Divide and Learn: Early Contracting with Endogenous Threat*

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Abstract

An economic agent may engage into an early negotiation with the sole purpose of gathering information to improve his bargaining position. This occurs when information is required to give credibility to early threats. We analyze this issue in the context of a buyer/seller relationship, where the seller has private information on the future gains from trade, and the buyer can bypass at some preliminary stage. We show how the buyer shapes the early contract offer creating an information leakage that makes bypass a credible threat. As a result, early contracting shifts the rent from the seller to the buyer.

1 Introduction

In most contracting situations, the parties can choose to set the deal either before the uncertainty is realized - ex-ante or early contracting - or after - ex-post of late

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contracting. Both for efficiency and insurance reasons, many papers starting from Borch (1962), Arrow (1963) or Pauly (1968) have advocated the use of early contracting.[1] With this paper, we show that the information gathering aspect of early contracting may be a significant strategic dimension and help the uninformed party in reaping most of the surplus.

As an illustration, consider the situation where a trade between B and S can generate a surplus of 1 with probability θ and 0 otherwise, with θ uniformly distributed on $[0, 1]$. If B and S wait for the realization of the surplus, we assume that whole surplus will be gained by S. Therefore, in the absence of early contract, B obtains a zero payoff. But suppose that, at a cost $k = 2/3$ paid ex-ante, B can bypass S and reap the whole surplus.

If neither S nor B knew the value of θ, they will not sign an early contract. Indeed, S can ensure a payoff of $\mathbb{E}(\theta) = 1/2$ by refusing to contract and B has no reason to bypass as the cost k is greater than the total expected surplus. If instead S privately knows the value θ, B can secure an expected payoff $v_B \approx 1/6$. Indeed, suppose that B proposes a early payment $T \gtrsim 1/3$ for the right to obtain the full surplus ex-post. Our analysis will imply that the unique equilibrium is for S to accept for all values of θ. Intuitively, only S with θ greater than T may have an incentive to refuse the contract. This cannot occur because, with the belief that $\theta > T$, the expected gain from bypassing is $\mathbb{E}(\theta|\theta > T) > 2/3 = k$, implying that B would bypass. With T close to $1/3$, B can therefore secure a payoff close to $\mathbb{E}(\theta) - 1/3 = 1/6$.

In the above example, due to the existence of an informed agent S, B is able to appropriate a share of the surplus without effectively paying the cost of bypass. Indeed, the strategic design of the contract exploiting implicit revelation of information allows him to raise the credibility of the threat of bypass. To increase this credibility, B designs the contract so as to partition the set the informed agents S in two subsets: those that accept irrespective of the credibility of the threat and the others. Then any rejection would reveal that the agent's type is in the latter sub-set. The sorting of the informed agent S is done so that the information conveyed by rejection makes the threat of bypass credible. Therefore, S has no other choice to accept the offer at any level of private information. The informed agent is caught into an information trap where the acceptance by some types forces acceptance by all types. We refer to this strategy as *divide and learn*.

Situations as the one described above are quite common. Examples include:

- A buyer considers a make-or-buy decision faced to a better informed supplier.

[1]See Laffont-Martimort (2002) for a modern treatment of the ex-ante complete contract approach.

Bypass consists in investing to produce in-house. In this case, the information of the supplier could be about the match value of its product with the buyer's needs, but it could also be about the likelihood that entry of competing sellers improve the buyer's ex-post bargaining (hence about the future market structure).[2]

- A subcontractor produces a good customized to the need of a buyer. The buyer is better informed about future demand. Bypass consists in making the product suitable for other potential clients. Even if it is inefficient, it may be carried on if it allows the producer to obtain better terms ex-post when negotiating with the buyer.

- An innovator must choose between contracting with an existing producer better informed about market demand and setting his own company. In the latter case, there would be competition in differentiated products ex-post, which constitutes the threat point in the former case. The credibility of this threat depends on the cost of setting the company, the type of competition and the belief about market demand.

In these situations, early contracting will help the uninformed party to raise his share of the surplus but to some limited extent. The limits of the divide and learn strategy is the cost of inducing types for which the threat is not credible to accept the offer, which generates a lower bound on the payoff the strategy can generate. To understand when the strategy is effective we focus on a very simple set-up.

We consider the relationship between a (zero cost) seller and a potential buyer, who will have a unit demand with some probability θ that is privately known by the seller. To highlight the rent shifting motive for early contracting, we focus in the first part of the paper on a situation where efficiency considerations are absent. For this purpose we assume that, conditional on the demand being positive, the reservation utility of the buyer is fixed at a known value. In this case, gains from trade are always realized ex-post and the only question is the division of the surplus between

[2] An initial motivation for the paper was the situation faced by a local government relying on a utility for the provision of a local public good. For instance municipalities that wish to upgrade the quality of their telecommunication infrastructure may contract with the historical operator or build their own infrastructure. The latter option is inefficient as it implies duplication. In these situations, superior information by telecommunication operators may backfire into low profit under the threat of duplication.

the buyer and the seller. We then extend the model to random reservation utility in the second part of the paper.

The buyer can wait until the demand is realized in which case a positive price will prevail, or he can invest in production facility and produce the good himself. Bypass is inefficient due to duplication of infrastructures. The buyer may also propose an advance purchase contract to the seller, under the threat of bypass. We assume that based on prior beliefs bypass is not a credible threat, but it would be if the buyer knew that θ is high. As discussed above, when a contract is proposed but refused by the seller, the buyer decides to bypass if his beliefs on demand are high enough. This interaction between the contract and post-refusal beliefs may be the source of a multiplicity of equilibria. We show that for each contract offered by the buyer, there will be at most two (pure strategy) continuation equilibria, one where refusal triggers bypass and one where it doesn't.

The ability of the buyer to reduce the rent of the seller depends on how this multiplicity is resolved. Before proceeding to the full characterization of the equilibrium set, we characterize extreme equilibrium payoffs, that obtain for particular selection rules among the multiple equilibria. In the *Minimal Rent Scenario*, any refusal induces high enough beliefs to trigger investment. Therefore, the rent of the seller is minimized and all the surplus in shifted to the buyer. In the *Maximal Rent Scenario*, the belief induced by a refusal minimizes the probability of bypass, which also minimizes extraction of the surplus by the buyer. Under the maximal rent scenario and absent ex-post inefficiencies, we show that the buyer and the seller either contract for a fixed price or do not contract at all, depending on the credibility of the threat.

We then conclude that any allocation that yields payoffs between the above two scenario's payoffs can be supported in equilibrium and that an early contract is always offered.

In the second part of the paper, we extend the model to the case with random reservation utility. Then early contracting entails efficiency gains due to the distortions associated with ex-post market power. Although the quantitative distortions are not fully eliminated, both scenarios result in less inefficiencies than if the buyer were not allowed to bypass.

The general idea that refusal of a contractual offer will induce agents to update their belief has been developed by Cramton-Palfrey (1995). They introduced the notion of ratifiability as the case where the contract is accepted because the game played in case of refusal is endogenously too unfavorable to the parties. We do not apply this notion directly but the contract we derive must naturally display this type of property. Even closer to our paper is the article by Celik-Peters (2011) where a Principal may enlarge the feasible set by proposing a mechanism that is refused by some types. In the latter approach, the idea is to use this rejection to transmit some information to others parties, in order to decrease the value of the outside option. We also use the idea that refusal convey information, but this is developed in an environment where the principal wants to get informed about a common value parameter. Even more importantly, because any refusal could be used against the agent, it does not happen at the equilibrium. [3]

Note also that in our paper, the outside option of the agent depends on the belief hold on his own type, in the tradition of the type-dependent reservation utility models analyzed by Jullien (2000). Following this tradition, Rasul-Sonderegger (2010) analyzed how the difference between ex-ante and ex-post outside options can help the principal. In our paper, the difference between ex-ante and ex-post outside options has mainly an impact through the investment choice of the principal, and not of the agent.

In the regulation literature, some articles have analyzed contract games where initial trading possibilities pre-exist the contract. In Auriol-Picard (2009) for example, the government can propose a contract to a monopoly to change the prevailing market price. We use a related setting, even if it is formally closer to Lewis-Sappington (1988) as the parameter is a common value one, but we focus on the impact of bypass on the terms of trade.

Our work can be also linked to the endogenous market literature, more specifically to Caillaud-Tirole (2004) or Dana-Spier (1994). In those articles, the market structure is endogenous, and the contract may generate different market configurations. In one interpretation of our paper, the contract may either lead to a monopoly, or leave open the possibility of competition ex-post.

One of the by-product of our analysis casts some light on the value of information in principal-agent models. Indeed, we show that the principal may benefit from negotiating with an superiorly informed agent. Kessler (1998) also found that, in a mixed model of regulation with both moral hazard and adverse selection, an agent may benefit from the possibility of remaining ignorant. In this approach, this result comes from the change in likelihood of low-cost projects while in our article, information influences the perceived outside options of both parties, hence the way surplus

[3]Note that in a complete contract setting, there should be no refusal at the equilibrium as the principal could propose in the contract the same allocation as the one obtained without contract. In the main example developed by Celik-Peters (2011), there is refusal at the equilibrium as the principal cannot influence the game played in case of refusal and needs refusal at the equilibrium to reduce the agents' rents.

is shared. This problem was also studied by Sobel (1993) in a moral hazard setting and, while participation is less costly when the agent is not informed, it may still be worth for the principal to face an informed agent. Indeed, the principal prefers to face an informed agent as the latter will choose the adequate actions while in our paper, since the agent does not take any action, the result is only driven by the impact of information on the outside options.[4]

Section 2 presents the model. Section 3 derives the equilibria and highlight the role played by the possibility of bypass both on the contract and on the way surplus is shared. Section 4 extends this analysis to the case of continuous demand while Section 5 concludes. Most of the proofs are relegated to an Appendix.

2 Basic model

Consider two risk-neutral agents , a buyer B and a seller S, where B may buy one unit of a good from S. There are two periods, interim and ex-post.

Consumption takes place in the second period. The production cost of the seller is normalized to zero while the utility from consumption of the buyer is $\eta.U$, where $U > 0$ is known while $\eta \in \{0, 1\}$ is positive with some probability $\theta \in [\underline{\theta}, \overline{\theta}]$.We will relax the assumption on U latter on. With some abuse, we refer to U as the reservation utility; when $\eta = 1$ there is positive demand while when $\eta = 0$ there is no demand.[5] At this second period, the buyer privately knows the realization of demand.

In the first period, the probability θ is known to the seller but not to the buyer whose beliefs are represented by a density $f(\theta)$ and a cumulative distribution functions $F(\theta)$. We assume that $f(.)$ is strictly positive and the hazard rate $F(\theta)/f(\theta)$is strictly increasing in θ.

During the first period, the buyer may approach the seller and offer a contract which stipulates a monetary transfer in exchange of the supply of good. If the negotiation fails, the buyer has two alternatives. One alternative is for B to invest a fixed cost k prior to observing the demand which allows him to produce the good at a zero marginal cost. We refer to this as bypass. The other alternative is to wait until he observes the realization of demand, in which case the buyer will be able to

[4] Note, contrarily to the Crémer-Khalil-Rochet (1998) for example, we do not discuss the incentives to acquire earlier some information that will be revealed ex-post.

[5] One formally equivalent alternative interpretation is that $\eta = 0$ corresponds to the entry of competitors. Competition then drives price to zero, in which case the buyer obtains the full surplus without relying on bypass facilities.

buy ex-post one unit of the good from the seller, at a perfectly anticipated price $\alpha \leq U$. This price may have several interpretations, depending on the context. For instance, it may be the monopoly price set ex-post, $\alpha = U$, or the outcome of the ex-post negotiation between the buyer and the seller (Nash bargaining would yield $\alpha = U/2$). It may also correspond to a catalogue price posted by the supplier that applies for all buyers.[6] As another example, in a regulatory context, the price α can be interpreted as a price-cap imposed on a seller of an essential input.[7]

The timing of the game of the first period is as follows:

1. The value of θ is realized and observed by the seller.
2. The buyer makes a contractual offer $\mathcal{C}$ to the seller. We discuss the nature of the contract $\mathcal{C}$ later on.
3. The seller accepts or rejects that offer.
   (a) If the offer is accepted, no bypass occurs;
   (b) If the offer is rejected, the buyer either bypasses the seller or not.

Then in the second period, the demand is realized and privately learned by the buyer. If a contract has been signed it is implemented, otherwise a spot transaction may occur.

The equilibrium concept will be Perfect Bayesian Equilibrium. Notice that we impose that early contracts preclude bypass. We justify this in appendix where we discuss the contract in more details and argue that since bypass is inefficient, a contract inducing some bypass would be dominated by a contract replacing bypass with an option to sell at zero price.

At stage 2, the buyer and the seller negotiate the future terms of trade. We view this negotiation as determining a two-part tariff (T, p) consisting in a fixed payment T by the buyer and an option to buy at p.[8]

[6] For instance, microprocessor suppliers propose a catalogue price for each processor that applies to all manufacturers, but negotiate specific rebates with large manufacturers.

[7] In the telecom industry, access regulation often constrains the price charged by the incumbent operator for accessing its local network infrastructure.

[8] As the realization of the demand is private information of the buyer, the contract cannot be made contingent on this final demand. As demand is unitary, any (non-stochastic) contract can be summarized by the transfer if the buyer does not consume and the transfer if he consumes, hence by a two-part tariff. Moreover we do not consider stochastic contracts or contracts that involve a third-party (a mediator), which we conjecture to be sub-optimal.

Hence the tariff is known to both parties. If the parties fail to agree on a tariff, then the buyer is free to bypass or to wait and buy ex-post. To capture this situation in a simple way, assume that the buyer offers a menu $\mathcal{C}$ of two-part tariffs. The seller then either rejects or chooses one tariff within the menu. W.l.o.g. we restrict attention to $0 \leq p \leq U$.

When θ is commonly known, the parties agree on trade with no bypass, as this maximizes the social surplus. The division of this surplus between the parties depends on whether bypass is a relevant threat for the buyer. Bypass is a credible alternative when it yields an expected gain larger than the gain associated with waiting and buying the good at the reference price, or:

$$\theta U - k \geq \theta(U - \alpha) \Leftrightarrow \theta \geq \frac{k}{\alpha} = \theta^B. \tag{1}$$

Thus, contracting under symmetric information about θ would result in a tariff $(T, p) = (0, 0)$ when $\theta > \theta^B$ as bypass is a credible threat. There would be no early contract when $\theta < \theta^B$, as the buyer would need to compensate the seller for the ex-post profit $\theta\alpha$.[9]

The same conclusion would hold, replacing θ by the expected value $\theta^e = \mathbb{E}(\theta)$, if no party were informed about the true value of θ.

Under asymmetric information about θ but absent the possibility of bypass, there is no space for contracting at the interim stage. This is because, since $\alpha \leq U$, the equilibrium allocation with no contract is efficient, i.e. trade takes place when demand is positive. Our benchmark model is thus one where introducing the alternative technology can only disrupt an otherwise efficient situation. The question is then how this will alter the equilibrium outcome.

Notice also that if the buyer could commit at the interim stage on her decision to bypass or not, then she would commit to bypass and negotiate a zero price. Thus the inability to commit is essential for our analysis. In our model there is no commitment on the decision wether to bypass or not if negotiation fails. As in Cramton and Palfrey (1995), this decision is taken considering the utility obtained in the continuation game. Since the seller has private information, rejection of the contractual offer may convey some information so that the decision of the buyer with regards to bypass depends on the revised beliefs after rejection. These revised beliefs, that are used to compute the payoff in this subgame, may then differ from the initial beliefs.

[9] In both cases, the outcome is efficient.

3 Contracting under the threat of bypass

Consider a contract $\mathcal{C}$ offered by the buyer. In the last stage of period 1, i.e. the continuation game after the contract proposal, the decision to bypass will depend on the seller's decision to accept the contract or not, which itself depends on the likelihood of bypass. This interaction problem opens the door to the possibility of multiple equilibria. We therefore start by solving the last stage equilibria of the first period.

The reservation utility obtained by the seller if he refuses the contract depends on his type and the anticipated behavior of the buyer. It is 0 if the buyer bypasses after rejection and it is given by $\pi^R(\theta) = \theta\alpha$ if there is no bypass.

We denote by $\left(T^{\mathcal{C}}(\theta), p^{\mathcal{C}}(\theta)\right)$ the preferred option of the seller when his information is θ, and $\pi^{\mathcal{C}}(\theta) = T^{\mathcal{C}}(\theta) + p^{\mathcal{C}}(\theta)\,\theta$. Notice that this optimal choice does not depend on beliefs about the set of types refusing the contract offered. The buyer then obtains a conditional expected utility $\theta U - \pi^{\mathcal{C}}(\theta)$. In what follows we restrict our analysis to offers that induce non-negative profit for the selling firm, $\pi^{\mathcal{C}}(\theta) \geq 0$.[10] Note also that for any equilibrium contract, we must have

$$\min_{\theta}\left(\pi^{\mathcal{C}}(\theta) - \pi^{R}(\theta)\right) \leq 0. \tag{2}$$

Indeed a contract with $\min_{\theta}\left(\pi^{\mathcal{C}}(\theta) - \pi^{R}(\theta)\right) > 0$ would be accepted with probability one and would leave to the buyer a payoff below the no-contract payoff. We restrict attention to such contracts in what follows.

Incentive compatibility requires that the profit obtained by a seller of type θ is maximal among the possible tariffs. Formally, with $T^{\mathcal{C}}(\theta) = \theta p^{\mathcal{C}}(\theta) - \pi^{\mathcal{C}}(\theta)$, the allocation is incentive compatible if only if

$$\dot{\pi}^{\mathcal{C}}(\theta) = p^{\mathcal{C}}(\theta) \text{ for all } \theta$$
$$p^{\mathcal{C}}(.) \text{ is non-decreasing}$$

To simplify the exposition, we assume that [11]

A.i) If rejection triggers no bypass and the seller is indifferent between accepting and rejecting, then the offer is rejected.

A.ii) If the buyer is indifferent between bypassing or not, then the buyer bypasses.

[10] In full generality, one may want to consider offers with negative profit for some types and positive profit for other types. In fact, these offers do not induce new equilibria since we can show that the offer is always dominated for the same out-equilibrium belief by an offer with $T = (0, 0)$.

[11] These assumptions are without consequence for equilibria but the proof would be more tedious without them.

The first assumption eliminates some innocuous multiplicity of sellers acceptance set, that arises when $\pi^C = \pi^R$ as the same allocation can be implemented with acceptance or rejection. The second assumption facilitates equilibrium selection, given that the buyer at indifference could always credibly induce himself to bypass by expanding slightly the set of seller's types rejecting the offer.

Let us now derive the allocation that follows a contract offer $\mathcal{C}$ by the buyer, thus the continuation equilibrium at stage 3. For this it is useful to define $\Theta^{\mathcal{C}}$ as the set of types that would reject the contract if they anticipated that the buyer would not bypass after rejection. Formally:

$$\Theta^{\mathcal{C}} = \left\{\theta \mid \pi^{\mathcal{C}}(\theta) \leq \pi^R(\theta)\right\}.$$

Types outside $\Theta^{\mathcal{C}}$ accept the offer in any continuation equilibrium. Types within $\Theta^{\mathcal{C}}$ reject the offer if they anticipate no bypass and accept it otherwise (since the profit in the contract offer is non-negative). Notice that, from (2), $\Theta^{\mathcal{C}}$ is non-empty.

Thus we can have at most two continuation equilibria, one with bypass and one without bypass. Formally we obtain the following equilibrium set, where θ^B is defined by equation (1):

Lemma 1 *Following a contract offer* $\mathcal{C}$*:*

- *If* $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$*, there a unique continuation equilibrium where* $\mathcal{C}$ *is accepted by all types of seller.*
- *If* $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} < \theta^B \leq \bar{\theta}$*, there are two continuation equilibria:*
  1. *Either* $\mathcal{C}$ *is accepted by all types of seller;*
  2. *or* $\mathcal{C}$ *is accepted by the seller with types* $\theta \notin \Theta^{\mathcal{C}}$ *only and no bypass follows a rejection.*
- *If* $\bar{\theta} < \theta^B$*, there is a unique continuation equilibrium where* $\mathcal{C}$ *is accepted by the seller with type* $\theta \notin \Theta^{\mathcal{C}}$ *and no bypass follows a rejection.*

Proof. See the Appendix. ■

The above lemma characterizes two possible classes of equilibria: One with 'full participation' of the seller to the contract and another one with 'partial participation'. These classes differ by the beliefs following rejection and the credibility of the threat of bypass. In both cases, however, no bypass occurs in equilibrium. As a side comment, since $\Theta^{\mathcal{C}}$ depends on α, for some range of α, there is a multiplicity of equilibria.

Equilibrium analysis

Lemma 1 enables to derive the optimal contract in two straightforward cases:

(i) If $\mathbb{E}\{\theta\} \geq \theta^B$, then the buyer can offer a contract $\mathcal{C} = \{0, 0\}$. This contract is such that $\Theta^{\mathcal{C}} = \left[\underline{\theta}, \overline{\theta}\right]$ and given that $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} = \mathbb{E}\{\theta\} \geq \theta^B$, all types of sellers accept in the unique continuation equilibrium. The optimal contract for the buyer is thus $\{0, 0\}$.

(ii) If $\theta^B > \overline{\theta}$, then bypass is never a credible threat. As the buyer must leave the rent $\pi^R(\theta)$ to the seller and there no efficiency gain, interim contracting becomes useless. The buyer thus offers no contract and trade occurs ex-post at the reference price α.

Consider now the remaining case in which:

$$\mathbb{E}\{\theta\} < \theta^B \leq \overline{\theta}. \tag{3}$$

Then, the continuation equilibrium may no longer be unique, giving rise to a multiplicity of equilibria. This arises in particular when $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} < \theta^B$, a condition which depends on the contract offer $\mathcal{C}$. In order to better understand the role of multiplicity, we start with the characterization of some economically-appealing equilibria suggested by Lemma 1.

3.1 The minimal rent scenario

In case of multiple continuation equilibria (i.e. condition (3) holds), consider the following selection rule:

Minimal rent scenario: Any contract $\mathcal{C}$ is accepted.

The motivation for such a selection is to focus on equilibria in which the compensation offered by the buyer to the seller is always lowered as much as possible. A tentative interpretation is that the buyer anticipates that all contract offers should be accepted, and interprets contract refusal as a signal that demand is high, which then induces bypass. From the buyer's point of view, the optimal contract is thus the contract with zero prices.

Proposition 1 *In the minimal rent scenario, the buyer offers $\mathcal{C}_0 = \{0, 0\}$ and the seller accepts.*

The equilibrium is supported by the buyer's beliefs $\overline{\theta}$ on θ in case of contract refusal.

3.2 The maximal rent scenario

Still focusing on situations where condition (3) holds, take any contract offer such that $\pi^{\mathcal{C}}(\theta) \leq \pi^R(\theta)$ for all θ. The equilibrium where $p^{\mathcal{C}}(\theta) \equiv 0$ and $\pi^{\mathcal{C}}(\theta) = 0$ is proposed and accepted by all types still exists but there is another equilibrium where all type of sellers refuse the contract and no bypass follows. Then the buyer will never make such an offer if this equilibrium prevails. Therefore, the seller will have strictly positive rents at the equilibrium, either by accepting the contract or by setting the initial price α. Formally, we define an equilibrium in the maximal rent scenario as one that satisfies the following condition.

Maximal rent scenario: For any contract $\mathcal{C}$:

(i) If $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} < \theta^B$, the contract is accepted by types $\theta \notin \Theta^{\mathcal{C}}$ only and there is no bypass following refusal;

(ii) If $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$, the contract is accepted by all types of seller.

With the restrictions implied by the maximal rent scenario, the buyer has to choose between a contract inducing full participation (case (ii)) and a contract with partial participation (case (i)). A contract inducing partial participation necessarily implements the price α for types $\theta \in \Theta^{\mathcal{C}}$. By contrast, a contract with full participation allows to reduce the price for the high types since it is compatible with a profit smaller than $\pi^R(.)$; it may require however to leave a positive net rent $\pi^{\mathcal{C}}(.) - \pi^R(.)$ to more types.

Notice that the buyer would not offer in equilibrium a contract that induces partial participation of the seller, i.e. such that rejection triggers no bypass. The reason is that with such a contract, the seller's payoff is at least $\pi^R(\theta)$. This implies that the expected payoff of the buyer when offering the contract is at most $\mathbb{E}\{\theta\} U - \mathbb{E}\left\{\pi^R(\theta)\right\} = \mathbb{E}\{\theta\}(U - \alpha)$, which is the minimal payoff that the buyer obtains without offering any contract. Thus either the buyer makes no offer or her offer is accepted by all types.

Consider a contract $\mathcal{C}$ such that $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$ with a continuation equilibrium where all types of seller accept the offer. Before we derive the optimal contract, a few preliminary results are required. First define $\pi(\theta)$ as the equilibrium expected payoff of the seller.

Lemma 2 *In any equilibrium where rejection triggers bypass,*

1. $\pi(\underline{\theta}) \geq \pi^R(\underline{\theta})$
2. $\pi(\overline{\theta}) \leq \pi^R(\overline{\theta})$
3. *the set of types obtaining no more than their reservation utility is an interval* $\Theta^{\mathcal{C}} = [\theta^*, \overline{\theta}]$.

Proof. See the Appendix. ■

Equipped with this lemma, we are know ready to characterize the optimal contract with full participation. For this, consider an equilibrium where the buyer strictly prefers to offer a contract to the seller than no contract. At a full participation outcome, the contract must ensure that the threat of bypass is credible, or $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$. At equilibrium with full participation of sellers, the buyer makes sure that this constraint binds. To see that, suppose $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} > \theta^B$. Then the buyer can propose the contract $\pi^{\mathcal{C}}(\theta) - \varepsilon$. By continuity and convexity of $\pi^{\mathcal{C}}(\theta)$, this would induce a marginal decrease in θ^* and thus would still be accepted by all types. Define $\tilde{\theta}$ as:

Definition 1 *For* $\theta^B \in \left(\mathbb{E}\{\theta\}, \overline{\theta}\right]$, $\tilde{\theta}$ *is the solution of* $E\{\theta \mid \theta \geq \tilde{\theta}\} = \theta^B$*.*

The parameter $\tilde{\theta}$ is uniquely defined because the density is positive. It is smaller than θ^B and increases from $\underline{\theta}$ to $\overline{\theta}$ on the relevant range of θ^B.

From above, if an equilibrium exists with a contract proposed and accepted by all, it must be the case that types below $\tilde{\theta}$ and only these types receive a strictly positive rent:

$$\Theta^{\mathcal{C}} = [\tilde{\theta}, \overline{\theta}] \text{ and } \mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} = \theta^B.$$

We can write the buyer's problem when choosing a contract to propose by solving the program **P1:**

$$\max_{\{p(.),\pi(.)\}} \mathbb{E}\left\{\theta U - \pi(\theta)\right\}$$
$$\text{subject to } \forall\theta : \dot{\pi}(\theta) = p(\theta), p(.) \text{ non-decreasing,}$$
$$\pi(\tilde{\theta}) = \pi^R(\tilde{\theta}),$$
$$\forall\theta > \tilde{\theta} : \pi(\theta) \leq \pi^R(\theta)$$
$$\forall\theta < \tilde{\theta} : \pi(\theta) > \pi^R(\theta).$$

In equilibrium the buyer proposes a contract if the value of this program is larger than $\mathbb{E}\{\theta\,(U-\alpha)\}$ since the buyer would not bypass under his prior beliefs. We show in the proof of the next proposition that, if this program has a solution, it must coincide with a contract offering to all types the same price $p < \alpha$. This implies an expected profit for the buyer $\mathbb{E}\left\{\theta\right\} U - T - p\mathbb{E}\left\{\theta\right\}$ where $T = \pi^R(\tilde{\theta}) - \tilde{\theta}p$. The total profit is then $\mathbb{E}\left\{\theta\,(U-\alpha)\right\} + (p-\alpha)\left(\tilde{\theta} - \mathbb{E}\left\{\theta\right\}\right)$. Given that $p - \alpha < 0$, the conclusion is that a contract is proposed if $\tilde{\theta}$ is small.

Proposition 2 *Assume that $\bar{\theta} \geq \theta^B > \mathbb{E}\left\{\theta\right\}$. In the 'maximal rent scenario', the buyer's expected payoff is $\mathbb{E}(\theta)U - \mathbb{E}\min\left(\mathbb{E}\left\{\theta\right\}, \tilde{\theta}\right)\alpha$. It is obtained by offering $\mathcal{C}^* = \left\{p = 0, T = \tilde{\theta}\alpha\right\}$ if $\tilde{\theta} < \mathbb{E}\left\{\theta\right\}$ (accepted by all θ) and no contract otherwise.*

Proof. See the Appendix. ■

Therefore, in the maximal rent scenario, the solution can be implemented either by a full participation contract or no contract. In the latter case, the buyer asks a fix rebate on the price α. This is credible because refusal by the seller would trigger bypass. The limit on the buyer's ability to shift the surplus in her favor lies in the credibility of bypass. If she asks for too large a rebate, an equilibrium emerges where it is not in her interest to bypass after a refusal. Whether the buyer is able to exploit this mechanism or not depends on the incentive to bypass thus on k and α. To be more precise, the bypass threat is credible for a price α above the critical level $\frac{k}{\mathbb{E}\{\theta|\theta\geq\mathbb{E}\{\theta\}\}}$.

3.3 The equilibrium set

So far we have restricted to two possible scenarios. For the sake of completeness let us discuss the general case. Then, to verify that a contract offer is an equilibrium offer, one has to specify a continuation equilibrium for all possible contracts and verify that this continuation equilibrium deters the buyer from deviating.[12] From lemma 1, the multiplicity issue arises only if θ^B lies between $\mathbb{E}(\theta)$ and $\bar{\theta}$, which we assume here.

Relying on lemma 1, an equilibrium of the game can be described as i) a contract offered by the buyer and ii) a mapping from contracts to continuation equilibrium allocations of the last stages.

For an allocation to be sustainable in equilibrium it must be the case that each party obtains at least as much as the minimal payoff in any equilibrium. For the seller this minimal payoff is equal to zero, whilst for the buyer it corresponds to the equilibrium payoff under the maximal rent scenario. The next proposition shows that indeed it is sufficient that these conditions are verified for the allocation to be an equilibrium allocation.

Proposition 3 *Suppose that $\bar{\theta} \geq \theta^B \geq \mathbb{E}(\theta)$. An allocation is an equilibrium if and only if it is incentive compatible and it generates interim payoffs $v \geq \mathbb{E}(\theta)U - \min\left(\tilde{\theta}, \mathbb{E}(\theta)\right)\alpha$ for the buyer and $\pi(\theta) \geq 0$ for the seller.*

Proof. See the Appendix. ■

Note that the same proof shows that inefficient allocations may be supported in equilibrium, where inefficiencies occur if there is some bypass or a probability of no consumption despite positive demand. An inefficient equilibrium would however always be interim Pareto dominated by another efficient equilibrium.

3.4 Discussion

The benchmark model highlights the impact of seller's private information on the sharing of the surplus. In the absence of private information the outcome would be straightforward: the contract would be $\mathcal{C}_0 = \{0, 0\}$ whenever the expected value of θ is above the threshold θ^B and there would be no contract otherwise.

[12] The best way to implement the equilibrium is to specify the continuation equilibria that are the least favorable to the deviating player. In our case, it is sufficient to state that for any out-of-equilibrium offer by the buyer, the maximal rent scenario is the continuation equilibrium.

Seller's private information affects the outcome whenever $\tilde{\theta} < \mathbb{E}\{\theta\} < \theta^B$. Then the equilibrium contract $\mathcal{C}^*$ leaves a maximal rent $\tilde{\theta}\alpha < \mathbb{E}\{\theta\}\alpha$ to the seller. This is the more remarkable that we start from a situation where the buyer has all the bargaining power.

While the threat of bypass would not be credible without seller's private information, the buyer is able to exploit information asymmetries so as to raise the credibility of the bypass threat, thereby increasing his bargaining position. This is done by offering favorable terms to low type sellers inducing them to reveal their type and achieving some form of separation between high and low types. Since under the contract terms, it is a dominant strategy for the low type sellers to accept, high types are trapped into a situation where refusing the offer would signal their type.

The key element for the reasoning is that the value of the bypass option for the buyer $\theta U - k$ is positively correlated with the opt-out value of the seller with no bypass $\theta\alpha$.

Thus in the equilibrium with private information, the seller's maximal ex-ante expected payoff is given by

$$\begin{aligned} \Pi^{\max} &= \min\left\{\tilde{\theta}, \mathbb{E}\{\theta\}\right\}\alpha \text{ if } \theta^B \geq \mathbb{E}\{\theta\} \\ &= 0 \text{ if } \theta^B < \mathbb{E}\{\theta\}. \end{aligned}$$

This maximal profit increases with θ^B and thus with the cost k of bypass, as θ^B increases with k. Notice that if $\underline{\theta} > 0$, there is a discontinuity at the point where $\theta^B < \mathbb{E}\{\theta\}$ as the rent jumps from 0 to $\underline{\theta}\alpha$. At this point, the buyer still proposes a price equal to zero but she must concede a fixed payment so as to ensure that the bypass threat is credible.

Notice that $\Pi^{\max}$ is not monotonic with the price α. Indeed in the range where $\tilde{\theta} < \mathbb{E}\{\theta\} \leq \theta^B$, the expected profit is given by[13]

$$\Pi^{\max} = k\frac{\tilde{\theta}}{\mathbb{E}\{\theta \mid \theta \geq \tilde{\theta}\}} = \alpha\tilde{\theta}, \tag{4}$$

which may increase or decrease with α. This reflects conflicting effects of the ex-post price α on the value of outside options and the credibility of bypass. Indeed raising α increases the credibility of bypass which tends to reduce the sellers' rent (in particular $\tilde{\theta}$ decreases). But it also raises the rent that has to be left to low type

[13] Using $\theta^B = \mathbb{E}\{\theta \mid \theta \geq \tilde{\theta}\}$ and $\theta^B = k/\alpha$.

sellers so as to maintain their unconditional participation to the contract (it raises $\pi^R(\tilde{\theta})$ for a given value of $\tilde{\theta}$).

As an illustration, consider the case of a uniform distribution on $[0,1]$. Then, we have $\tilde{\theta} = 2k/\alpha - 1$, $\mathbb{E}\{\theta\} = 1/2$ and $\theta^B = k/\alpha$. The expected profit is

$$\Pi^{\max} = \begin{cases} \frac{1}{2}\alpha & \text{if } \alpha \leq \frac{4k}{3}; \\ \max\{2k - \alpha, 0\} & \text{if } \alpha > \frac{4k}{3}. \end{cases}$$

In this case, the expected profit is decreasing as soon as bypass in case of refusal is credible.

Consider now the case of symmetric and perfect information on the parameter θ. In this case the threat of bypass is credible for $\theta > \theta^B$. The ex-ante expected profit of the seller is then $\int_0^{\theta^B} \theta f(\theta)\, d\theta$. Clearly the buyer prefers no information or asymmetric information in the case where $\theta^B < \mathbb{E}\{\theta\}$. In this case information can only remove the bypass option as a potential threat. In the range where $\Pi^{\max}$ is positive but smaller than with no information, the comparison is ambiguous. Symmetric information allows the buyer to extract the full rent for high types (above θ^B) while private sellers information leaves higher rents to these types as well as to low types (below $\tilde{\theta}$) but allow to reduce the rent left to intermediate types (between $\tilde{\theta}$ and θ^B).

The next figure shows the various rent under the three situations when $\tilde{\theta} < \mathbb{E}\{\theta\} \leq \theta^B$. The plain horizontal line is the rent with asymmetric information, the dotted horizontal line is the rent without information while the dashed triangle is the rent under perfect information.

4 The case of continuous demand

4.1 Presentation and benchmark case

The previous analysis can be extended to the case where there is inefficient trade ex-post by assuming that the value of U is uncertain. To capture the situation in a simple way let us assume that the information θ of the seller is still the probability that the demand is positive, $\eta = 0$ or $\eta = 1$ but, conditional on demand being positive, U is now a continuous random variable on $(0, \bar{U}]$. We denote by $D(p)$ the probability that U is larger than p and by $W(p) = \int_p^{\bar{U}} D(t)dt$ the expected surplus

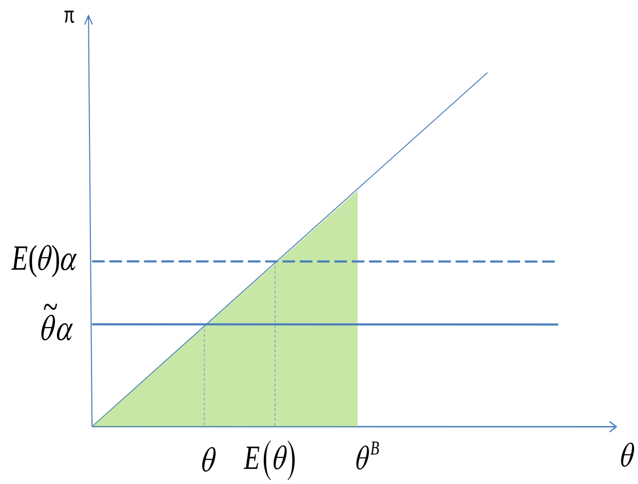

Figure 1: Rents under full and asymmetric information

when the unit price is p. We assume that $\varepsilon(p) = -pD'(p)/D(p)$ is increasing.[14] The profit $pD(p)$ is then maximal at p^M assumed to be larger than α.

In the previous setting, the only impact of introducing bypass was to affect the way the surplus was shared. In this new setting, there is also an inefficiency resulting from the both asymmetric information and the possibility of bypass. With random reservation utility, a contract can again be simply represented by a fixed fee $T(\theta)$ and a price $p(\theta)$ if the unit is consumed by the buyer, as the demand remains unitary.

To start with, let us consider the case without bypass. The buyer offers a contract $\mathcal{C}$ which induces a final allocation $(T(\theta), p(\theta))_{\theta\in\Theta}$ where $(T(\theta), p(\theta)) = (T^{\mathcal{C}}(\theta), p^{\mathcal{C}}(\theta))$ if the contract is accepted and $(T(\theta), p(\theta)) = (0, \alpha)$ otherwise. The constraints on the final allocation that the buyer can induce are then given by the classical

[14] Note that this model is equivalent to the case of multiple units if the contract is restricted to a two part-tariff. In this case $D(.)$ is the standard demand function and $W(p)$ the consumer surplus function.

incentive compatibility and individual rationality conditions. Formally, with $\pi(\theta) = \theta p(\theta)D(p(\theta)) + T(\theta)$, the allocation $(T(\theta), p(\theta))_{\theta\in\Theta}$ is incentive compatible if

$$\dot{\pi}(\theta) = p(\theta) \text{ for all } \theta \text{ and } p(\theta) \text{ is non-decreasing.}$$

When bypass is not possible, the analysis is formally quite close to the model of regulation with unknown demand developed by Lewis and Sappington (1988) but for the type-dependent reservation utility considered in Auriol-Picard (2008). Indeed a seller accepts to participate only if he obtains more than with a price α, thus the final allocation must verify the participation constraint.

$$\pi(\theta) \geq \pi^R(\theta) \text{ for all } \theta \text{ and } \tilde{\theta}$$

with

$$\pi^R(\theta) = \theta\alpha D(\alpha).$$

This informational structure with type-dependent reservation utility will shape the optimal contract from the buyer's point of view.[15]

A first remark is that the buyer will always offer a contract such that $\pi(\theta) = \pi^R(\theta)$ for at least one type.[16] Second it is immediate that the final allocation will have only prices $p(\theta)$ below the monopoly price p^M.

Lemma 3 *Assume that bypass is not possible. The contract offered is such that all types above some threshold θ^p rejects the offer and all types below θ^p accept and choose a price $p(\theta) < \alpha$.*

Proof. See the Appendix. ■

Neglecting the sufficient condition for incentive compatibility [17], and using routine computations, the problem faced by the buyer is then to choose θ^p and the allocation $(p(.), \pi(.))$ for θ below θ^p, which can be stated as follows:

$$\max_{\{p(.),\pi(.),\theta^p\}} \int_{\underline{\theta}}^{\theta^p} [\theta W(p(\theta)) + \theta p(\theta)D(p(\theta)) - \pi(\theta)] f(\theta) d\theta + \int_{\theta^p}^{\bar{\theta}} \theta W(\alpha) f(\theta) d\theta$$

$$\text{subject to } \forall\theta \in [\underline{\theta}, \theta^p] : \dot{\pi}(\theta) = p(\theta)D(p(\theta)),$$

$$\pi(\theta^p) = \pi^R(\theta^p).$$

[15] See Jullien (2000) for a general analysis of the Principal-Agent model with type-dependent outside options.

[16] The argument is the same as before, a contract strictly above π^R for all types is accepted by all types so that it would be possible to reduce the transfer to the seller without affecting its incentives.

[17] We show in the Appendix that the second order condition, $p(.)$ non-decreasing, is implied by MLRP and the assumption on the elasticity of demand.

Notice that we do not include the constraints $\pi(\theta) \geq \pi^R(\theta)$ and $0 < p(\theta) < \alpha$ since they are implied by the last two constraints. The optimal contract is derived formally in the Appendix.

Lemma 4 *Assume that bypass is not possible. Define the price $\phi(\theta)$ as the solution of $\varepsilon(\phi(\theta)) = \left[1 + \frac{\theta f(\theta)}{F(\theta)}\right]^{-1}$ and define $\theta^{**} \equiv \min\{\overline{\theta}, \phi^{-1}(\alpha)\}$. Then, the allocation implemented by the buyer is such that $\theta^p = \theta^{**}$ and $p(\theta) = \min\{\phi(\theta), \alpha\}$*

Proof. See the Appendix. ■

Thus, this allocation verifies:

- For $\theta \in [\underline{\theta}, \theta^{**}]$, the price is $\phi(\theta)$.
- For $\theta \in]\theta^{**}, \overline{\theta}]$, the price is α.
- The seller's profit $\pi(\theta) = \pi^R(\theta^{**}) - \int_\theta^{\theta^{**}} p(s)D(p(s))ds$ is such that $\pi(\theta) - \pi^R(\theta)$ is decreasing for $\theta \in [\underline{\theta}, \theta^{**}]$ and vanishes for $\theta \in [\theta^{**}, \overline{\theta}]$.

To focus on the intuition underlying the previous lemma, one must recall that the price offered without any outside option for the seller is zero. Accounting now for the seller's outside option, the latter can refuse the contract and stick to the price α, so new incentives emerge. Low-type sellers want to mimic high-type ones in order to pretend having a high outside option. Paradoxically, sellers knowing that the demand is low will be able to command some informational rents (defined as $\pi - \pi^R$). For those types, the price proposed by the buyer will be close to the null price while, as the expected level of gains from trade level increases, the contractual price will increase and tend to α.

In contrast with the simple binary model, "partial participation" may emerge. While for low-type sellers, the buyer gives up some informational rents in return for a smaller price, for the high-type sellers, no change is made compared to the initial price. Moreover, some intermediate prices between the efficient price 0 and the status quo price α are proposed in the contract.

4.2 Bypass with continuous demand

We now turn to the general analysis of the game by allowing the possibility of bypass. We keep the same form of contract as in section 3, where T is an up-front fee and p

is the price when the option to buy is exerted. We also keep the same assumption on the way indifference is broken at the acceptance stage.

Notice that it is still the case that the buyer would duplicate in the absence of contract if the expected value θ is above some threshold θ^B defined as the solution of

$$\theta^B \int_0^{\bar{U}} D(u)\, dU - k = \theta^B \int_\alpha^{\bar{U}} D(u)\, du$$

or

$$\theta^B = \frac{k}{\int_0^\alpha D(u)\, du} = \frac{k}{W(0) - W(p)}.$$

Remark that the main methodological results derived in the binary case still apply in this continuous case. In particular, lemma 1 did not rely on the specific structure of demand and therefore it is still valid in the case of continuous random reservation utility. As a result, the multiplicity issue and the ways this multiplicity is dealt with is unchanged. In particular under the minimal rent scenario, the buyer offers the contract $(0, 0)$ that is accepted by all. Since the contract in the minimal rent scenario is unchanged, we now look only at the maximal rent scenario.

Recall that, if this equilibrium prevails, the buyer has to choose between a contract inducing full participation and a contract with partial participation, the latter contract being now sometimes optimal to increase total efficiency. By the same argument as in the previous cases, a contract optimally implements prices below α .

Lemma 5 *The contract offered is such that all types above some threshold θ^p rejects the offer and all types below θ^p accept and choose a price $p(\theta) < \alpha$.*

Proof. See the Appendix. ■

A contract inducing partial participation optimally implements the price α for types θ above θ^p. By contrast, a contract with full participation allows to reduce the price for the high types but implies that higher transfers (so higher profits) should be left to the seller. In fact, those two types of contract are associated to different profiles of profit.

Indeed, take first a contract inducing partial participation. Since in this case, we have $\mathbb{E}[\theta \mid \theta > \theta^p] < \theta^B$, it is clear that $\theta^p < \tilde{\theta}$. So, for any type θ below θ^p, $\pi(\theta) < \pi^R(\tilde{\theta})$. Therefore, in a contract with partial participation, the final allocation is such that $\pi(\theta) \leq \max\{\pi^R(\tilde{\theta}), \pi^R(\theta)\}$. Let us consider now a contract with full participation. In this case, it is the case that $\theta^p \geq \tilde{\theta}$ which implies that $\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta})$. It is then direct to see that, for any type θ, $\pi(\theta) \geq \min\{\pi^R(\tilde{\theta}), \pi^R(\theta)\}$. To sum up, a contract with partial participation is a contract where the profits are bounded above whereas a contract with full participation is a contract where the profits are bounded below.

Even if the allocations generated by the two possible contracts are "characterized" by different constraints on the profile of profit, a common condition can be written to encompass those constraints. More precisely, for any contract and equilibrium allocation generated by this contract, we have $\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta})$. We use this common property and show that the previous contracts are the two possible implementations of a general allocation problem with this property.

Lemma 6 *Consider the maximal rent scenario and take any allocation that is feasible and incentive compatible. Then there exists a contract offer by the buyer and a continuation equilibrium that implements this allocation if and only if* $\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta})$*;*

Proof. See the appendix. ■

The previous lemma provides a set of allocations that may be obtained in equilibrium. We can thus search for the preferred allocation of the buyer within the set of allocations that satisfies the properties of the lemma. Formally, it amounts to finding the solution to the following problem :

P

$$\begin{aligned} &\max_{\{p(.),\pi(.)\}} \mathbb{E}_\theta \left\{ \theta W(p(\theta)) + \theta p(\theta) D(p(\theta)) - \pi(\theta) \right\}, \\ \text{subject to } \forall\theta, \ &\dot{\pi}(\theta) = p(\theta) D(p(\theta)) \\ &p(\theta) \in [0, \alpha] \text{ and it is non-decreasing} \\ &\pi(\tilde{\theta}) \geq \pi^R(\tilde{\theta}). \end{aligned}$$

Notice that the constraints imply that $\pi(\theta) \geq 0$ for all θ (because the slope is less than α) and $\pi(\theta) \geq \pi^R(\tilde{\theta})$ for θ above $\tilde{\theta}$.

The problem would be a standard Principal-Agent problem if it were not for the constraint at $\tilde{\theta}$. Using results in Jullien (2000, Theorem 3 and 4), we can state the following proposition.

Proposition 4 *Assume that* $\mathbb{E}(\theta) < \theta^B \leq \bar{\theta}$*. Then the solution to the program (**P**) is unique and such that* $\pi(\tilde{\theta}) = \pi^R(\tilde{\theta})$ *and there exists* $\theta^* < \tilde{\theta}$ *and* $p^* = \phi(\theta^*) \leq \alpha$ *such that*:

- $p(\theta) = \min\left(\phi\left(\theta\right), p^*\right),$
- *If* $\tilde{\theta} \leq \mathbb{E}\left(\theta\right)$ *then* $\theta^* = \underline{\theta}$ *and* $p^* = 0,$
- *If* $\tilde{\theta} > \mathbb{E}\left(\theta\right)$ *then* $\underline{\theta} < \theta^* \leq \theta^{**}$*, and*

$$\left(1-\varepsilon\left(p^*\right)\right)\left(\int_{\theta^*}^{\tilde{\theta}} F\left(\theta\right)d\theta - \int_{\tilde{\theta}}^{\bar{\theta}}\left(1-F\left(\theta\right)\right)d\theta\right) \geq \varepsilon(p^*)\int_{\theta^*}^{\bar{\theta}} \theta f\left(\theta\right)d\theta \qquad (5)$$

with equality if $\theta^* < \theta^{**}$*.*

Proof. See the Appendix. ■

The equilibrium offer thus includes "bunching" on the top, all firms with type above θ^* being offered the same price p^*. The intuition behind this result can be derived by expanding profit around $\pi(\tilde{\theta})$.

Indeed

$$\begin{aligned}\pi\left(\theta\right) &= \pi^R(\tilde{\theta}) + \int_{\tilde{\theta}}^{\theta} p\left(s\right)ds \text{ for } \theta > \tilde{\theta},\\ \pi\left(\theta\right) &= \pi^R(\tilde{\theta}) - \int_{\theta}^{\tilde{\theta}} p\left(s\right)ds \text{ for } \theta < \tilde{\theta}.\end{aligned}$$

As in the benchmark model, to minimize the seller's profits, the buyer would like to set low price above $\tilde{\theta}$ and high prices below. This violates incentive compatibility and leads the buyer to offer a constant price. A difference with the linear case is that the level of price matters for efficiency which leads to lower prices when θ is below θ^*.

The level of the price p^* also reflects a trade-off between rent and efficiency. To see that consider the effect of increasing p^* while maintaining $\pi(\tilde{\theta})$. This is depicted in Figure 1.

Increasing the price p^* has two effects. First total surplus decreases by an amount that corresponds to the right hand side of equation (5) . Second, as shown in the figure the expected rent of the seller is affected positively for high types and negatively for low types. The left hand side of equation (5) corresponds to the expected decrease of the seller's profit. Whenever the latter is larger than the former, it is in the

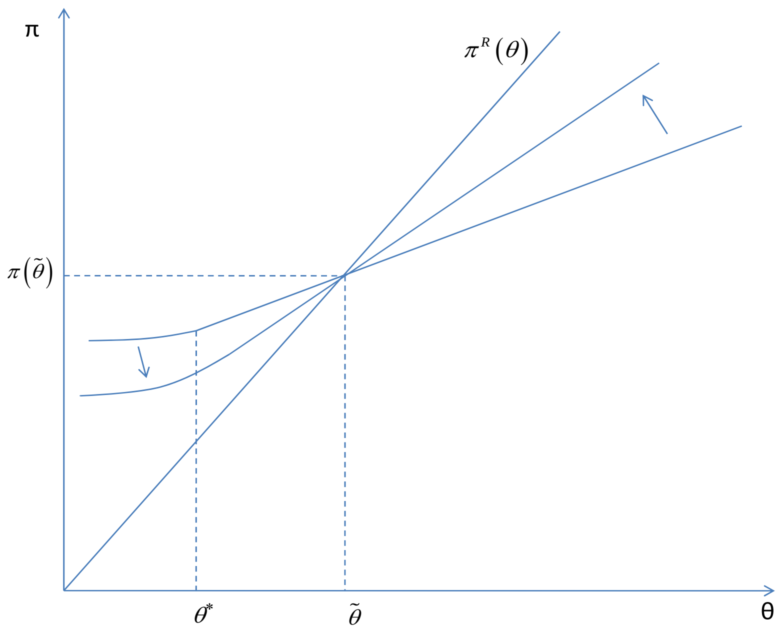

Figure 2: Effect of an increase in α

best interest of the buyer to increase the price targeted to sellers informed that the expected demand is high.

For $\mathbb{E}(\theta) \leq \theta^B < \overline{\theta}$, the solution can take three different forms. First, if $\tilde{\theta} \leq \mathbb{E}(\theta)$, all types of firms are proposed the (efficient) null price and get the same profit equal to $\pi^R(\tilde{\theta})$. Now, if $\tilde{\theta}$ is high enough, $\theta^* = \theta^{**}$ and the solution corresponds to the case without bypass, with profit as in Figure 1. At last, in the intermediate case, with $\theta^* < \theta^{**}$, prices are uniformly below the level α and there is bunching starting at θ^* which yields the profit schedule represented in Figure 1.

5 Concluding Remarks

We have shown that one party may engage into an advance negotiation with the sole purpose of gathering information to improve his bargaining position. When an agent has some superior information on a common value parameter, a principal can play some types against other and reduce the minimal level of utility that must be left to some agents. More surprisingly, as this method can only be implemented when the agents have private information, the principal benefits from this asymmetry of information. We derive the optimal contract in this setting and extend the analysis to the case where some efficiency issues are also at stake.

In this paper, we have used as a leading example a buyer-seller relationship where the future demand of competition is unknown. But many other applications can be made of such a framework, with an ex-ante uncertainty on the demand [18], on the cost or on the likelihood of future competition.

To take a more general perspective, most of the principal-agent literature assumes that if an agent refuses the contract, no game is played. In many cases, for example in contracting between equal agents or sovereign States,[19] this assumption does not hold and there is no way to commit or control the outside opportunities. It is therefore important to have a better understanding of the game played when the contract may be refused. Our article bring some news elements to this debate but much work remains to fully understand this issue.

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[18] In a slightly simpler setting, Jullien-Pouyet-Sand-Zantman (2011) look at the case of a innovator selling a product to a downstream firm when the latter has a better knowledge of the demand than the former.

[19] The negotiations over climate change are another good example of such a situation where the failure of a contract does not lead to no action.

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Appendix

Proof of lemma 1. To begin with, consider the case where $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$. There exists an equilibrium such that all types of seller accept; in case of (out-of equilibrium) contract refusal, the buyer holds beliefs $\theta = \overline{\theta}$ and bypasses after after refusal.

Let us show now that there is no other type of equilibrium. Suppose on the contrary that some types θ reject the contract and that refusal is not followed by bypass. First, a seller with type $\theta \notin \Theta^{\mathcal{C}}$ has no reason to refuse since that seller's profit is greater with the contract than without, even if the buyer bypasses. On the contrary sellers with type $\theta \in \Theta^{\mathcal{C}}$ would then have an incentive to refuse the contract since they would then gain more, and would do so by assumption A.i. But with beliefs after rejection $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$, the buyer would bypass. Therefore, when $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$, the only possible equilibrium is such that all sellers accept and there is bypass in case of out-of-equilibrium refusal.

Consider the case $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} < \theta^B \leq \bar{\theta}$. There still exists a continuation equilibrium in which all sellers accept $\mathcal{C}$ and in the (out-of-equilibrium) event of contract refusal, the buyer holds beliefs $\theta = \bar{\theta} \geq \theta^B$ and bypasses. But there exists a second continuation equilibrium in which $\mathcal{C}$ is rejected by types $\theta \in \Theta^{\mathcal{C}}$ and accepted by types $\theta \notin \Theta^{\mathcal{C}}$. In the event of contract refusal, the buyer's beliefs about θ are pinned down by Bayes' Law and must be equal to: $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\}$. Since $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} < \theta^B$, the buyer does not bypass in the event of rejection, thereby implying that types $\theta \in \Theta^{\mathcal{C}}$ reject $\mathcal{C}$ and types $\theta \notin \Theta^{\mathcal{C}}$ accept $\mathcal{C}$.

At last, if $\overline{\theta} < \theta^B$, there are no beliefs that can sustain bypass by the buyer in case of contract refusal. Therefore, only sellers with type $\theta \notin \Theta^{\mathcal{C}}$ accept the contract at the equilibrium. ■

Proof of lemma 2. First notice that from the first-order incentive constraint, $\pi(\theta) - \pi^R(\theta)$ is a convex function.

1. To show the first result, suppose instead that $\pi(\underline{\theta}) < \pi^R(\underline{\theta})$. Because $\pi(\theta) - \pi^R(\theta)$ is convex, then $\Theta^{\mathcal{C}}$ is an interval $[\underline{\theta}, \theta^*]$, with $0 < \theta^* \leq \bar{\theta}$. This implies that $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \leq \mathbb{E}\{\theta\}$, which contradicts $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B > \mathbb{E}\{\theta\}$.

2. To show the second result, suppose this is not the case. Since $\pi(\theta) - \pi^R(\theta)$ is convex and $\dot{\pi}(\theta) - \dot{\pi}^R(\theta) = p(\theta) - \alpha$, it must be the case that $p(\overline{\theta}) > \alpha$. Therefore, there exists a maximal subset of types $]\theta', \overline{\theta}]$ such that $p(\theta) > \alpha$. Moreover $\theta' > \underline{\theta}$ because $\theta' = \underline{\theta}$ and (2) would imply that $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} = \underline{\theta} < \theta^B$ which contradicts our initial condition on the contract. At this point it must also be the case that $\pi(\theta') \leq \pi^R(\theta')$ because of claim 1. For those types in $[\theta', \overline{\theta}]$, let us change the contract as follows: $p(\theta) = \alpha$ and $\pi(\theta) = \pi(\theta') + \pi^R(\theta) - \pi^R(\theta') \leq \pi^R(\theta)$. This change of contract strictly benefits the buyer. Moreover, bypass in case of refusal remains a credible threat as

$\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}} \cup [\theta', \overline{\theta}]\} > \mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$. This contradicts the optimality of the contract under the maximal rent scenario.

3. The last result follows directly from Claims 1 and 2 and the fact that $\pi(\theta) - \pi^R(\theta)$ is convex.

■

Proof of proposition 2. Consider program **P1.** Notice that incentive compatibility along with the last constraint and $\pi(\tilde{\theta}) = \pi^R(\tilde{\theta})$ imply that $p(\theta) \leq \alpha$ for θ below $\tilde{\theta}$. Let us solve the relaxed problem obtained by replacing the last two constraints by $p(\theta) \leq \alpha$ for $\theta \leq \tilde{\theta}$. Using $\pi(\theta) = \pi^R\left(\tilde{\theta}\right) + \int_{\tilde{\theta}}^{\theta} p(\theta)\, d\theta$ and Fubini theorem, standard computations lead to the following relaxed program:

$$\max_{p(.)} \int_{\underline{\theta}}^{\tilde{\theta}} p(\theta)F(\theta)d\theta - \int_{\tilde{\theta}}^{\overline{\theta}} p(\theta)(1-F(\theta))d\theta + \pi^R(\tilde{\theta})$$
$$\text{subject to } \forall\theta : p(.) \text{ non-decreasing.}$$
$$\forall\theta \leq \tilde{\theta} : p(\theta) \leq \alpha.$$

It is straightforward to show that $p(.)$ must be constant. Hence, the buyer's problem can be further simplified as $\max p(\tilde{\theta} - \mathbf{E}\{\theta\})$ s. t. $p \leq \alpha$. If $\tilde{\theta} > \mathbf{E}\{\theta\}$, then the buyer cannot do better than with no contract at all so that no contract is an equilibrium. If $\tilde{\theta} < E\{\theta\}$, then the buyer chooses $p = 0$ and obtains more than with no contract. The equilibrium offer is then:

$$T(\theta) = \tilde{\theta}\alpha \text{ and } p(\theta) = 0$$

which is accepted by all θ. ■

Proof of proposition 3. Clearly any equilibrium allocation must be incentive compatible and individually rational for the seller. Consider now the condition on v.

The necessity is immediate because the offer $\mathcal{C} = \left\{\tilde{\theta}\alpha + \varepsilon, 0\right\}$ is accepted by all types of sellers and the buyer can also make no offer. Thus the buyer can ensure an expected payoff at least equal to $\mathbb{E}(\theta)\, U - \min\left(\tilde{\theta}, \mathbb{E}(\theta)\right)\alpha$.

For sufficiency consider the initial stage. Assume that $v \geq \mathbb{E}(\theta)\, U - \min\left(\tilde{\theta}, \mathbb{E}(\theta)\right)\alpha$. Define the contract $\bar{\mathcal{C}}$ as a contract implementing the candidate allocation (it exists since the allocation is incentive compatible). We build the equilibrium by choosing the last stage selection mapping as follows:

i) if $\mathcal{C} = \bar{\mathcal{C}}$, the seller accepts for all type and rejection triggers bypass;

ii) If $\mathcal{C} \neq \bar{\mathcal{C}}$ and $\mathbb{E}\{\theta \mid \theta \in \Theta^{\mathcal{C}}\} \geq \theta^B$, then the seller accepts for all type and rejection triggers bypass;

iii) Any other offer $\mathcal{C}$ is rejected by the seller with types $\theta \in \Theta^{\mathcal{C}}$ and no bypass follows a rejection.

Thus the last stage continuation equilibrium coincides with the maximal rent scenario for all contracts except $\bar{\mathcal{C}}$ which coincide with the Minimal rent scenario. This implies that the maximal surplus that the buyer can expect by offering $\mathcal{C} \neq \bar{\mathcal{C}}$ is $\mathbb{E}(\theta) U - \min\left(\tilde{\theta}, \mathbb{E}(\theta)\right) \alpha$. Thus it is optimal to offer $\bar{\mathcal{C}}$. Then condition i) ensures that the seller accepts for all θ. Finally $\theta^B \leq \bar{\theta}$ implies that bypass is credible. ■

Proof of lemma 3. To prove that, we show that it is not possible that the contract is accepted by a type θ with $p^{\mathcal{C}}(\theta) > \alpha$.

Suppose this is the case. Since $p^{\mathcal{C}}(\theta)$ is increasing, there exists some minimal value θ' such that $p^{\mathcal{C}}(\theta) > \alpha$ for $\theta > \theta'$. Let us modify the contract offered by setting $p^{\mathcal{C}}(\theta) = \alpha$ for $\theta \geq \theta'$. Then on $(\theta', \bar{\theta})$ the final allocation is $\left(\pi^R(\theta), \alpha\right)$ (with rejection). Since $\pi^R(\theta) \leq \pi^{\mathcal{C}}(\theta)$ and $\alpha \leq p^{\mathcal{C}}(\theta) \leq p^M$ it is immediate that such a deviation would increase the buyers payoff, hence a contradiction.

To conclude the proof, it suffices to notice that with $p^{\mathcal{C}}(\theta) \leq \alpha$, the rent $\pi^{\mathcal{C}}(\theta) - \pi^R(\theta)$ is non-increasing, implying that it is non-positive on an interval $\left[\theta^*, \overline{\theta}\right]$. ■

Proof of Lemma 4. We solve this problem in two steps, first by looking at the interval $[\theta, \theta^p]$ and then optimizing with respect to θ^p.

The first part is solved using Pontryagin Principle. We define the Hamiltonian of the problem, with μ the co-state variable, as:

$$H = [\theta W(p(\theta)) + \theta p(\theta) D(p(\theta)) - \pi(\theta)] f(\theta) + \mu(\theta) [p(\theta) D(p(\theta))],$$

Using the sufficient theorems for concave objectives derived by Seierstad and Sydsaeter (1977), the following conditions must hold:

- $p(.)$ should maximize H so the first-order condition is:

$$p(\theta) \in \arg\max_p \theta W(p) + \left(\theta + \frac{\mu(\theta)}{f(\theta)}\right) pD(p).$$

- $\dot{\mu} = -\frac{\partial L}{\partial \pi} = f(\theta)$.

- $\dot{\pi} = p(\theta)D(p(\theta))$.
- $\mu(\underline{\theta}) = 0$.

The conditions stated above imply $\mu(\theta) = F(\theta)$. The first condition then implies that $p(\theta) = \phi(\theta)$ where $\phi(\theta)$ is implicitly defined by:

$$\theta\phi(\theta)D'(\phi(\theta)) + \frac{F(\theta)}{f(\theta)}\left[D(\phi(\theta)) + \phi(\theta)D'(\phi(\theta))\right] = 0 \Leftrightarrow \varepsilon(\phi(\theta)) = \frac{1}{1 + \frac{\theta f(\theta)}{F(\theta)}}.$$

Since $\varepsilon(p)$ and $\frac{1}{1+\frac{\theta f(\theta)}{F(\theta)}}$ are non-decreasing, $\phi(\theta)$ is non-decreasing.

The second part of the proof consists in optimizing with respect to the cut-off type θ^p. Using classical results in dynamic control (see Seierstad-Sydsaeter (1987, chapter 5, Theorem 17)), this cut-off is such that

$$\max_{\theta^p} \int_{\underline{\theta}}^{\theta^p} \left[\theta W(\phi(\theta)) + \theta\phi(\theta)D(\phi(\theta)) - \pi(\theta)\right] f(\theta)\, d\theta + \int_{\theta^p}^{\bar{\theta}} \theta W(\alpha) f(\theta)\, d\theta$$

The first derivative is given by:

$$\theta^p W(\phi(\theta^p) + \theta^p \phi(\theta^p) D(p(\theta^p)) - \pi(\theta^p) - \theta^p W(\alpha)$$

Since by continuity, $\pi(\theta^p) = \theta^p \alpha D(\alpha)$, it can be written as

$$\theta^p \left[W(\phi(\theta^p) + \phi(\theta^p) D(\phi(\theta^p)) - W(\alpha) - \alpha D(\alpha)\right]$$

Using the fact that $W(p) + pD(p)$ and $\phi(\theta)$ are monotonic, the objective is quasi-concave in θ^p. Canceling the derivative leads to $\phi(\theta^p) = \alpha$ for an interior solution. The solution is then $\theta^p = \phi^{-1}(\alpha)$ if it is less than $\overline{\theta}$, $\theta^p = \overline{\theta}$ otherwise (notice that $\phi(\underline{\theta}) = 0$ hence $\underline{\theta}$ cannot be solution).

Notice that the price $p(\theta)$ is continuous and monotonic, equal to $\min\{\phi(\theta), \alpha\}$. Thus, the incentive compatibility conditions are globally satisfied. Moreover, the profit $\pi(\theta)$ is larger than $\pi^R(\theta)$ which ensures that the participation constraints are also satisfied. Hence, the solution to the relaxed problem is the optimal allocation proposed by a buyer. ■

Proof of lemma 5. To prove that we show that it is not possible that the contract is accepted by a type θ with $p^{\mathcal{C}}(\theta) > \alpha$.

Suppose this is the case. Since $p^{\mathcal{C}}(\theta)$ is increasing, there exists some minimal value θ' such that $p^{\mathcal{C}}(\theta) > \alpha$ for $\theta > \theta'$. Since $\pi^{\mathcal{C}}$ is convex there exists $\theta'' \geq \theta'$

such that $\theta > \theta$" accepts the contract, while moreover $p^{\mathcal{C}}(\theta) > \alpha$ on $\left(\theta'', \bar{\theta}\right)$. Let us modify the contract offered by setting $p^{\mathcal{C}}(\theta) = \alpha$ for $\theta \geq \theta'$. Suppose the initial contract induces partial participation. Then with the new contract the set Θ expand toward higher levels of θ. If the new contract still induces partial participation, the allocation changes to $\left(\pi^{R}(\theta), \alpha\right)$ on $(\theta', \bar{\theta})$. Since on $\theta > \theta''$, $\pi^{R}(\theta) < \pi^{\mathcal{C}}(\theta)$ and $\alpha < p^{\mathcal{C}}(\theta) \leq p^{M}$, such a deviation would increase the buyers payoff, hence a contradiction. If the new contract induces full participation, the new allocation has $\pi(\theta) < \max\left(\pi^{R}(\theta), \pi^{\mathcal{C}}(\theta)\right)$ and $p(\theta) < \alpha$ for all types that were rejecting the initial allocation or were above θ''. Again the deviation is profitable.

Suppose the initial contract induces full participation. Since the set Θ expand toward higher levels of θ, the new contract still induces full participation. The new allocation has $\pi(\theta) \leq \pi^{\mathcal{C}}(\theta)$ and $p(\theta) \leq p^{\mathcal{C}}(\theta)$ for all types, with strict inequalities above θ'. Again the deviation is profitable.

To conclude the proof, it suffices to notice that with $p^{\mathcal{C}}(\theta) \leq \alpha$, the rent $\pi^{\mathcal{C}}(\theta) - \pi^{R}(\theta)$ is non-increasing, implying that it is non-positive on an interval $\left[\theta^{*}, \overline{\theta}\right]$. ■

Proof of lemma 6. We have seen above that any contract under the maximal rent scenario corresponds to an allocation that is feasible, incentive compatible and $\pi(\tilde{\theta}) \geq \pi^{R}(\tilde{\theta})$. Let us then show that the condition is sufficient.
Let us assume first that $\pi(\tilde{\theta}) > \pi^{R}(\tilde{\theta})$. Then, consider the contract $\mathcal{C} = (p(\theta), \pi(\theta))$ with full participation and bypass out-of-equilibrium. This contract with full participation implements the allocation. Indeed, since $\pi(\tilde{\theta}) > \pi^{R}(\tilde{\theta})$, for all $\theta \leq \tilde{\theta}$, $\pi(\theta) > \pi^{R}(\theta)$ so $\theta^{P} > \tilde{\theta}$. It is then direct to see that the contract is accepted by all and so implements the initial allocation.

Let us now assume that $\pi(\tilde{\theta}) = \pi^{R}(\tilde{\theta})$ in the allocation. We define the cut-off τ by $\tau = \max\{\theta \mid p(\theta) < \alpha\}$.
Consider the case where $\tau > \tilde{\theta}$ then the arguments are similar as above. Indeed, since $p(\theta)$ is increasing by incentive compatibility, then for all $\theta < \tilde{\theta} < \tau$, we have $p(\theta) < \alpha$. Since the slope of $\pi(\theta)$ is increasing with $p(\theta)$, for $\theta < \tilde{\theta}$, $\pi(\theta) > \pi^{R}(\theta)$ so $\tilde{\theta} = \theta^{P}$ and the contract with full participation implements the allocation.
Now, consider the case where $\tau < \tilde{\theta}$. Notice that in this case, we have $p(\theta) = \alpha$ for all $\theta > \tau$ which implies that $\pi(\theta) = \pi^{R}(\theta)$ for all these types. Also, for the same reason as above, we have for $\theta < \tau$, $\pi(\theta) > \pi^{R}(\theta)$. As before, any contract that replicates the allocation for $\theta < \tau$ implements the allocation. For example, consider the contract $\mathcal{C}$ with profit schedule $\pi^{C}(\theta) = \pi(\theta)$ for all types. Then, $\theta^{P} = \tau$, $\mathcal{C}$ is accepted by types less then τ and the allocation is implemented.
Finally, consider the case where $\tau = \tilde{\theta}$. Then, combining the above reasoning, when the contract $\pi^{C}(\theta) = \pi(\theta)$ is offered for all types, then there are two continuation

equilibria: one where all types accept and rejection triggers bypass and another one where only types below $\tilde{\theta}$ accept and there is no bypass in case of rejection. Thus the first equilibrium implements $\pi(\theta)$. ■

Proof of proposition 4. Notice that $\pi(\tilde{\theta}) = \pi^R\left(\tilde{\theta}\right)$ is the only binding constraint. From Jullien, Theorems 3 and 4 (adapting the constraint $q \geq 0$ to $p \leq \alpha$), the solution is characterized by (where in Theorem 4)

- $p\left(\theta\right)$ is continuous;
- $\gamma^*\left(\theta\right) = 0$ for $\theta < \tilde{\theta}$ and $\gamma^*\left(\theta\right) = 1$ for $\theta \geq \tilde{\theta}$
- There exists θ^* such that:
  - $p(\theta) = \phi\left(\theta\right) < \alpha$ if $\theta < \theta^*$, $p\left(\theta\right) = p^* \leq \alpha$ if $\theta \geq \theta^*$
  - When evaluated at p^*, if $p^* > 0$ for all $\tau \in \left[\theta^*, \overline{\theta}\right]$

$$\int_{\theta^*}^{\tau} \frac{\partial}{\partial p}\left(\frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)}W(p) + (\theta + \frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)})pD(p)\right)_{p=p^*} f\left(\theta\right)d\theta \geq 0 \tag{A1}$$

  - and if $p^* < \alpha$, for all $\tau \in \left[\theta^*, \overline{\theta}\right]$

$$\int_{\tau}^{\bar{\theta}} \frac{\partial}{\partial p}\left(\frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)}W(p) + (\theta + \frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)})pD(p)\right)_{p=p^*} f\left(\theta\right)d\theta \leq 0. \tag{A2}$$

We write

$$\begin{aligned} & \frac{\partial}{\partial p}\left(\frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)}W(p) + (\theta + \frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)})pD(p)\right) \\ = & \frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)}D(p) + (\theta + \frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)})pD'(p) \\ = & D(p)\left(\frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)} - (\theta + \frac{F\left(\theta\right)-\gamma^*\left(\theta\right)}{f\left(\theta\right)})\varepsilon(p)\right). \end{aligned}$$

Consider the case where $\underline{\theta} < \theta^* < \bar{\theta}$. Continuity implies that $p^* = \phi(\theta^*) > 0$. Then

$$\begin{aligned} \frac{F\left(\theta\right)}{f\left(\theta\right)} - (\theta + \frac{F\left(\theta\right)}{f\left(\theta\right)})\varepsilon(p^*) &> 0 \text{ for } \theta > \theta^* \text{ as } p^* < \phi\left(\theta\right) \\ \frac{F\left(\theta\right)-1}{f\left(\theta\right)} - (\theta + \frac{F\left(\theta\right)-1}{f\left(\theta\right)})\varepsilon(p^*) &< 0 \text{ for } \theta > \theta^* \text{ as } \varepsilon(p^*) < 1 \end{aligned}$$

Thus the LHS of $(A1)$ is quasi concave in τ, while the LHS of $(A2)$ is quasi convex. The condition $(A1)$ then holds for all τ if it holds for $\tau = \bar{\theta}$:

$$\int_{\theta^*}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} - (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)})\varepsilon(p^*) \right) f(\theta)\, d\theta \geq 0;$$

while the condition $(A2)$ holds for all τ if

$$\int_{\theta^*}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} - (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)})\varepsilon(p^*) \right) f(\theta)\, d\theta \leq 0.$$

Thus we have condition (5). Notice that this implies $\theta^* < \tilde{\theta}$, since otherwise the integral is negative.

Similarly if $p^* = \alpha$ we have

$$\int_{\theta^{**}}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} - (\theta + \frac{F(\theta) - \gamma^*(\theta)}{f(\theta)})\varepsilon(r) \right) f(\theta)\, d\theta \geq 0;$$

and for $p^* = 0$ we have

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{F(\theta) - \gamma^*(\theta)}{f(\theta)} \right) f(\theta)\, d\theta = \tilde{\theta} - \mathbb{E}(\theta) \leq 0.$$

■