_{ij}$ is observed while its expectation $M_{ij}$ is not, we define the multiplicative error $\widetilde{R}_{ij}$ as:

$$\widetilde{R}_{ij} \equiv \begin{cases} \frac{\widetilde{M}_{ij}}{M_{ij}} \text{ if } M_{ij} > 0, \text{ and} \\ 0, \text{ otherwise.} \end{cases}, \tag{12}$$

so that by construction, $\widetilde{M}_{ij} = M_{ij}\widetilde{R}_{ij}$. Replacing the expressions of $\widetilde{M}_{ij}$ and $M_{ij}$ into (12) yields:

$$\widetilde{R}_{ij} = \begin{cases} \frac{1}{N_j V_{ij}} \sum_{k=1}^{N_j} a_{(kj)}^{1-\varepsilon} 1\left(a_{(kj)} \in [a_L, a_{ij}]\right) \text{ if } V_{ij} > 0, \text{ and} \\ 0, \text{ otherwise.} \end{cases}. \tag{13}$$

Hence, realized trade flows are random even when trade costs are constant over time.

### 3.2 Trade Costs and Aggregate Productivity Shocks

Reformulating the HMR model with a finite number of firms allows us to introduce in an intuitive manner a multiplicative productivity shock $\widetilde{R}_{ij}$ in the expression of trade flows. By definition, $\widetilde{R}_{ij}$ is a supply side shock as it stems from the aggregation of firm-level productivity shocks. If these productivity shocks are independent across firms, then the probability of $\widetilde{R}_{ij} = 0$ shrinks to zero as the number of candidate exporting firms increases to infinity. Furthermore, a law of large number applies such that:

$$\widetilde{R}_{ij} = \frac{1}{N_j V_{ij}} \sum_{k=1}^{N_j} a_{(kj)}^{1-\varepsilon} 1\left(a_{(kj)} \in [a_L, a_{ij}]\right) \to 1 \text{ as } N_j \to \infty.$$

Hence, the multiplicative shock $\widetilde{R}_{ij}$ vanishes as $N_j$ increases to infinity. Based on this observation, one might argue that aggregate productivity shocks are irrelevant for large economies.

However, the productivity of candidates exporting firms may be contingent to domestic macroeconomic shocks. For example, Eaton and Kortum (2002) posited that country $j$'s efficiency in producing good $k$ is the realization of a random variable $Z_j$, where the distribution of $Z_j$ is country-specific, independent and identically distributed across goods. This assumption could be formalized by specifying the productivity of a country $j$ firm as:

$$b_{(kj)} = a_{(j)}^{\rho} a_{(kj)}^{1-\rho}, \tag{14}$$

where $a_{(j)}$ and $a_{(kj)}$ are independent Pareto random variables with support $[a_L, a_H]$, $a_{(j)}$ is common to all firms of country $j$ and $a_{(kj)}$ is specific to the firm $k$ of country $j$. For trade to occur, $b_{(kj)}$ must lie between the bounds $a_L$ and $a_{ij}$. The corresponding bounds for the firm-specific shock $a_{(kj)}$ can be derived for a given common productivity shock $a_{(j)}$. Indeed, trade flows are given by:

$$\widetilde{M}_{ij} = a_{(j)}^{\rho(1-\varepsilon)} \sum_{k=1}^{N_j} x_{ij}\left(a_{(kj)}\right) q_{ij}\left(a_{(kj)}\right) 1\left(a_{(kj)} \in [lb, ub]\right), \tag{15}$$

with $lb = \left(a_L / a_{(j)}^{\rho}\right)^{1/(1-\rho)}$ and $ub = \left(a_{ij}/a_{(j)}^{\rho}\right)^{1/(1-\rho)}$. As depicted by the bounds of $a_{(kj)}$, a strong common shock compensates a lower probability of export conditional on the realization of the common shock.

Let us examine the implications of (14) when trade costs are non-random. Straightforward calculations show that:

$$E\left(\widetilde{M}_{ij} | a_{(j)}\right) = M_{ij} a_{(j)}^{-\rho\widetilde{k}},$$

where $\widetilde{k} = k/(1-\rho)$,

$$M_{ij} = N_j \left(\frac{\tau_{ij} c_j}{\alpha P_i}\right)^{1-\varepsilon} Y_i V_{ij}, \tag{16}$$

$$V_{ij} = \frac{\widetilde{k}\left(a_L\right)^{\widetilde{k}+1-\varepsilon}}{\left(\widetilde{k}+1-\varepsilon\right)\left(a_H^k - a_L^k\right)} W_{ij} \quad \text{and}$$

$$W_{ij} = \max\left\{\left(\frac{a_{ij}}{a_L}\right)^{\widetilde{k}+1-\varepsilon} - 1, 0\right\}. \tag{17}$$

Thus, we can write $\widetilde{M}_{ij} = E\left(\widetilde{M}_{ij}\right)\widetilde{R}_{ij}$ where $\widetilde{R}_{ij} = \widetilde{R}_{1,ij}\widetilde{R}_{2,ij}$,

$$E\left(\widetilde{M}_{ij}\right) = \frac{a_H^{k-\rho\widetilde{k}} - a_L^{k-\rho\widetilde{k}}}{\left(a_H^k - a_L^k\right)\left(1-\rho/(1-\rho)\right)} M_{ij}, \tag{18}$$

$$\widetilde{R}_{1,ij} = \frac{\widetilde{M}_{ij}}{E\left(\widetilde{M}_{ij}|a_{(j)}\right)} = \begin{cases} \frac{a_{(j)}^{\rho(\widetilde{k}+1-\varepsilon)}}{N_j V_{ij}} \sum_{k=1}^{N_j} a_{(kj)}^{(1-\varepsilon)(1-\rho)} 1\left(a_{(kj)} \in [lb, ub]\right) & \text{if } V_{ij} > 0, \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \tag{19}$$

$$\widetilde{R}_{2,ij} \equiv \frac{E\left(\widetilde{M}_{ij}|a_{(j)}\right)}{E\left(\widetilde{M}_{ij}\right)} = \frac{\left(a_H^k - a_L^k\right)\left(1-\rho/(1-\rho)\right)}{a_H^{k-\rho\widetilde{k}} - a_L^{k-\rho\widetilde{k}}} a_{(j)}^{-\rho\widetilde{k}}. \tag{20}$$

As shown by the equations above, the error term stemming from the productivity shocks has two parts. The first part ($\widetilde{R}_{1,ij}$) converges to one as the number of firms increases to infinity whilst the second part ($\widetilde{R}_{2,ij}$) remains strictly positive and non-vanishing.

The term $M_{ij}$ that appears in (18) and the error term $\widetilde{R}_{ij}$ depend on the threshold $a_{ij}$ given by (4). In turn, $a_{ij}$ depends on the price index ($P_i$) and income ($Y_i$) of the importing country as well as on the costs incurred by country $j$ firms to serve country $i$ ($\tau_{ij}$). Hence, $a_{ij}$ is not immunized against the macroeconomic volatility of the destination country as it captures all demand side shocks. Assuming that either of the constituent of $a_{ij}$ is random leads to the conclusion that $M_{ij}$ is also random. Following, HMR we posit that only trade costs are random. This allows us to interprete (18) as the expectation of trade flows conditional on $a_{ij}$:

$$E\left(\widetilde{M}_{ij}|a_{ij}\right) = \frac{a_H^{k-\rho\widetilde{k}} - a_L^{k-\rho\widetilde{k}}}{\left(a_H^k - a_L^k\right)\left(1-\rho/(1-\rho)\right)} M_{ij}. \tag{21}$$

Overall, realized trade flows can be written as

$$\widetilde{M}_{ij} = E\left(M_{ij}\right) R_{ij}\widetilde{R}_{ij}, \tag{22}$$

where

$$R_{ij} \equiv \frac{E\left(\widetilde{M}_{ij}|a_{ij}\right)}{E\left(\widetilde{M}_{ij}\right)} \text{ and } \widetilde{R}_{ij} \equiv \frac{\widetilde{M}_{ij}}{E\left(\widetilde{M}_{ij}|a_{ij}\right)}.$$

As shown below, this representation of $\widetilde{M}_{ij}$ allows us to decompose the extensive margin into a part explained by trade costs and demand side shocks and another part due to the finiteness of the number of firms and supply side shocks.

### 3.3 Extensive Margin Decomposition

According to (22), the probability of export from $j$ to $i$ is:

$$\Pr(\widetilde{M}_{ij} > 0) = \Pr(R_{ij} > 0)\Pr(\widetilde{R}_{ij} > 0|R_{ij} > 0), \tag{23}$$

where $R_{ij} \equiv \frac{E\left(\widetilde{M}_{ij}|a_{ij}\right)}{E\left(\widetilde{M}_{ij}\right)}$, $E\left(\widetilde{M}_{ij}|a_{ij}\right)$ is given by (21) and $\widetilde{R}_{ij} = \widetilde{R}_{1,ij}\widetilde{R}_{2,ij}$ is defined at (19)-(20). From this, we see that $R_{ij} = 0$ if and only if $E\left(\widetilde{M}_{ij}|a_{ij}\right) = M_{ij} = 0$. In turn, $M_{ij} = 0$ if and only if:

$$\begin{aligned} W_{ij} &= \max\left\{\left(\frac{a_{ij}}{a_L}\right)^{\widetilde{k}+1-\varepsilon} - 1, 0\right\} = 0, \\ &\Leftrightarrow \; a_{ij} \le a_L. \end{aligned}$$

This happens when an adverse demand side shock or a large realization of trade costs impedes on the ability of the most productive firm of country $j$ to serve country $i$. Hence, $\Pr(R_{ij} > 0)$ is the probability of the most efficient firm of country $j$ being able to export to country $i$.

Even if the most productive firm has the potential to export *ex ante,* a too low realization of productivity (i.e., an adverse supply side shock) may cause the ex-post aggregate trade flow to be zero. This is featured by $\Pr(\widetilde{R}_{ij} > 0|R_{ij} > 0)$, which measured the probability of aggregate trade flows from $j$ to $i$ being positive ex-post conditional on the most productive firm of country $j$ having the potential to serve country $i$ ex-ante. In this reasoning, note that we have conditioned the probability of $\left\{\widetilde{R}_{ij} > 0\right\}$ on the event $\{R_{ij} > 0\}$ only for convenience. This does not imply that a timing is imposed on the realizations of $\widetilde{R}_{ij}$ and $R_{ij}$ as the reasoning is also valid the other way around. In fact, an increase in the productivities of all country $j$ firms can be offset by a rise in the costs for serving country $i$ so that the overall number of firms exporting to that destination remains unchanged. Below, we attempt to segregate the probability of trade into its part due to productivity shocks and its part imputable to trade costs volatility.

Taking the logarithm of both sides of (23) and summing over all countries yields:

$$\sum_{i \neq j} \log \Pr(\widetilde{M}_{ij} > 0) = \sum_{i \neq j} \log \Pr(R_{ij} > 0) + \sum_{i \neq j} \log \Pr(\widetilde{R}_{ij} > 0|R_{ij} > 0).$$

We propose to measure the contribution of productivity shocks to the extensive margin by the ratio:

$$\lambda = \frac{\sum_{i \neq j} \log \Pr(\widetilde{R}_{ij} > 0|R_{ij} > 0)}{\sum_{i \neq j} \log \Pr(\widetilde{M}_{ij} > 0)}. \tag{24}$$

As shown by (19)-(20), $\widetilde{R}_{2,ij} > 0$ is the limit of $\widetilde{R}_{ij}$ as the number of candidate exporting firms increases to infinity. Consequently, the probability of $\widetilde{R}_{ij}$ being strictly positive is equal to one when $N_j$ is infinite. In this case, aggregate productivity shocks do not contribute to the selection of countries into trading relationship (i.e., $\lambda = 0$). On the other hand, $a_{ij}$ is deterministic in the absense of demand side shocks. In the latter case, $R_{ij} = 1$ and $\Pr(\widetilde{M}_{ij} > 0) = \Pr(\widetilde{R}_{ij} > 0)$, meaning that the extensive margin is solely determined by aggregate productivity shocks (i.e., $\lambda = 1$). Thus, $0 \leq \lambda \leq 1$ by construction and the closer $\lambda$ is to one, the stronger is the contribution of productivity shocks to the extensive margin.

# 4 Heterogeneity and Sample Selection

Recall that realized trade flows are given by $\widetilde{M}_{ij} = E(M_{ij}) R_{ij} \widetilde{R}_{ij}$ where $M_{ij}$, $R_{ij}$ and $\widetilde{R}_{ij}$ are defined in Section 3.2. The term $M_{ij}$ inherits its randomness from $a_{ij}$ which, among others, depends on trade costs ($\tau_{ij}$). HMR (2008, p.453) assumed that $\tau_{ij}^{\varepsilon-1} = D_{ij}^{\gamma} \exp(-u_{ij})$, where $D_{ij}$ measures the distance between countries $i$ and $j$ (including other trade resistance factors) and $u_{i,j} \sim N(0, \sigma_u^2)$. Substituting into Equation (16) yields:

$$M_{ij} = \frac{\widetilde{k} a_L^{\widetilde{k}+1-\varepsilon}}{\left(a_H^k - a_L^k\right)\left(\widetilde{k}+1-\varepsilon\right)} N_j Y_i D_{ij}^{\gamma} \left(\frac{c_j}{\alpha P_i}\right)^{1-\varepsilon} \exp(-u_{ij}) W_{ij},$$

where $\widetilde{k} = \frac{k}{1-\rho}$. HMR (2008, p.455) further assumed that the fixed input quantity required to export is specified as $f_{ij} = \exp\left(\phi_{Ex,j} + \phi_{IM,i} + \kappa\phi_{ij} - v_{ij}\right)$, where $\phi_{Ex,j}$ and $\phi_{IM,i}$ are respectively related to export costs and trade barriers imposed by the importing country, $\phi_{ij}$ includes any additional fixed trade costs tied to the country pair $i$ and $j$ and $v_{i,j} \sim N(0, \sigma_v^2)$. This implies that $W_{ij}$ is also random through the zero profit condition to export:

$$\left(\frac{a_{ij}}{a_L}\right)^{\varepsilon-1} = \frac{(1-\alpha)}{c_j}\left(\frac{\alpha P_i}{a_L c_j}\right)^{\varepsilon-1} Y_i D_{ij}^{-\gamma} \exp\left(-\phi_{Ex,j} - \phi_{IM,i} - \kappa\phi_{ij} + u_{ij} + v_{ij}\right). \tag{25}$$

By assuming that $u_{ij}$ and $v_{ij}$ are independent of $\widetilde{R}_{ij}$, we obtain:

$$\widetilde{M}_{ij} = \exp\left(x_{ij}\beta\right) E\left[\exp\left(-u_{ij}\right) W_{ij}\right] \epsilon_{ij}, \tag{26}$$

where $\exp\left(x_{ij}\beta\right)$ is a proxy defined as:

$$\begin{aligned}
\exp\left(x_{ij}\beta\right) &\equiv \frac{k a_L^{\widetilde{k}-\varepsilon+1}}{\left(a_H^k - a_L^k\right)\left(\widetilde{k}-\varepsilon+1\right)} N_j Y_i D_{ij}^{\gamma}\left(\frac{c_j}{\alpha P_i}\right)^{1-\varepsilon}, \\
R_{ij} &\equiv \frac{M_{ij}}{E\left(M_{ij}\right)} = \frac{\exp\left(-u_{ij}\right) W_{ij}}{E\left[\exp\left(-u_{ij}\right) W_{ij}\right]}, \\
\epsilon_{ij} &\equiv R_{ij}\widetilde{R}_{ij}
\end{aligned}$$

and $x_{ij}$ is a set of regressors including a constant.

If we had to estimate Model (26) in multiplicative form by relying only on observations with positive trade flows, we would have to control for the sample selection bias, which in this context amounts to normalize the error term as follows:

$$\widetilde{M}_{ij} = \exp\left(x_{ij}\beta\right) E\left(\exp\left(-u_{ij}\right) W_{ij}\right) E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)\left(\frac{\epsilon_{ij}}{E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)}\right), \tag{27}$$

where $\epsilon_{ij}/E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)$ is an error term with unit mean on the domain $\{\epsilon_{ij}>0\}$ and $E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)$ is a correction for the sample selection effect. Log-linearizing (27) yields:

$$\log\widetilde{M}_{ij} = x_{ij}\beta + \log E\left(\exp\left(-u_{ij}\right) W_{ij}\right) + \log E\left(\epsilon_{ij}|\epsilon_{ij}>0\right) + \log\frac{\epsilon_{ij}}{E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)}.$$

where

$$\log\frac{\epsilon_{ij}}{E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)} \equiv E\left(\log\epsilon_{ij}|\epsilon_{ij}>0\right) - \log E\left(\epsilon_{ij}|\epsilon_{ij}>0\right) + \widetilde{\epsilon}_{ij},$$

$\log E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)$ is the sample selection bias-correction term, $E\left(\log\epsilon_{ij}|\epsilon_{ij}>0\right) - \log E\left(\epsilon_{ij}|\epsilon_{ij}>0\right)$ is the residual heterogeneity and $\widetilde{\epsilon}_{ij}$ is a zero mean error.

An explicit expression is obtained for $E\left[\exp\left(u_{ij}\right) W_{ij}|x_{ij}\right]$ by using Equation 6 of Santos Silva and Tenreyro (2009). We have:

$$\begin{aligned}
E\left[\exp\left(u_{ij}\right) W_{ij}|x_{ij}\right] &= \exp\left(\sigma_u^2/2\right)\left[\exp\left(\delta^2/2 + \delta r + \delta z_{ij}^*\right)\Phi\left(z_{ij}^* + \delta + r\right) - \Phi\left(z_{ij}^* + r\right)\right], \\
&\equiv \overline{W}\left(z_{ij}^*, \delta, r\right)
\end{aligned} \tag{28}$$

where $\Phi$ is the CDF of the standard normal distribution, $z_{ij}^*$ is a latent variable such that $\Phi\left(z_{ij}^*\right) = \Pr\left(R_{ij}>0\right)$, $\delta = \left(\frac{k-\varepsilon+1}{\varepsilon-1}\right) Var(u_{ij}+v_{ij})$ and $r = \frac{Var(u_{ij})}{\sqrt{Var(u_{ij})+Var(v_{ij})}}$. Finally, substituting into the expression of $\log\widetilde{M}_{ij}$ yields:

$$\log\widetilde{M}_{ij} = x_{ij}\beta + \log\overline{W}\left(z_{ij}^*, \delta, r\right) + E\left(\log\epsilon_{ij}|\epsilon_{ij}>0\right) + \widetilde{\epsilon}_{ij}, \tag{29}$$

where $\log\overline{W}\left(z_{ij}^*, \delta, r\right)$ captures the heterogeneity in log-expected trade flows and $E\left(\log\epsilon_{ij}|\epsilon_{ij}>0\right)$ controls the sample selection bias and the error term heterogeneity.

# 5 Empirical Specifications and Estimation

The aim in this section is to design proxies for $\overline{W}\left(z_{ij}^*, \delta, r\right)$ and $E\left(\log \epsilon_{ij} | \epsilon_{ij} > 0\right)$ for the purpose of taking equations (29) to the data. To this end, we assume that $\epsilon_{ij}$ follows a log-normal distribution with a point-mass at zero:

$$\begin{aligned}
\log \epsilon_{ij} | \epsilon_{ij} &> 0 \sim N\left(\mu_{ij}, \sigma_{ij}^2\right) \text{ and} \\
\Pr\left(\epsilon_{ij} = 0\right) &= 1 - \Phi\left(\widetilde{z}_{ij}^*\right)
\end{aligned}$$

where $\widetilde{z}_{ij}^*$ is a latent variable to be estimated from the data, $\sigma_{ij}^2 \equiv \log\left(1 + \exp\left(\theta_0 + \theta_1\left(x_{ij}\beta\right)\right)\right)$ and $(\theta_0, \theta_1)$ are real parameters.

Under these assumptions, we have:

$$\log E\left(\epsilon_{ij} | \epsilon_{ij} > 0\right) = \mu_{ij} + \sigma_{ij}^2/2$$

The unconditional mean of $\epsilon_{ij}$ is equal to one. This implies that:

$$E\left(\epsilon_{ij} | \epsilon_{ij} > 0\right) = \Pr\left(\epsilon_{ij} > 0\right)^{-1} = \Phi\left(\widetilde{z}_{ij}^*\right)^{-1},$$

so that finally:

$$E\left(\log \epsilon_{ij} | \epsilon_{ij} > 0\right) \equiv \mu_{ij} = -\log \Phi\left(\widetilde{z}_{ij}^*\right) - \sigma_{ij}^2/2 \tag{30}$$

It remains to find proxies for $z_{ij}^*$ (which enters in $\overline{W}\left(z_{ij}^*, \delta, r\right)$) and $\widetilde{z}_{ij}^*$ (which is needed to evaluate $\Pr\left(\epsilon_{ij} > 0\right) = \Phi\left(\widetilde{z}_{ij}^*\right)$). For this purpose, we need to fit a Probit model to the indicator of trade. There is a positive trade flow from $j$ to $i$ if and only if $\frac{a_{ij}}{a_L} > 1$. Thus, we define:

$$\widetilde{z}_{ij}^* = \frac{1}{\sqrt{\sigma_u^2 + \sigma_v^2}} \log\left(\frac{a_{ij}}{a_L}\right)^{\varepsilon - 1},$$

so that $\widetilde{M}_{ij} > 0$ if and only if $\widetilde{z}_{ij}^* > 0$. Equation (25) implies that $\widetilde{z}_{ij}^* = x_{ij}\gamma + \eta_{ij}$, where $x_{ij}\gamma$ collects all fixed effects and $\eta_{ij} = \frac{u_{ij} + v_{ij}}{\sqrt{\sigma_u^2 + \sigma_v^2}}$ is a standard normal error. Hence:

$$\Pr\left(\widetilde{M}_{ij} > 0 | x_{ij}\right) \equiv \Phi\left(\widetilde{z}_{ij}^*\right) = \Phi\left(x_{ij}\gamma\right). \tag{31}$$

Finally, the proxy of $\widetilde{z}_{ij}^*$ is given by $\widehat{\widetilde{z}}_{ij}^* = x_{ij}\widehat{\gamma}$, where $\widehat{\gamma}$ is a Probit estimator of $\gamma$.

Next, note that the following inequality links $\widetilde{z}_{ij}^*$ to $z_{ij}^*$:

$$\begin{aligned}
\Phi\left(\widetilde{z}_{ij}^*\right) &= \Pr\left(R_{ij} > 0\right) \Pr\left(\widetilde{R}_{ij} > 0 | R_{ij} > 0\right), \\
&\leq \Pr\left(R_{ij} > 0\right) = \Phi\left(z_{ij}^*\right).
\end{aligned}$$

This inequality is imposed by assuming that:

$$\Phi\left(z_{ij}^*\right) = \frac{1}{2}\left(\widetilde{z}_{ij}^*\right) + \frac{1}{2}\left(\widetilde{z}_{ij}^* + \exp\left(\alpha\right)\right).$$

The proxy for $z_{ij}^*$ is deduced from above as:

$$\widehat{z}_{ij}^* = \Phi^{-1}\left(\frac{1}{2}\Phi\left(\widehat{\widetilde{z}}_{ij}^*\right) + \frac{1}{2}\Phi\left(\widehat{\widetilde{z}}_{ij}^* + \exp\left(\alpha\right)\right)\right). \tag{32}$$

The parameter $\alpha$ is estimated along with trade elasticities by imposing (32) in the following gravity estimating equation:

$$\log \widetilde{M}_{ij} = x_{ij}\beta + \log \overline{W}\left(\widehat{z}^*_{ij}, \delta, r\right) + \log \Phi\left(\widehat{\widetilde{z}}^*_{ij}\right) - \sigma^2_{ij}/2 + \widetilde{\epsilon}_{ij}, \tag{33}$$

where $\widetilde{\epsilon}_{ij} \sim N\left(0, \sigma^2_{ij}\right)$ on the subsample of interest.

Note that $z^*_{ij}$ converges to $\widetilde{z}^*_{ij}$ as $\alpha \to -\infty$. Hence, Equation (32) nests the two cases of interest. A large and negative value of $\alpha$ is suggestive that the number of firm is virtually infinite so that productivity shocks have little or no impact on the probability of trade. In contrary, a positive value of $\alpha$ is suggestive that the number of firms is infinite and that productivity shocks matter for the determination of the extensive margin.

## 6 Empirical Application

For this application we use the world trade data available on Elhanan Helpman's website, which describes trade flows between 158 countries during the 80's. We focus on the years 1980 and 1989 and restrict the data to countries that import from at least one origin or that export to at least one destination. This amounts to a total of 24649 bilateral trade flows for each year, of which 10975 and 11203 are strictly positive in 1980 and 1989 respectively. For each of the two years, we estimate a Probit model that reveals the determinants of the selection of countries into trading relationship. The following regressors are included in the Probit models along with importer and exporter specific fixed dummies:

(i) Log-distance between the country pair;

(ii) Land border: equals to 1 if the partners pair has a common land border;

(iii) Island: equals 1 if at least one partners in the pair is an island;

(iv) Landlock: equals 1 if at least one partners in the pair is landlocked;

(v) Legal system: equals 1 if the two partners have same legal system;

(vi) Language: equals 1 if the two partners have at least one common language;

(vii) Colonial ties: equals 1 if the two partners have historical colonial ties;

(viii) Currency union: equals 1 if the two partners are in a common currency union zone;

(ix) FTA: equals 1 if the two partners have a free trade agreement;

(x) Religion: an index that is increasing in the percentage of population sharing a common religion.

Table 1 shows the Probit estimation results. Factors that negatively impact the probability of trade are the log-distance, a common land border and whether one of the partners is an island or landlocked. The negative impact of the existence of a common land border on the probability of trade is quite puzzling. HMR suggested that this may be reflecting the existence of border conflicts. Interestingly, the negative coefficients associated with the previous factors have decreased over the decade. In particular, the disadvantage of landlocked countries to trade creation is not significant for 1989. All other explanatory variables have positive impact on the probability of trade. The effects of a common legal system and colonial ties are not significant for 1980. Also, all positive elasticities (except for the coefficient of "religion") have increased between 1980 and 1989.

Subsequently, two gravity equations are considered. The first equation (henceforth, Model 1) assumes that the number of firms is potentially finite so that:

$$\log \widetilde{M}_{ij} = x_{ij}\beta + \log \overline{W}\left(\widehat{z}^*_{ij}, \delta, r\right) + \log \Phi\left(\widehat{\widetilde{z}}^*_{ij}\right) - \sigma^2_{ij}/2 + \widetilde{\epsilon}_{ij},$$

where $\widehat{z}_{ij}^{*} = \Phi^{-1}\left(\frac{1}{2}\Phi\left(\widehat{\widetilde{z}}_{ij}^{*}\right) + \frac{1}{2}\Phi\left(\widetilde{z}_{ij}^{*} + \exp(\alpha)\right)\right)$. The second equation (henceforth, Model 2) assumes that the number of firms is infinite so that we have $\widehat{z}_{ij}^{*} = \widehat{\widetilde{z}}_{ij}^{*}$:

$$\log \widetilde{M}_{ij} = x_{ij}\beta + \log \overline{W}\left(\widehat{\widetilde{z}}_{ij}^{*}, \delta, r\right) + \log \Phi\left(\widehat{\widetilde{z}}_{ij}^{*}\right) - \sigma_{ij}^{2}/2 + \widetilde{\epsilon}_{ij},$$

Model 2 is an heteroscedastic version of the one estimated in HMR (2008). Both models should deliver similar predictions if $\alpha$ is negative and large.

**Table 1: Probit Estimation Results (extensive margin)**

| | 1980 | 1989 |
|---|---|---|
| variables / parameters | Estimate | Estimate |
| Log distance | -0,740*** | -0,548*** |
| Land border | -0,408*** | -0,227** |
| Island | -0,377*** | -0,325*** |
| Landlock | -0,269*** | -0,139 |
| Legal system | 0,031 | 0,085*** |
| Common language | 0,253*** | 0,268*** |
| Colonial ties | 0,430 | 0,766** |
| Currency union | 0,903*** | 0,913*** |
| FTA | 1,290*** | 1,837*** |
| Religion | 0,398*** | 0,320*** |

**Table 2: Gravity estimation results.**

Model 1: Potentially finite number of firms. Model 2: Infinite number of firms.

| | 1980 | | 1989 | |
|---|---|---|---|---|
| variables | Model 1 | Model 2 | Model 1 | Model 2 |
| Log distance | **-1,191*** ** | **-0,757*** ** | **-1,001*** ** | **-0,981*** ** |
| Land border | 0,586*** | 0,833*** | 0,555*** | 0,378*** |
| Island | -0,594*** | -0,303** | -0,205* | -0,199** |
| Landlock | -0,307* | -0,504** | -0,308* | -0,525*** |
| Legal system | 0,357*** | 0,376*** | 0,386*** | 0,319*** |
| Common language | 0,291*** | 0,027 | 0,360*** | 0,354*** |
| Colonial ties | 1,375*** | 1,094*** | 1,241*** | 0,769*** |
| Currency union | 1,258*** | 0,936*** | 1,782*** | 1,820*** |
| FTA | **1,195*** ** | 1,083*** | **1,353*** ** | 0,377*** |
| $\theta_0$ (heterosced.) | 2,169*** | 2,421*** | 1,286*** | 4,673*** |
| $\theta_1$ (heterosced.) | **0,072*** ** | 0,122*** | **0,083*** ** | -0,192*** |
| $\delta$ (heterogen.) | $5 \times 10^{-4}$*** | 0,806*** | $1 \times 10^{-4}$*** | 0,064 |
| $r$ (heterogen.) | 0,361 | 0,698*** | -0,369* | 0,920*** |
| $\alpha$ | **0,366*** | - | **0,608*** ** | - |

Table 2 shows the estimation results. The parameter $\alpha$ is estimated to be positive ($\widehat{\alpha} = 0.366$ for 1980 and $\widehat{\alpha} = 0.608$ for 1989), which suggests that aggregate productivity shocks determine the selection of countries into trading relationship in synergy with the randomness in trade costs. Based on this estimate of $\alpha$, we find that aggregate productivity shocks have contributed to $\widehat{\lambda} = 51\%$ of the extensive margin in 1980 and to $\widehat{\lambda} = 57\%$ in 1989. This suggest that aggregate productivity (and other supply side) shocks have been more important than trade costs (and other demand side) shocks in determining the selection of countries into trading relationship.

# 7 Conclusion

We modify the Helpman-Melitz-Rubinstein (HMR) model to account for firm-level productivity shocks that translate into aggregate effects on trade flows. First, we allow the number of candidate exporting firms to be finite and the productivity of each individual firm to be random. Firms of the same country are homogenous ex-ante regarding the distribution of productivity but heterogenous ex-post regarding realized productivity. Candidates exporting firms of the same country share a common productivity shock and as in HMR (2008), trade costs are random. These assumptions imply that the selection of countries into trading relationship is determined by productivity shocks in synergy with shocks with trade costs shocks. When the number of firms is infinite, productivity shocks do not contribute to the extensive margin.

Using bilateral world trade data for the year 1980 and 1989, we estimate the model with and without assuming that the number of firms is infinite. We find supportive evidence for the finiteness of the number of firms and the presence of aggregate productivity shocks. The empirical results suggest that on average, aggregate productivity shocks accounted for 51% of the sample selection effect in 1980 and about 57% in 1989.

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