Equalization of Opportunity: Definitions and Implementable Conditions*

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October 30, 2013

Abstract

This paper develops a criterion of opportunity equalization, that is consistent with theoretical views of equality of opportunity. Our analysis rests on the characterization of inequality of opportunity as a situation where some groups in society enjoy an illegitimate advantage. In this context, equalization of opportunity requires that the extent of the illegitimate advantage enjoyed by the privileged groups falls. Robustness requires that this judgement be supported by the broadest class of individual preferences. We formalize this criterion by resorting to a decision theory perspective and we derive an empirical condition for equalization of opportunity that is defined on the sole basis of observed opportunity distributions. We discuss observability constraints and offer an empirical testing procedure to implement this condition. Lastly, we apply these criteria to the study of the equalizing impact of educational policy in France.

Keywords: Equality of opportunity, public policy, inverse stochastic dominance, economic distance, income distribution.

JEL Codes: D63, J62, C14.

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*We are indebted to Claudio Zoli for valuable comments and discussions. We are also grateful to Marc Fleurbaey, Eugenio Peluso, Alain Trannoy, Dirk Van de gaer and participants at the LAGV#11 conference (Marseille 2012), 11th SSCW (New Delhi 2012), EEA-ESEM (Malaga 2012), GRASS V (Roma 2012), ECINEQ (Bari 2013) and seminars at THEMA, Verona, CEPS for discussion. This research received financial support from the French National Research Agency, under the grant TRANSINEQ (ANR-08-JCJC-0098-01). Francesco Andreoli kindly acknowledges the financial support from the Università Italo-Francese (UIF/UFI), Bando Vinci 2010. The usual disclaimer applies.

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1 Introduction

Equality of opportunity has gained popularity, in scholarly debates as well as among policy-makers, for defining the relevant objective for distributive justice. Nowadays, public policy often explicitly seeks to level the playing field among citizens and to equalize opportunities for a broad range of individual social and economic outcomes (e.g. education, health, income). Assessing whether public intervention succeeds at equalizing opportunities thus represents a key issue for policy evaluation. But what criterion should we use to conduct such an evaluation? Unfortunately, while an abundant literature has been devoted to the definition of *equality* of opportunity, the economic evaluation of situations where equality of opportunity is not satisfied has been much less analyzed. As a consequence, the literature offers little guidance for measuring the *equalization* of opportunity, understood as a reduction in the extent of inequality of opportunity. The objective of this paper is to define a criterion of opportunity equalization, that would be both consistent with theoretical views of equality of opportunity and empirically implementable.

The equality of opportunity (henceforth EOP) perspective amounts to draw a distinction between fair and unfair inequality.[1] Fairness judgements, according to the EOP approach, require to take into account the determinants of individual outcomes. This leads to distinguish between two sets of determinants: on the one hand, *effort* gathers the legitimate sources of inequality among individuals. On the other hand, *circumstances* correspond to the set of morally-irrelevant factors fostering inequalities across individuals that call for compensation. Define a *type* as the set of individuals with similar circumstances. Equality of opportunity defines a situation where, given effort, no *type* is advantaged compared to others, in the sense of having access to a more favorable opportunity set.

This principles translates into different formal definitions of equality of opportunity, depending on the way opportunity sets and advantage are defined. For instance, in the models of Roemer (1998) and Roemer *et al.* (2003), individual outcomes are fully determined by circumstances and effort. Conditional on effort, the opportunity set of a type is a singleton. And equality of opportunity requires that individuals experience the *same* outcome, regardless of their circumstances. In the model of Lefranc, Pistolesi and Trannoy (2009), outcome is not fully determined by circumstances and effort, owing to the influence of a third set of determinants, *luck* in their terminology. The opportunity set offered to a given individual can be characterized by the distribution function of outcome conditional on his circumstances and effort. In this context, requiring that no type is advantaged over the others amounts to require that, given effort, the conditional outcome distributions of the different types cannot be ranked by stochastic dominance tools.

To some extent, these definitions of equality of opportunity can be used to rank social states, characterized by different sets of outcome distributions conditional on type and effort. However, these definitions lead to binary rankings: equality of opportunity is satisfied or not. Situations where equality of opportunity is not satisfied in all states cannot be ranked. For instance, assessing the equalizing impact of policy intervention obviously calls for such a ranking, especially when the policy under scrutiny does not allow to reach full equality of opportunity. More generally, a ranking of social states, consistent with the equality of opportunity principle, is necessary to compare different countries or to study changes in inequality of opportunity over time.

Several papers have relied on inequality of opportunity indices in order to rank social

[1] For a comprehensive discussion, see Dworkin (1981), Roemer (1998) and Fleurbaey (2008).

states when equality of opportunity is not satisfied.[2] The dominant approach among these papers is to isolate, within total outcome inequality, the amount of unfair inequality, i.e. that which is driven by circumstances. Some authors rely on decomposable inequality indices and use inequality between types as a measure of inequality of opportunity.[3] While consistent with the EOP principles, this approach raises concerns of robustness as it relies on several restrictive assumptions. First, as is always the case with inequality indices, it relies on a specific welfare function used to aggregate outcome differentials between different types. Second, it requires to summarize by a single scalar the opportunity set offered to individuals. Many authors use the mean income conditional on circumstances and effort, which amounts to assume that individuals are risk neutral, with respect to the effect of luck. Lefranc et al. (2008) assume risk aversion with respect to luck but rely on specific preferences. Again, this lacks generality.

The contribution of this paper is to offer an ordinal ranking of social states when equality of opportunity does not prevail. It allows to make statements such as: ”Inequality of opportunity is higher in social state 0 than in social state 1”, where different states might correspond to different countries, time periods or policy regimes. Our concern is to develop a criterion that is robust to the specific individual or social welfare functions used in the evaluation. When equality of opportunity does not prevail, individuals are not indifferent between the opportunity sets offered to different types. Furthermore, they are able, given their individual preferences, to rank the different types in society, by order of advantage, in both state 0 and state 1. Our equalization principle requires that individuals, regardless of their preferences, agree that the advantage conferred to the “privileged” types falls, when moving from state 0 to state 1.

Turning this principle into a formal criterion raises several issues. First, it requires a *measure* of the advantage granted to one type relative to other types. We rely on economic measures of the distance between outcome distributions, as developed by Shorrocks (1982) and Chakravarty and Dutta (1987). In this setting, the equalization principle requires that the distance between the outcome distributions of the different types fall. Of course, the distance metric depends on individual preferences. Robustness requires that the distance between distribution falls for the broadest possible class of preferences. A key question in this respect, as we discuss below, is whether a consensus can be reached in judging that the economic distance has fallen. When consensus cannot be reached, a related issue is to characterize the subset of preferences over which individuals unanimously agree in the assessment of the change in the economic distance.

The second issue pertains to identification. In practice, we only observe (at best) the outcome distribution of each type but we do not observe individual preferences. Since verifying the distance condition for all possible preferences is not feasible, we would like to define a tractable condition, involving only the observed outcome distributions, that would be equivalent to the distance reduction condition. We show that such a condition can be formulated provided that individuals agree in the ranking of types both in state 0 and in state 1. It requires that the gap in the cumulative outcome distribution between two types falls when moving from state 1 to state 0. We refer to this condition as outcome

[2]A general discussion of inequality of opportunity indices can be found in Ramos and Van de gaer (2012). Examples of this approach can be found, *inter alia*, in Ferreira and Gignoux (2011) Checchi and Peragine (2010) Bourguignon, Ferreira and Menendez (2007) Almås, Cappelen, Lind, Sørensen and Tungodden (2011) Lefranc, Pistolesi and Trannoy (2008).

[3]Checchi and Peragine (2010) and Ramos and Van de gaer (2012) discuss a dual approach where inequality of opportunity is measured by the difference between total inequality and fair inequality. The two limitations discussed here also apply to the dual approach.

gap dominance. On the contrary, when individuals disagree on the ranking of types, they cannot unanimously agree that the distance between types has fallen. However, in this case, it is possible to identify subclasses of preferences within which individuals agree on the ranking of types in each state and to single out a necessary and sufficient condition for equalization within this subclass of preferences. This can only be performed within a restricted class of preferences. In this paper, we mainly focus on the class of rank-dependent preferences (Yaari 1987), although we discuss extensions of our results to other classes.

The third issue pertains to the aggregation of distance measures across types. It arises from the fact that, on the one hand, the distance measure can be defined for any pair of types, yet on the other hand, the ranking of social states should take all types into consideration. Lastly, several issues arise that pertain to the empirical implementation of the equalization criterion. Our ranking is based on the assumption that individual effort and circumstances are fully observable, which might not be the case in practice. We discuss the consequences of imperfectly observing the relevant determinants of outcome for our ranking. Next, we discuss issues of statistical inference and provide an application of our equalization condition in the evaluation of educational policy in France, using quantile treatment effects estimates of the impact of schooling expansion.

The rest of the paper is organized as follows. Section 2 lays out the main notations and reviews the definitions of equality of opportunity previously given in the literature. Section 3 discusses our main equalization principle and provides a tractable condition in the case of a simplified setting with only two types and one effort level. In section 4 we consider the general case with multiple effort and multiple circumstances. In this setting, we discuss how pair-wise equalization conditions can be aggregated into a global ranking of social states. We also discuss the consequences of partial observability of the relevant determinants of outcomes. Lastly section 5 develops a statistical framework for testing our equalization condition and provides an empirical application.

2 Equality of opportunity and social states ranking

2.1 Determinants of outcome

Our analysis builds upon the framework developed by Lefranc et al. (2009), elaborating on Roemer (1998). We let y denote individual outcome. The determinants of outcome are assumed to be partitioned into four components: circumstances, denoted by c; effort, denoted by e; luck, denoted by l and social state denoted by π.

Circumstances capture the determinants of individual outcomes that are not considered a legitimate source of inequality. Define a *type* as the set of individuals with similar circumstances. *Effort*, on the contrary, includes the determinants of outcome that are seen as a legitimate source of inequality. Following LPT, *luck* comprises the factors that are perceived as a legitimate source of inequality as long as they affect individual outcomes in a neutral way, given circumstances and effort.

Lastly, we allow individual outcome to be contingent on a binary social state, denoted $\pi \in \{0, 1\}$. All individuals in a society share a common social state, although the realization of the social state does not affect all individuals in the same manner. For instance, the social state π may indicate a policy regime where $\pi = 0$ can be interpreted as a state without a specific policy intervention and $\pi = 1$ the state with the specific policy being implemented. The two states may also, more generally, correspond to two periods or two countries, that one would like to compare. The analysis can be extended to comparisons involving more than two policy regimes.

Given their type, their level of effort and the realization of the social state, the outcome prospects offered to individuals can be summarized by the conditional distribution function $F_\pi(y|c,e)$, where $F(z|t)$ denotes the cumulative distribution function, conditional on some covariates of value t, evaluated at outcome z. We define $F^{-1}(p|t)$ the conditional quantile distribution associated with $F(.|t)$, for all population shares p in [0,1].[4]

2.2 Definitions of equality of opportunity

EOP theories emphasize that inequality due to differences in circumstances are morally or politically objectable, while inequality originating from differential effort are legitimate. Several definitions of equality of opportunity can be offered that agree with these two principles. In a context where outcomes are not fully determined by effort and circumstances, LPT distinguish between a strong and weak form of equality of opportunity.

Strong equality of opportunity, which we label EOP-S, corresponds to the situation where the opportunity sets of individuals with similar effort are *identical* regardless of circumstances. Hence, for a given social state π, EOP-S requires that $F_\pi(.|c,e) = F_\pi(.|c',e)$, for any effort e, and for any pair of circumstances (c,c'). Although outcome distributions may still vary with effort, luck or social state, EOP-S appears as a demanding condition as it requires that the effect of circumstances be nullified.

Situations where outcome distributions differ across types do not necessarily imply that one type is advantaged over the others. In fact, there might be cases where it is not possible to unanimously rank circumstances according to the advantage they confer, over all possible individual preferences. In such cases, it may be argued that a weak form of equality of opportunity prevails. This corresponds to the notion of *weak equality of opportunity* (EOP-W) discussed in LPT, which requires that no consensus be reached in the ranking of types among all preferences displaying risk aversion. In other words, for a given social state π, EOP-W requires that, for any effort e, and for any pair of circumstances (c,c'), the outcome distributions $F_\pi(.|c,e)$ and $F_\pi(.|c',e)$ cannot be ranked according to second-order stochastic dominance. [5]

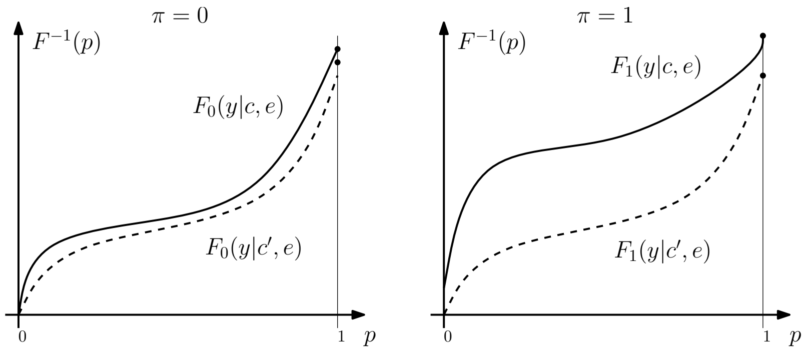
When neither EOP-S nor EOP-W are satisfied, individuals can unanimously rank the outcome distribution attached to the different types, and *inequality of opportunity* (denoted IOP) prevails.

2.3 Ranking social states

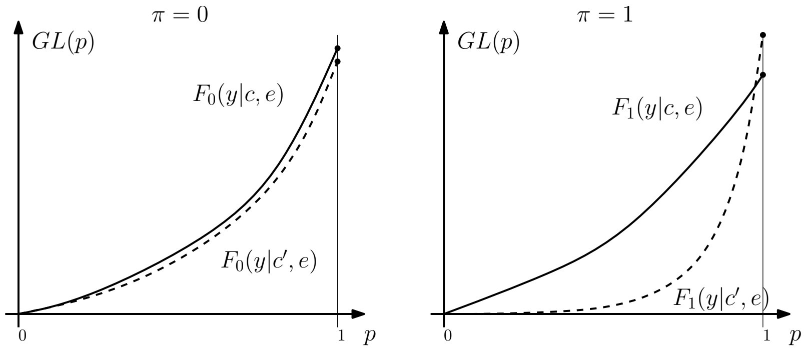
These different conceptions of equality of opportunity define a taxonomy that allows to rank social states. Since EOP-S is more demanding than EOP-W, one may claim that states where EOP-S is satisfied are better, from the perspective of equality of opportunity, than states where only EOP-W is satisfied. Furthermore, one may also argue that IOP

[4] If the cumulative distribution function is only left continuous, we define $F^{-1}()$ by the left continuous inverse distribution of F: $F_\pi^{-1}(p|c,e) = \inf\{y \in \mathbb{R}_+ : \ F_\pi(y|c,e) \geq p\}$, with $p \in [0,1]$.

[5] When the distribution with c.d.f. F dominates distribution F' for second-order stochastic dominance, all risk-averse preferences prefer F over F' in the expected utility framework. Formally, first-order stochastic dominance requires that for all y in the support of F and F', $F(y) \leq F'(y)$. Second-order stochastic dominance requires that $\int_0^y F(u)du \leq \int_0^y F'(u)du$.



(a) First order stochastic dominance



(b) Second order stochastic dominance

Figure 1: EOP-W and IOP with two circumstances

represents the worst case from the perspective of equality of opportunity, as individuals unanimously agree that one type is advantaged over others. Hence, the analysis in LPT leads to the following implicit ranking of social states: EOP-S $\succ$ EOP-W $\succ$ IOP.

This ranking can be challenged on two main grounds. First, it only provides a (very) partial-order and is unable to discriminate among social states that all satisfy IOP or EOP-W. Second, the ranking is exclusively based on the *existence* of a distributional advantage enjoyed by some types over other types: in EOP-S, all types face similar outcomes; in IOP, all preferences agree that one type is advantaged over other types; EOP-W is an intermediate situation where there is no consensus. But beyond the sheer existence of an advantage, the *size* of the advantage is ignored by the above ranking, although it is obviously relevant for inequality of opportunity.

To illustrate the first limitation, consider panel (a) of figure 1. The figure presents the outcome distribution for two types c and c', for a common effort level e, for two social states ($\pi = 0$ and $\pi = 1$). Since the distribution of type c dominates that of type c' at the first order, IOP prevails in both states. Hence they cannot be distinguished according to the above ranking. However, the extent of the advantage enjoyed by type c is much higher for $\pi = 1$ than for $\pi = 0$. It would thus seem reasonable to claim that state 0 is better than state 1 from the perspective of equality of opportunity.

One may further question the implicit ranking of LPT by considering the situation illustrated in panel (b) of the same figure 1. In state 0, the distribution of type c dominates that of type c' at the second order. Hence IOP prevails, although the gap between the two curves is small. In state 1, the generalized Lorenz (GL) curves of the two types intersect at the very top of the distribution so EOP-W is satisfied. According to the above ranking, state 1 would be considered better than state 0. Yet, over most percentiles of the outcome distribution the gap between the two GL curves is much larger in state 1 than in state 0. As a consequence, not all preferences would agree that the advantage of type c decreases by moving from state 0 to state 1. In fact, one may even wonder to what extent a consensus could be reached on the opposite statement that state 0 is better than state 1 from the perspective of equality of opportunity.

This discussion reveals important issues for the ranking of social states. First, this ranking calls for a *difference-in-differences* comparisons of distributions: the first difference measures the gaps in outcomes between different types; the second one measures the differences across social states in the extent of these gaps. Second, the discussion also emphasizes that assessing the advantage or disadvantage of a type should draw on a subjective evaluation function. Furthermore, the ranking should be robust to the diversity of subjective evaluation functions. Lastly, the ranking of social states should also be sensitive to the size of the distributional advantage enjoyed by some types with respect to others. These key ingredients should be taken into consideration when building an criterion for equalization of opportunity, as we do in the next section.

3 Equalization of opportunity: a simplified setting

We start by considering a simplified setting with only two types, c and c', who exert a common effort level e. To simplify notations, we abbreviate by $F_\pi(.)$ (resp. $F'_\pi(.)$) the c.d.f. of outcome for type c (resp. c') at effort e in social state π, *i.e.* $F_\pi(.|c, e)$ (resp. $F_\pi(.|c', e)$).

3.1 A criterion for equalization of opportunity

Equalization of opportunity and economic distance reduction We assume that each individual is endowed with preferences W over risky outcomes. As a result, when different types are offered different opportunity sets, each individual is able to rank types, on the basis of the welfare they provide, as well as to compute the economic advantage offered by type c relative to type c'. Obviously, this measure of the economic advantage depends on the individual preferences W, as well as the conditional distributions F_π and F'_π. We let $\Delta_W(F_\pi, F'_\pi)$ denote this measure of economic advantage, where a positive value indicates that the value of the opportunity set is greater for type c than for type c'. We defer to the next section the discussion of how the economic advantage $\Delta_W()$ should be precisely defined.

The economic advantage $\Delta_W()$ underlies the characterization of equality of opportunity discussed in the previous section. The principle behind EOP-S is that $\Delta_W(F_\pi, F'_\pi)$ should be zero for all possible W. The requisite of EOP-W is that individuals should not agree on the sign of $\Delta_W(F_\pi, F'_\pi)$. In this paper, we are not concerned with assessing equality of opportunity but with assessing equalization of opportunity when strong equality of opportunity is not satisfied. EOP theories emphasize that inequality of outcomes due to circumstances is morally offensive and call for compensation. In line with this principle, our equalization principle requires, for ranking state 1 preferable to state 0, that the economic advantage enjoyed by the most privileged type be smaller in state 1 than in state 0.

Given that inequality across types is equally offensive, regardless of the identity of the advantaged type, one should further require that the equalization principle satisfy a principle of *anonymity*, in the sense that only the absolute value of the economic advantage, but not the sign of the advantage should matter for assessing equalization of opportunity. We refer to the absolute value of economic advantage as the economic distance between types.

Differences across social states in the economic distance between types depend upon the individual preferences W that are used in the evaluation. While it is possible to define equalization for a specific choice of preferences, this criterion would lack robustness. In fact, it is an important issue for distributive justice to take into account the diversity of individual welfare functions. In order to reach a robust criterion, one may require that the economic distance be smaller, for a broad class of evaluation functions. We let $\mathcal{P}$ denote the class of individual preferences.

The following definition summarizes our notion of equalization of opportunity:

Definition 1 (ezOP: equalization of opportunity between two types) *Moving from social state $\pi = 0$ to $\pi = 1$ equalizes opportunity between circumstances c and c' at effort e on the set of preferences $\mathcal{P}$ if and only if for all preferences $W \in \mathcal{P}$, we have:* $|\Delta_W(F_0, F_0')| \geq |\Delta_W(F_1, F_1')|$.

Note that this definition is contingent on the choice of the class of preferences $\mathcal{P}$. When $\mathcal{P}$ includes all possible individual preferences, ezOP amounts to require a consensus in the population over the statement that the economic advantage of most privileged type falls when moving from state 0 to state 1. If consensus does not prevail within the class $\mathcal{P}$ it might be interesting to identify the sub-class of preferences of $\mathcal{P}$ over which individuals agree with the equalization of opportunity statement.

Measuring economic distance So far, we have not discussed how the economic distance between the opportunity sets could be measured. In assessing EOP, many authors have used the mean outcome gap between types. In the presence of luck, this ignores risk associated with the distribution of outcomes within types, which might not be consistent with individual preferences.

Various measures of the economic distance between outcome distributions can be defined on the basis of individual preferences.[6] For two distributions F and F' and for preferences W, the distance measure has to reflect differences in the welfare of the two distributions and is usually expressed as $|\Delta_W(F, F')| = f(W(F), W(F'))$, where $W(F)$ denote the expected welfare of distribution F. This approach requires that individual be endowed with a cardinal welfare representation, so that quantities such as $W(F) - W(F')$ can be compared in a meaningful way. Assuming this is the case, the desirable properties of the function f have to be clarified. They are discussed in Chakravarty and Dutta (1987).

One natural candidate for a distance measure is the *absolute welfare gap* between the distribution: $|\Delta_W(F, F')| = |W(F) - W(F')|$. In the case where individual preferences have an expected utility representation, this is simply the difference in the expected utility between F and F'. An undesirable feature of this measure of distance is that it does not satisfy the translation invariance axiom of Chakravarty and Dutta (1987). In particular, adding a fixed amount to both distributions will change the distance measure.

Shorrocks (1982) proposed to measure the economic distance by *certain equivalent gap* between the two distributions, where the certain equivalent of F is the certain outcome that

[6] See in particular Shorrocks (1982), Ebert (1984), Chakravarty and Dutta (1987) and Magdalou and Nock (2011).

yields the same welfare level as the distribution F. Let the $CE_W(F)$ denote this quantity. An alternative measure of the economic distance is thus: $|\Delta_W(F, F')| = |CE_W(F) - CE_W(F')|$.

The distinction between these two measures of distance echoes a well-known divide between absolute and relative approaches in the measurement of inequality. In the rest of the paper, we will consider both the absolute welfare gap and the certain equivalent gap as candidate measures of economic distance.

Beyond translation invariance, other desirable properties of the economic distance measure, as discussed in Chakravarty and Dutta (1987), are (i) that the distance between a distribution and its average provide an absolute measure of distributional inequality, and (ii) that the distance measure satisfy linear homogeneity [7]. It turns out that these two properties and translation invariance are satisfied by the certain equivalent gap distance only in the case where individual preferences W belong to the Rank-Dependent Expected Utility family.[8] As a consequence, in this family, the absolute welfare gap and the certain equivalent measure coincide.

The identification problem Once individual preferences W are known, the economic distance measures of the previous section can be straightforwardly computed for both social states and it is possible to check the equalization condition. However, our equalization condition is not defined for a single W but for an entire class of preferences $\mathcal{P}$. For a sufficiently general class, it is practically impossible to verify the economic distance condition for all preferences.

In practice, the equalization condition previously defined will only be relevant if it can be reformulated in terms of a restriction that only involves the outcome distributions of the different types under the different social states.

This cannot be achieved in the most general case where no restriction is imposed on the class $\mathcal{P}$ of individual preferences. Two possible alternative representations of preferences under risk have been widely studied and adopted in decision theory: the *expected utility* model and the Yaari's (1987) *rank-dependent* model. In the rest of the paper, we focus on the rank-dependent expected utility class, which we denote by $\mathcal{R}$. One of its advantages is that it is consistent with the properties and representation of economic distance measures proposed in the literature. The analysis can be nevertheless extended to the traditional expected utility setting when distance is assumed to reflect absolute welfare gaps. In the rest of this section we concentrate on the following question: What minimal conditions need to be imposed on the set of distributions F_0, F'_0, F_1, F'_1 to ensure that equalization is satisfied for all preferences in $\mathcal{R}$?

3.2 Identification under the rank dependent utility model

Properties The rank-dependent expected utility model assumes that the welfare derived from a risky distribution F can be written as a weighted average of all possible realizations where the weights are a function of the rank of the realization in the distribution of outcome. Formally, let $w(p) \geq 0$ denote the weight assigned to the outcome at percentile

[7] The distance should be multiplied by λ when all outcomes are multiplied some scalar λ.

[8] See below for a definition.

p, the welfare derived from F can be written as[9]:

$$W(F) \;=\; \int_0^1 w(p)F^{-1}(p)dp,$$

Given that the rank-dependent expected utility representation is linear in the realization of outcomes, the certain equivalent of a distribution F, for preferences $W \in \mathcal{R}$, $CE_W(F)$ is simply equal to the expected welfare $W(F)$.[10] As a consequence, the certain equivalent gap between two distributions, as defined in the previous section, is also equal to the welfare gap. In the end, the economic distance between distributions F and F' can be written as:

$$|\Delta_W(F,F')| = |\int_0^1 w(p)\Gamma(F,F',p)dp|. \tag{1}$$

where $\Gamma(F,F',p) = F^{-1}(p) - F'^{-1}(p)$ is the *cumulative distribution gap* between F and F'. In the rest of the paper, we refer to the graph of $\Gamma(F,F',p)$ as the *gap curve* and to the graph of $|\Gamma(F,F',p)|$ as the *absolute gap curve.*

Necessary condition for ezOP When assessing whether ezOP is satisfied, only the distribution curves under the two social states are observed, but not individual preferences. Our objective is to provide a condition on these observables warranting that ezOP holds for all preferences in the class of rank-dependent utility functions. Equation (1) establishes the relationship between economic distance $\Delta_W(F,F')$ and the cumulative distribution gap $\Gamma(F,F',p)$. A necessary condition for ezOP is that the cumulative distribution gap should be smaller, in absolute value, at any percentile, under $\pi = 1$ than under $\pi = 0$.

Proposition 1 *If ezOP is satisfied on the set of preferences $\mathcal{R}$ then for all $p \in [0,1]$, we have: $|\Gamma(F_1,F_1',p)| \leq |\Gamma(F_0,F_0',p)|$.*

Proof. See appendix A.2. ■

This proposition shows that a necessary condition for ezOP is that the gap between the cdfs of types c and c' falls at all percentiles when moving from social state 0 to social state 1. In other terms, the absolute gap curve under $\pi = 0$ should always be larger than under $\pi = 1$. We refer to this situation as a situation where the absolute gap curve for $\pi = 0$ dominates the absolute gap curve for $\pi = 1$

Note also that absolute gap curve dominance is *not* a *sufficient* condition for ezOP. Appendix A.2 provides a detailed counter-example. The reason is that a reduction in the gap between the cumulative distribution functions of types c and c' cannot be unambiguously interpreted in the general case. For instance, assume that the distribution of type c dominates the distribution of type c' over some small interval $[a,b]$. It does not imply, in the general case, that type c dominates c' over the entire support of the distribution. Henceforth, some preferences will rank c better c' and other preferences will rank c' better. Now assume that social state 1 is identical to social state 0, except that under $\pi = 1$, the advantage of type c over $[a,b]$ has been reduced. In this case, gap curve dominance will be satisfied. At the same time, preferences that ranked c' better than c will conclude that the

[9]Formally, one requires that $w(p) \geq 0 \; \forall p \in [0,1]$ and $\widetilde{w}(p) = \int_0^p w(t)dt \in [0,1]$ is such that $\widetilde{w}(1) = 1$. For a discussion, see Zoli (2002).

[10]By definition of the certain equivalent, we have: $\int_0^1 w(p)F^{-1}(p)dp = \int_0^1 w(p)CE_W(F)dp$. But since weights sum to unity the second term is also equal to $CE_W(F)$.

cardinal advantage of c' has increased since the local advantage of c over $[a, b]$ has fallen. This contradicts ezOP.

Necessary and sufficient condition under stochastic dominance A corollary of the previous discussion is that if individuals agreed on the ranking of types, one would expect them to interpret gap curve dominance in a unanimous way. We now examine this specific case.

As discussed in Muliere and Scarsini (1989) unanimity in ranking distributions F_π better than F'_π will be achieved for all preferences in $\mathcal{R}$ if and only if distribution F_π dominates distribution F'_π for order-one inverse stochastic dominance (which we denote $F_\pi \succ_{ISD1} F'_\pi$), *i.e.* whenever the graph of F_π^{-1} lies above the graph of $F_\pi'^{-1}$.

Within this section, we shall assume that this condition is satisfied. Since c and c' play a symmetric role in the definition of ezOP, which type dominates the other is irrelevant. Hence we make the neutral assumption that the distribution of type c dominates the distribution of type c', under both policy regimes.

When all preferences unanimously rank type c better than type c', a fall in the cumulative distribution gap has unambiguous consequences for the change in the economic distance between types. In fact, since the sign of the cumulative distribution gap is constant over all percentiles, the economic distance can be expressed as an increasing function of the absolute income gap: $|\Delta_W(F, F')| = \int_0^1 w(p)|\Gamma(F, F', p)|dp$. This leads to the following proposition:

Proposition 2 *If* $\forall\pi\ F_\pi \succ_{ISD1} F'_\pi$ *then: ezOP over the set of preferences* $\mathcal{R} \ \Leftrightarrow\ \forall p \in [0, 1],\ \Gamma(F_0, F'_0, p) \geq \Gamma(F_1, F'_1, p)$.

Proof. See appendix A.3. ■

This proposition establishes that when agents agree on the ranking of types, gap curve dominance provides a necessary and sufficient condition for equalization of opportunity. This contrasts with the situation where dominance does not prevail: in this latter case, gap curve dominance only provides a necessary condition and equalization of opportunity might not prevail even when the dominance condition is satisfied. This undecisiveness can be alleviated at the cost of considering a restricted set of preferences, as we discuss in the next section.

Restricted consensus on ezOP We now focus on cases where types cannot be ranked according to first-order inverse stochastic dominance. In this case, cumulative distribution gap can no longer be used to infer ezOP. We now discuss how necessary and sufficient conditions for equalization of opportunity can be produced by considering restricted sets of preferences.

It is to some extent obvious that using a restricted set of preferences makes the assessment of ezOP easier. At the extreme, if one is willing to consider a single preference function, it is always possible to determine whether social state 0 is better then social state 1. However, in this case, the greater accuracy of the equalization judgements will come at the cost of lower generality. Our objective is to identify the minimal set of restrictions on individual preferences that allow to form unambiguous predictions on equalization of opportunity. We show that it is always possible to find a subset of $\mathcal{R}$ over which individuals agree on the ranking of types. Furthermore, on this subset, one can establish a necessary and sufficient condition for equalization of opportunity.

We first illustrate how necessary and sufficient conditions for ezOP can be achieved in the absence of first-order dominance, in the special case where distributions F_π and F'_π can be ranked by *the second-order* inverse stochastic dominance. Without loss of generality, assume that for all π $F_\pi \succ_{ISD2} F'_\pi$. Define $\mathcal{R}^2 \subseteq \mathcal{R}$, the set of risk-averse rank-dependent preferences.[11] Under our dominance assumption all preferences in $\mathcal{R}^2$ will unanimously rank type c better than c' under both policies. Furthermore, over the class $\mathcal{R}^2$, the economic distance can be expressed as an increasing function of the integral of the cumulative distribution gap. Hence a necessary and sufficient condition for ezOP, over $\mathcal{R}^2$, under inverse second-order stochastic dominance, is that the integrated cumulative distribution gap falls at all percentiles. This is established in the following proposition:

Proposition 3 *If* $\forall \pi$ $F_\pi \succ_{ISD2} F'_\pi$ *then: ezOP over the set of preferences* $\mathcal{R}^2$ $\Leftrightarrow$ $\forall p \in [0,1], \int_0^p \Gamma(F_0, F'_0, t)dt \ \geq \int_0^p \Gamma(F_1, F'_1, t)dt$

Proof. See appendix A.4. ■

Now, consider the general case where distributions cannot be ranked by ISD2. In this case, consensus cannot be reached in the class $\mathcal{R}^2$ over the ranking of types. However, it is possible to define a more restricted set of preferences over which individuals agree on the ranking of types. Following Aaberge (2009), consider the subset of preferences $\mathcal{R}^k$ defined by:

$$\mathcal{R}^k \ = \ \left\{ W \in \mathcal{R} \mid (-1)^{i-1} \cdot \frac{d^i \widetilde{w}(p)}{dp^i} \geq 0, \quad \frac{d^i \widetilde{w}(1)}{dp^i} = 0 \ \forall p \in [0,1] \text{ and } i = 1, \dots, k \right\},$$

where $\widetilde{w}(p)$ is the cumulative weighting scheme. The sequence of subsets of the type $\mathcal{R}^k$ defines a nested partition of $\mathcal{R}$ and we have: $\mathcal{R}^k \subset \mathcal{R}^{k-1} \subset ... \subset \mathcal{R}$.[12]

As discussed in Zoli (2002) and in appendix A.1, when distributions F_π dominates distribution F'_π for order-k inverse stochastic dominance, all preferences in $\mathcal{R}^k$ will prefer F_π over F'_π.

Furthermore, as established in the appendix, any pair of distribution can always be ranked by inverse stochastic dominance, for a sufficiently high order. We now define κ the *minimal order* at which F_π and F'_π can be ranked for the inverse stochastic dominance order. Without loss of generality, we have that for all π $F_\pi \succ_{ISD\kappa} F'_\pi$. All preferences in $\mathcal{R}^\kappa$ agree on the ranking of types under both social states. Furthermore, it is worth emphasizing that the set $\mathcal{R}^\kappa$ is endogenously defined and represents the largest set in the partition $\{\mathcal{R}^k, \ k \in \mathbb{N}\}$ over which consensus is reached on the ranking of types.

We now introduce, for $k \in \mathbb{N}^+$, Λ^k_π the integral of order $k-1$ of the inverse distribution functions of F_π. It is recursively defined, for all $p \in [0,1]$ by:

$$\Lambda^2_\pi(p) = \int_0^p F_\pi^{-1}(u)du \ \text{ and } \ \Lambda^k_\pi(p) = \int_0^p \Lambda^{k-1}_\pi(u)du$$

Similarly, define Λ'^k_π the integral of order $k-1$ of the inverse distribution functions of F'_π. Define $\Gamma(\Lambda^k_\pi, \Lambda'^k_\pi, p) = \Lambda^k_\pi(p) - \Lambda'^k_\pi(p)$, the cumulative distribution gap integrated at order $k-1$.

[11] This set contains all evaluation functions that assign decreasing weights to increasing outcomes realizations.

[12] Note that k is a measure of the effect of a precise sequence of restrictions on all possible cumulative weighting schemes $\widetilde{w}(p)$ defined on $\mathcal{R}$. Hence, k embodies information on the risk attitude of preferences isolated by the class $\mathcal{R}^k$.

If for all π $F_\pi \succ_{ISD\kappa} F'_\pi$, then for all preferences $W \in \mathcal{R}^\kappa$, the economic distance $|\Delta_W|$ under policy π is an increasing function of $\Gamma(\Lambda^\kappa_\pi, \Lambda'^\kappa_\pi, p)$. As a consequence, ezOP will be satisfied in the set $\mathcal{R}^\kappa$ if and only if $\Gamma(\Lambda^\kappa_\pi, \Lambda'^\kappa_\pi, p)$ is smaller under $\pi = 1$ than under $\pi = 0$. This is established in the following proposition:

Proposition 4 *ezOP over the set of preferences* $\mathcal{R}^\kappa \;\Leftrightarrow\; \forall p \in [0,1],\; \left|\Gamma(\Lambda^\kappa_0, \Lambda'^\kappa_0, p)\right| \geq \left|\Gamma(\Lambda^\kappa_1, \Lambda'^\kappa_1, p)\right|$.

Proof. See appendix A.5. ∎

Proposition 4 establishes a necessary and sufficient condition for ezOP under a less stringent dominance condition than in propositions 2 and 3. At the same time, the set of preferences over which it allows to identify ezOP is more restrictive. Last, since κ always exist, proposition 4 also establishes a necessary condition for ezOP over the entire class $\mathcal{R}$.

3.3 Discussion

Several features of the equalization criterion defined in this section are worth discussing further. First, the criterion laid out in definition 1 does not resort to an external social welfare function in order to evaluate the opportunity sets offered to the different types in society. On the contrary, our criterion relies on the individuals' *own* preferences in order to assess whether equalization of opportunity is achieved. Second, our criterion is general in the sense that it does not place any restriction on the preferences of the individuals with respect to the opportunity set of the different types. Third, our criterion does not even require a priori that individuals agree in their ranking of the various types. It simply requires a consensus between agents over the reduction in the gap between the value of the opportunity sets of the different types. In other words, our criterion requires a consensus on the reduction of the advantage but not on the identity of the advantaged type. Lastly, our criterion does not require to summarize the opportunity sets of the different types by a scalar measure, such as the mean income, as is often done in the literature on the measurement of inequality of opportunity.

Of course, while the generality of the criterion leads to a robust assessment of equalization of opportunity, this robustness comes at the cost of tractability. As we noted, it is not possible, in practice, to verify whether the condition of equalization is satisfied without considering a restricted set of preferences. In the rest of our analysis, we considered the family of rank-dependent expected utility functions and showed that it is possible to derive equalization conditions that only depend on the distribution functions of the lotteries offered to the various types in society. However, our framework is not confined to the rank-dependent family and could be extended to other families of preferences. For instance, in the same spirit, equalization conditions could be derived for preferences within the Von Neumann expected utility framework.

The results obtained under the rank-dependent assumption also call for further comments. They lead to distinguish between two cases: the case where individuals agree in the ranking of types under each social state, and the case where they do not agree. When individuals agree on the ranking of types, assessing equalization of opportunity is straightforward, as proposition 2 provides a necessary and sufficient condition.

The case where individuals do not agree on the ranking of types does not allow such a clear cut judgment on equalization. Proposition 1 provides a necessary condition of equalization. Violation of this condition rules out equalization of opportunity. If not, proposition 4 allows to endogenously identify a restricted set of preferences over which unanimity might be reached regarding equalization of opportunity. Of course, this only provides a partial judgment over equalization of opportunity. In fact, the higher the order of restriction κ that must be placed, the less general the judgement will be.

However, the extent of the restrictions on preferences that need to be placed to achieve a consensus on the ranking of types is, in itself, informative. When little restrictions need to be placed to achieve a consistent ranking, individuals largely agree on which type is advantaged in society. On the contrary, when strong restrictions need to be placed, there is widespread disagreement on which type is advantaged in society. In this case, following LPT, one might argue that a weak form of equality of opportunity already prevails. In fact, by capturing the degree of consensus on the advantaged type, κ helps generalize the notion of weak equality of opportunity introduced in LPT. To summarize, when there is a large disagreement on which type is advantaged (high κ), our criterion provides a very partial condition for consensus on equalization of opportunity, although this admittedly corresponds to a case of weak inequality of opportunity. On the contrary, when there is large agreement on which type is advantaged (low κ), our equalization condition becomes least partial and turns into a necessary and sufficient condition for ezOP in the case where there is full consensus on identifying the advantaged type ($\kappa = 1$).

4 Equalization of opportunity: generalization

In the general case, opportunity equalization has to be assessed with more than two circumstances across many effort levels. When effort is observable, one possibility is to extend the ezOP comparisons on all pairs of circumstances at every effort level, or to study meaningful aggregations of these judgements. Identification criteria when effort is not observable are also discussed, in order to provide relevant notions of equalization that can still be used in applied analysis, under observability constraints.

4.1 Extending the ezOP criterion to multiple circumstances

We consider the case in which there are T types. Let $C = \{c_1, ..., c_i, ...c_T\}$ denote the set of possible circumstances. For simplicity, we assume a single effort level e. The results of this section can be easily extended to multiple effort levels by requiring that equalization holds for every effort level.

A straightforward extension of definition 1 to multiple circumstances is to require that for every possible pair of circumstances, the distance falls when moving from social state $\pi = 0$ to $\pi = 1$. This is given by the following definition:

Definition 2 (Non-anonymous ezOP between multiple types) *Moving from social state $\pi = 0$ to $\pi = 1$ equalizes opportunity over the set of circumstances C at effort e on the set of preferences $\mathcal{P}$ if and only if for all preferences $W \in \mathcal{P}$, for all $(i, j) \in \{1, \ldots, T\}$, we have: $|\Delta_W(F_0(.|c_i, e), F_0(.|c_j, e))| \geq |\Delta_W(F_1(.|c_i, e), F_1(.|c_j, e))|$.*

Again, this generalized form of ezOP cannot be verified, in practice, without resorting to a specific class of preferences. In the class $\mathcal{R}$, the results of propositions 2 and 4 generalize easily to the multivariate case. For every pair (i, j), let κ_{ij} denote the *minimal order* at which $F_\pi(.|c_i, e)$ and $F_\pi(.|c_j, e)$ can be ranked according to inverse stochastic dominance, for all π. According to proposition 4, integrated gap curve dominance provides a necessary and sufficient condition for ezOP between types c_i and c_j over the subclass $\mathcal{R}^{\kappa_{ij}}$.

Considering all possible pairs yields the following necessary condition for non-anonymous ezOP between multiple circumstances:

Proposition 5 *A necessary condition for non-anonymous ezOP between multiple types over the set of preferences $\mathcal{R}$ is :*
$\forall (i,j) \in \{1,\ldots,T\}, \forall p \in [0,1], \left|\Gamma\left(\Lambda_0^{\kappa_{ij}}(p|c_i,e), \Lambda_0^{\kappa_{ij}}(p|c_j,e)\right)\right| \geq \left|\Gamma\left(\Lambda_1^{\kappa_{ij}}(p|c_i,e,1), \Lambda_1^{\kappa_{ij}}(p|c_j,e)\right)\right|.$

The proof is based on the same arguments used in the proof of proposition 4.

Definition 4 assumes that the quantum of interest is the extent of the advantage gap between a specific circumstance c_i compared to another c_j. It requires that every such gaps fall when moving from social state 0 to 1. This makes the "identity" of each type relevant for defining equalization of opportunity. One may challenge this view and claim that only the magnitude of the gaps (and not the identity of the types involved) is relevant for defining equalization of opportunity. Consider a simple example in which there are three types c_1, c_2 and c_3. Assume that there is only one effort level and luck plays no role. Under each of the three social states $\pi = A, B, C$, each type is assigned with an outcome given by the following table:

	Outcomes		
	$\pi = A$	$\pi = B$	$\pi = C$
c_1	6	6	6
c_2	3	4	2.5
c_3	1	2.5	4

When moving from social state A to B, the gap between each type and the other two falls and the condition in definition 4 is satisfied. On the contrary, when moving from social state A to C, the gap between c_1 and c_2 increases and equalization is not satisfied, although state C is obtained from state B by permuting the outcomes of groups c_2 and c_3. This inconsistency arises from the fact that assessment of equalization of opportunity in definition 4 is sensitive to the identity of the groups associated to a given opportunity gap.

This counterexample echoes a well-know *anonymity principle* used in the assessment of inequality. According to this principle, the measurement of inequality of outcome should be unsensitive to a permutation of the outcomes of individuals within the distribution. This principle can be incorporated to our definition of equalization of opportunity by making it unsensitive to a permutation of the opportunity sets across types.

Let us introduce some additional notation. Let $r_\pi^W(c)$ be the *rank function* assigning to circumstance c its rank, $r(.) \in \{1,\ldots,T\}$ in the ranking of types, in social state π, according to preferences W. Given W, all circumstances can be ranked. But the rank of a specific circumstance c might change across social states and differs across preferences.

The anonymous principle of equalization of opportunity between multiple types requires that the opportunity gap between two types sitting at given ranks falls when moving from social state 0 to 1. This should hold for every pair of ranks and every utility function in $\mathcal{P}$.

Definition 3 (Anonymous ezOP between multiple types) *Moving from social state $\pi = 0$ to $\pi = 1$ equalizes opportunity over the set of circumstances C at effort e on the set of preferences $\mathcal{P}$ if and only if for all preferences $W \in \mathcal{P}$, for all $(i,j,h,\ell) \in \{1,\ldots,T\}^4$ such that $r_0^W(c_i) = r_1^W(c_h)$ and $r_0^W(c_j) = r_1^W(c_\ell)$ we have:*
$\left|\Delta_W(F_0(.|c_i,e), F_0(.|c_j,e))\right| \geq \left|\Delta_W(F_1(.|c_h,e), F_1(.|c_\ell,e))\right|.$

Implementation of anonymous equalization requires first to rank circumstances according to each specific preference in $\mathcal{P}$, and, second, to check if advantage between circumstances occupying a similar rank is reduced when changing social state. In practice, testing whether the second condition is satisfied requires that the ranking of circumstances be identified empirically. Hence, the condition in definition 3 can only be tested empirically for classes of preferences where there is agreement on the ranking of circumstances in both social states. This subset of $\mathcal{R}$ corresponds to the intersection of all the sets $\mathcal{R}^{\kappa_{ij}}$, for all pairs (i, j). This is equal to $\mathcal{R}^{\kappa_{\max}}$, where $\kappa_{\max}$ is defined as: $\kappa_{\max} = \max_{i,j\in\{1,\ldots,T\}} \{\kappa_{ij}\}$. Once this set is identified, gap curve dominance can be tested. It is thus possible to get a necessary and sufficient condition for anonymous ezOP over the set of preferences $\mathcal{R}^{\kappa_{\max}}$. This condition is also necessary for anonymous ezOP over $\mathcal{R}$ since $\mathcal{R}^{\kappa_{\max}} \subseteq \mathcal{R}$. This is established in the following proposition.

Proposition 6 *A necessary condition for anonymous ezOP between multiple types over the set of preferences $\mathcal{R}$ is that:*
$\forall p \in [0, 1]$, $\forall (i, j, h, \ell) \in \{1, \ldots, T\}^4$ *such that* $r_0^W(c_i) = r_1^W(c_h)$ *and* $r_0^W(c_j) = r_1^W(c_\ell)$,
$\left|\Gamma\left(\Lambda_0^{\kappa_{\max}}(p|c_i, e), \Lambda_0^{\kappa_{\max}}(p|c_j, e), p\right)\right| \geq \left|\Gamma\left(\Lambda_1^{\kappa_{\max}}(p|c_h, e), \Lambda_1^{\kappa_{\max}}(p|c_\ell, e), p\right)\right|$.

The proof is based on the same arguments used in the proof of proposition 4.

4.2 Aggregation across circumstances

Propositions 5 and 6 require that advantage gaps fall for all possible pairs of circumstances, either ranked or not. However, one might consider that some gaps are more worth compensating than others. For instance, one might assign priority to the bottom of the distribution of types (i.e. reduce the gap between the bottom type and other types) or to the top of the distribution. This would amount, in definition 4, to restrict the scope of inter-type comparisons to pairs involving either the bottom type or the top type. This criterion remains, nevertheless, disaggregated and demanding: it requires to perform a large number of comparisons of pairs of types that must be all validated by all preferences in a sufficiently heterogeneous class.

It might be argued, however, that a small increase in the opportunity gap between two types might be compensated by a fall in the opportunity gap between another pair of types. This view suggests aggregating welfare gaps across pairs of circumstances, i.e. aggregate across types measures of $|W(F_\pi(.|c_i, e)) - W(F_\pi(.|c_j, e))|$. This leads to a scalar measure of inequality of opportunity. Of course, implementing such a scalar measure requires to select a particular preference function W. It also requires to take into account the size of the various types when aggregating welfare gaps. Define p_c the relative frequency of type c in the population. One can define, for a function W, an *Inequality of Opportunity Indicator* (IO):

$$IO(\pi) \ = \sum_{i=1}^{T} \sum_{j=i+1}^{T} p_{c_i}\, p_{c_j} \left|W\left(F_\pi(.|c_i, e)\right) - W\left(F_\pi(.|c_j, e)\right)\right|$$

IO equals the average absolute welfare gap, across all pairs of circumstances, computed for function W. This appears as a generalization of several inequality of opportunity indices suggested in the literature.[13] Lefranc et al. (2008) introduce the *Gini Opportunity* index

[13] Checchi and Peragine (2010) undertake a similar approach. They define, in a ex-post setting with

defined as :

$$GO(\pi) \ = \ \frac{1}{\mu}\sum_{i=1}^{T} \ \sum_{j=i+1}^{T} p_{c_i} \ p_{c_j}\big|\mu_{c_i}(1-G_{c_i}) \ - \ \mu_{c_j}(1-G_{c_j})\big\|.$$

This amounts to take in the evaluation of IO, the function:

$$W\left(F_\pi(.|c,e)\right) \ = \ \frac{\mu_c}{\mu}(1-G_c),$$

where μ_c/μ is the ratio between the average outcome associated to the distribution conditional on circumstance c and the population average, while G_c is the Gini coefficient of circumstance c's distribution.[14]

For every W, the IO index always allows to rank social states, although the conclusion is not robust with respect to the evaluation of advantage. Yet, the index is consistent with an anonymous opportunity equalization criterion: if anonymous ezOP is satisfied, one should have $IO(0) \geq IO(1)$ for all preferences W.

4.3 Aggregation in the effort dimension

Let us now consider a situation where effort can be summarized by a scalar indicator $e \in \mathbb{R}^+$. We refer to the distribution of effort within a type by $G(e|c,\pi)$.

Consider the anonymous or non-anonymous equalization principles. Assume first that effort is realized and observable. This corresponds to what has been referred to in the EOP literature as an *ex post* situation.[15] A straightforward extension of definitions 4 and 3 to the multiple effort setting can be made by requiring equalization to hold at every effort level. With ideal data, the anonymous or non-anonymous equalization criteria can be implemented and separately tested at every effort level.

In most existing data sets, however, information on effort is missing. In this context, it is only possible to observe for each type its outcome distribution, given by :

$$F_\pi(y|c) \quad = \quad \int_E F_\pi(y|c,e)dG(e|c,\pi). \tag{2}$$

In the presence of luck, the distribution of outcome of a given type arises from a *mixture of luck and effort* factors. Hence, contrary to Roemer (1998), it is not possible to identify effort with the quantiles of this distribution.

The ex-ante approach Although they do not allow to test of ex post equalization, the distributions $F_\pi(.|c)$ are interesting in their own right, in order to define equalization of opportunity. Each distribution captures the opportunity sets associated to different types

degenerate luck, indicators of inequality of opportunity. Their indicators measure relative inequality among individual realizations, under the assumption that all individuals in a type exerting similar effort receive similar outcomes. Their indicator aggregates outcome differentials not only across types, but also across effort levels, something that is not necessarily imposed in the $IO(\pi)$ index. A different approach is instead undertaken in Peragine (2002, 2004), where the objects of interest are social evaluation functions and types are ordered. Overall welfare depends on the evaluation of how much dispersed are the average realizations associated to every type.

[14] For a complete survey of Gini-type indices for Equality of Opportunity sets, see Weymark (2003).

[15] See for instance Fleurbaey and Peragine (2013)

in an *ex ante* perspective, i.e. before the effort choices are made. If individuals make equalization judgements without knowing in advance what their effort choice will be, their ex post level of effort could be treated as luck. This amounts to assume that all individuals in a type exert similar effort. One may further assume that effort levels are comparable across types, as discussed below. This comes close in spirit to the analysis of Van de gaer (1993). In this case, equalization should be decided on the basis of the outcome distributions of each type, $F_\pi(y|c)$. This leads to an alternative notion of equalization, which we refer to as ex ante equalization, defined by :

Definition 4 (ex ante non-anonymous ezOP between multiple types) *Moving from social state $\pi = 0$ to $\pi = 1$ equalizes opportunity ex ante over the set of circumstances C on the set of preferences $\mathcal{P}$ if and only if for all preferences $W \in \mathcal{P}$, for all $(i, j) \in \{1, \ldots, T\}$, we have: $|\Delta_W(F_0(.|c_i), F_0(.|c_j))| \geq |\Delta_W(F_1(.|c_i), F_1(.|c_j))|$.*

According to this definition, opportunities are equalized if every preference agrees that the gap between the expected opportunity sets associated to every pair of circumstances falls by effect of the change in social state. Here, opportunity sets are "expected" in the sense that they are evaluated before individuals make their effort choice.

When $\mathcal{P} = \mathcal{R}$, proposition 5 can be used to identify ex ante non-anonymous ezOP. The same approach can be used to derive the anonymous approach.

The Roemerian setting We now consider the special case of the Roemerian setting.[16] In this setting, luck plays no role: individual outcome only depends on circumstances and effort. Individuals with circumstances c and effort e, in social state π are assigned a single value of outcome $Y_\pi(c, e)$. Since luck plays no role, requiring ex post equalization amounts to require that for all (c, c') and all e: $|Y_1(c, e) - Y_1(c', e)| \leq |Y_0(c, e) - Y_0(c', e)|$

Roemer further requires, on a priori grounds, that effort be defined in such a way that its distribution be independent of type. The argument is that since individuals cannot be held responsible for their circumstances, they should not be held accountable for the association between their "effort" and their circumstances. One may in fact push the argument further and require that the distribution of effort be independent of type and social state. In this case, we have that for all c and π, $G(e|c, \pi) = G(e)$. Furthermore, under the assumption that the outcome function $Y(c, e)$ is strictly increasing in e, individual effort, within a type, can be identified by the rank in the type-specific outcome distribution: hence, an individual with outcome y and circumstances π in state π will have exerted effort $F_\pi(y|c)$.

In the Roemerian setting, can thus be normalized to take values in the set $[0, 1]$ and uniformly distributed and $Y_\pi(c, e)$ is simply given by $F_\pi^{-1}(e|c)$. Thus requiring ex post ezOP in this setting amounts to require that, for all $p \in [0, 1]$: $|F_1^{-1}(p|c) - F_1^{-1}(p|c')| \leq |F_0^{-1}(p|c) - F_0^{-1}(p|c')|$. This shows that the absolute gap curve dominance condition, defined in proposition 1 turns out to be a necessary and sufficient condition for ex post ezOP in the Roemerian setting. Furthermore, as a consequence of proposition 1, this condition is necessary for ex ante ezOP. As a result, ex ante ezOP implies ex post ezOP in the Roemerian setting.

[16]For a complete discussion of the conditions of identification of equality of opportunity in Roemer's model, see O'Neill, Sweetman and Van De Gaer (2000) and Lefranc et al. (2009)

The general case In the general case where luck and effort distributions are not degenerate, the relationship between ex ante and ex post equalization cannot be established without further assumptions. This can be illustrated by a simple example. Consider two circumstances, c and c', and many effort levels. Assume that for all effort level, type c dominates c' at the first order. In this case, ex post ezOP requires that for all e, $|F_1(y|c,e) - F_1(y|c',e)| \leq |F_0(y|c,e) - F_0(y|c',e)|$. Assuming further that effort is distributed independently of type and social state, and maintaining our dominance assumption, we have, using (2) : $|F_1(Y|c) - F_1(y|c')| = \int |F_1(y|c,e) - F_1(y|c',e)|dG(e)$. This allows to establish that ex post ezOP implies ex ante ezOP. However, this is only valid under the two maintained assumption. Unfortunately, these assumptions cannot be tested empirically, without observing effort.

This shows that in the general case, ex post equalization cannot be identified using ex ante comparisons. Furthermore, failing to accept ex ante equalization provides little guidance on the fact that the ex post criteria also fails to be accepted, unless one is willing to make non-testable assumption of monotonicity of the ranking of circumstances with respect to effort. The question of identification and separation of luck and effort components requires a more dedicated treatment that we leave for future research.

5 Empirical implementation

We conclude with an empirical illustration of the validity of the ezOP criterion for assessing the potential of different educational policies in equalizing opportunities for income acquisition. Here, the social state indicator corresponds to a policy variable, depicting two situations before/without and after/with the policy implemented.

Before proceeding to the application, we develop an algorithm that allows to test whether the ex ante ezOP condition is satisfied.

5.1 Implementation algorithm

Assume that individual outcome and circumstances are observed for a representative sample of the population. The following algorithm operationalizes the ex ante ezOP criterion. The inference procedure for gap curve dominance are discussed in an appendix, while the inference procedure to test inverse stochastic dominance relations are investigated in Andreoli (2013) and references therein.

The algorithm defines a procedure for comparing *pairs of distributions* made conditional on circumstances c_i, c_j with $i, j \in \{1, \ldots, T\}$. The algorithm can be extended to multiple circumstances considering all pairs i, j, while a similar procedure can be used to test ex post approaches, iterating the algorithm across all effort levels. We denote with the scalar $\kappa_{ij}(\pi)$ the minimal degree of ISD at which the two distributions can be ranked in a given social state π.

Algorithm 1 (Implementable ezOP for two varieties) *The following sequence of estimations and tests implements ezOP:*

(i) $\forall c_i \in C$, $\forall \pi$, *estimate* $F_\pi^{-1}(p|c_i)$ *and its integrals* $\Lambda_\pi^k(p|c_i)$.

(ii) For each (c_i, c_j, π) *with* $i, j \in \{1, \ldots, T\}$ *compute* $\kappa_{ij}(\pi)$ *as follows:*

  (a) Consider $k \in \mathbb{N}_{++}$, *with* $k = 1$ *for the first iteration;*

(b) *Given k, define and test the following pair of null hypothesis:*
$\{H_0 : F_\pi(y|c_i) \succ_{ISDk} F_\pi(y|c_j)$ *vs.* $H_a : F_\pi(y|c_i) \not\succ_{ISDk} F_\pi(y|c_j)\}$
and
$\{H_0 : F_\pi(y|c_j) \succ_{ISDk} F_\pi(y|c_i)$ *vs.* $H_a : F_\pi(y|c_j) \not\succ_{ISDk} F_\pi(y|c_i)\}$.

(c) *Define* $I_k = (a, b)$ *the result of this pair of tests, where* a, b *is equal to 1 if the null hypothesis is rejected and 0 otherwise, respectively for both null hypothesis.*

(d) *Compute* I_k*:*

- *if* $I_k = (0, 0)$*:* $\kappa_{ij}(\pi) = \infty$ - **stop**.
- *if* $I_k = (0, 1)$ *or if* $I_k = (1, 0)$*:* $\kappa_{ij}(\pi) = k$ - **stop**.
- *if* $I_k = (1, 1)$*: let* $k = k + 1$ *and* **iterate** *from step (b).*

(iii) *Define* $\kappa_{ij} := \max_\pi\{\kappa_{ij}(0), \kappa_{ij}(1)\}$.

(iv) *Verify gap curve dominance at order* κ_{ij}*, where* c *and* c' *represent respectively the dominating and dominated distribution out of the pair* c_i, c_j*:*
$\big\{H_0 : \Gamma\big(\Lambda_0^{\kappa_{ij}}(p|c), \Lambda_0^{\kappa_{ij}}(p|c')\big) \geq \Gamma\big(\Lambda_1^{\kappa_{ij}}(p|c), \Lambda_1^{\kappa_{ij}}(p|c)\big) \; \forall p \in [0, 1]$ *and* $H_a : H_0$*is false*$\big\}$.

- *If* $\kappa = 1$*:*
  - *If* H_0 *accepted: ezOP is verified.*
  - *If* H_0 *rejected: inconclusive, ezOP is rejected.*
- *If* $\kappa \geq 2$*:*
  - *If* H_0 *accepted: Necessary conditions for equalization are satisfied.*
  - *If* H_0 *rejected: inconclusive, ezOP is rejected.*

5.2 Application: Evaluation of educational policies in France

We implement the opportunity equalization criterion for evaluating an educational policy expanding accessibility to the secondary education system, and we test if this policy fosters equalization of opportunity. To do so, we go beyond the calculation of *average treatment effects* (Angrist and Krueger 1991, Card 1993), by simulating the impact of such policies on the whole distributions of earnings of the treated group.

Educational policies are often considered by economists and policy makers as the means *par excellence* to equalize opportunities among children with different social and family backgrounds (Meghir and Palme 2005, Björklund and Salvanes 2011) and to promote intergenerational mobility (Hanushek and Woessmann 2011). We focus in particular on a policy aiming at expanding access to secondary and higher education through a compulsory schooling requirement. Expansion affects only a well defined subset of the students population, those that would drop out if the policy is not implemented. We illustrate the French case. For a comprehensive survey and a comparative analysis of the educational policies that took place in Europe in the last 70 years, see Braga, Checchi and Meschi (2011).

5.2.1 The impact of widening access to secondary education

A policy widens accessibility to the secondary education when it provides additional years of schooling to those who would have otherwise dropped out from the system. If the drop out students, those that are more likely to benefit from increasing high school access, are the ones coming from more disadvantaged families, we expect that after policy implementation their earnings profiles should look more similar to the earnings profiles of the students coming from more advantaged backgrounds.

The *simulation* of an increase in high school access is carried out in two steps. The first step consists in estimating the distribution of benefitting from high school participation (the treatment variable) on earnings. The second step consists in treating the income distribution of the *target group*, i.e. the drop out students, with the policy treatment effects estimated in the first step.

5.2.2 First step: identification of the treatment effect of the *Loi Berthoin*

The first step consists in identifying the causal impact of participation to high school on earnings quantiles, that is the *quantile treatment effects* (QTE hereafter) of this educational treatment. We analyze educational profiles and simulate policy implementation within the French case.

In the application discussed here, educational attainment is captured by an indicator D, where $D = 1$ whenever the student's educational attainment is higher than a specified level, here denoting if the students have passed at least one year in the educational system from the age 15 or above. We use $D = 0$ to identify all students who dropped out the educational system just at (or before) the limit minimum schooling age (14 years old).

The simple differences of income distributions conditional on $D = 1$ versus $D = 0$ does not measure the causal effect of being treated with D, since individuals may sort into the treated and non treated group according to observable and unobservables, notably ability or family background characteristics. We rely on an instrumental variable (IV) strategy to estimate causal QTE of education on earnings. Our identification information rests on an exogenous change in the underlying *institutional background* occurring at a certain point in time and captured by an indicator variable Z. This institutional change is represented by the educational reform introduced by the *Loi Berthoin* (1959) in France. Identification rests on a *regression discontinuity design*. With the Berthoin's reform, the students in their 14th year of life born after 1953 who would have dropped out of the school are now compelled to take two additional years of education, presumably during the high school period. Therefore, Z takes value $Z = 1$ for all students born after the cutoff date January 1, 1953 and $Z = 0$ otherwise.[17]

The rationale for Z being an instrument for schooling attainment is that, conditional on the treatment D received, the distribution of potential earnings profiles is independent on shift in education of roughly one year induced by the Berthoin's reform (Card 2001), at least for those cohorts born in proximity of the reform date. The instrument is coherent with the type of unobserved heterogeneity that we would like to control for: ability, family background effects, "hard" and "soft" skills and parent investments are likely to be similarly distributed across adjacent cohorts, while these factors are likely to differ substantially for people self-selecting into different schooling attainment levels. A second condition for identification is that the IV has a causal impact on educational choices. This is granted by the universal coverage of the Berthoin's reform.

With an analogous identification strategy, Grenet (2012) identifies the average returns from age left full time schooling (in years of education) only at the discontinuity, when the Loi Berthoin is introduced. Estimates of the average returns from education reveal

[17]The Loi Berthoin applied to all students born from January 1, 1953 onwards. A description of the changes after the introduction of the law can be found in Grenet (2012).

that age left full time schooling has a very narrow and statistically insignificant impact on earnings. We move beyond average treatment effect to estimate the effect of the treatment on the *whole* distribution of earnings

To retrieve a measure comparable to the effects in Grenet (2012), we trim the estimating sample, considering only individuals with an observed educational level lower than high school diploma. With this operation, we are sure to capture the sole effect of high school participation by differentiating out the earnings profiles associated to individuals that completed the secondary education versus those who did not. Trimming is not problematic in this case, because it allows to preserve the group of compliers, which corresponds to the target of the simulation analysis.

Estimation of QTE is possible through the conditional IV model in Abadie, Angrist and Imbens (2002). We condition the model for a time trend variable (which captures time fixed effects) and other selected covariates such as polynomials of the years left full time schooling.

5.2.3 Second step: simulation of the expansion in high school access

The simulation consists in combining together the estimated QTE and the observed distribution of earnings of the group of marginal students born before 1953, also referred to as the target group. In our applications, the target group can be exactly identified in the data.

The simulation is developed in three stages. In the first stage, we detrend the observed earnings by the impact of the cohort of birth and the time trend and we use these earnings to estimate the empirical distribution of earnings of the target group.

In a second stage, we partition the earning distribution of the treatment group into twenty quantiles and we assign observations to their appropriate quantile. We then treat those in the treatment group in the same quantile with the estimated QTE associated to that quantile. In this way, we obtain the earning distribution they would have had if they were treated with additional years of schooling.

In the third and final stage we construct the earning distributions conditional on circumstances by using the *whole sample*. When the observed values of the target group are used, along with the rest of the observed earnings, we obtain the empirical earning distributions before policy implementation ($\pi = 0$). When the simulated earnings of the target groups are used, along with the observed earnings of the remaining population, we obtain the empirical earning distributions after policy implementation ($\pi = 1$). The equalization test is performed using these distributions.

Finally, we compare the opportunity equalization potential of a policy simulating the expansion in secondary education accessibility with a policy simulating an expansion in the higher education system, extensively treated in Andreoli (2012), Chapter 3.

5.2.4 Data

Educational and labor market outcomes for the cohorts considered in this study can be illustrated using the French Labor Force Survey (LFS, *Enquête Emploi* distributed by INSEE) for the years 1990, 1993, 1996, 1999, 2004, 2006, 2008 and 2010. The sample is a rotating panel, therefore we select only particular years of the survey to preserve exclusively the cross sectional information.[18] The LFS is a large representative sample of the French population of age 15 and above. There are on average 15,000 respondent per cohort in our pooled sample.

The LFS database reports, for each observed individual, the information on the socioeconomic characteristics of the father when the observed individual was a child. We consider four *family background circumstances*: *Circumstance 1* collects the individuals whose father is non French, nearly 9% of the overall sample. The remaining circumstances are obtained by partitioning the sample of those with a French father according to her socioeconomic background, thus giving: *Circumstance 2* if father was a farmer or a manual worker; *Circumstance 3* if the father was an artisan or non-manual worker; *Circumstance 4* if the father was involved in a professional activity.

The LFS sample is restricted to French male workers born between 1950 and 1955, for a total of 26,421 observations, equally distributed across cohorts. As motivated by Grenet (2012), the Berthoin's reform induced a significant increase of roughly one year in age left full time schooling for cohorts born after 1953, with respect to older cohorts. This result is also illustrated in table 4 in the appendix, where differences in education and age of leaving school are significantly different between the treatment and the comparison groups. The proportion of students who received the policy treatment (longer staying in secondary education) is also significantly higher in the treatment group, thus explaining the reduction in the size of the target group (which shrinks from 27% to 16% of the students population). Treatment and control groups are otherwise similar according to a variety to characteristics reported in table 4.

To estimate the quantile treatment effects, we make use of a trimmed sub-sample of 17,779 observations, corresponding to those who at most received an high school diploma (incomes for the subsamples are reported in table 1).

The outcome used to measure opportunities is monthly earnings after taxes. We partition the distribution of earnings in the sample of interest into twenty groups of 5% population mass each and for each of these quantiles we estimate the treatment effects from policy treatment. Selected quantiles of the overall earnings distributions, as well as for distributions made conditional upon treatment groups status (IV) and policy treatment (High/Low education) are reported in table 1 for the sample of cohort 1950 to 1955.

5.2.5 Estimation results

As shown in table 1, the differences between earnings quantiles of treated and non treated observed individuals with higher education, i.e. columns (2) versus (3) and (4) versus (5), are sizable. However, these differences are similar across treatment and control groups. This indicates that the treatment effect that can be identified is low and statistically not significant. This conjecture is verified by the results in table 2. For a selected number of quantiles we report the IV estimates of the quantile treatment effects for the overall sample and for the sub-samples defined by background circumstances. Despite the important share of compliers, it is not possible to identify a significant effect of the educational indicator for population percentiles that range out of the 40% to the 80% quantiles intervals. The

[18] The panel rotation frequency was of three years before 2003 and earnings information are available only after 1990. This explains the choice of the years 1990, 1993, 1996 and 1999. Moreover, the rotation frequency after 2003 changed to one year and a half (that is, one-sixth of the sample is replaced every trimester). Picking up information every two years allows to deal with a renewed sample, as in years 2004, 2006, 2008 and 2010. The year 2002 is not exploited due to imperfections in the data collected.

Table 1: Descriptive statistics: earnings by treatment (IV) and policy treatment

Earnings (Monthly)	Overall	Treatment (IV=1)		Control (IV=0)	
		H-educ	L-educ	H-educ	L-educ
	(1)	(2)	(3)	(4)	(5)
Q5%	426.9	387.2	416.0	450.0	472.6
Q10%	762.2	762.2	686.0	800.4	731.8
Q25%	985.6	990.9	911.1	1,067.1	914.7
Q50%	1,219.6	1,250.0	1,092.8	1,311.1	1,112.9
Q75%	1,550.0	1,585.0	1,402.5	1,676.9	1,402.5
Q90%	2,058.1	2,000.0	2,200.0	2,134.3	1,900.0
Q95%	2,500.0	2,400.0	3,000.0	2,500.0	2,591.6
Mean	1,395.0	1,383.9	1,436.6	1,458.5	1,285.4
	[2,160.3]	[1,977.2]	[3,771.0]	[2,185.2]	[886.9]
Overall sample size	26,421	7,357	6,276	5,513	7,275
Estimation sample size	17,779	7,357	1,785	5,513	3,124

Source: Labor Force Survey 1990, 1993, 1996, 1999, 2004, 2006, 2008 and 2010.

Notes: Trimmed sample reduced to French male earners where circumstances have been recorded, cohorts 1950 to 1955. Income quantiles are measured in Euro. IV is a dummy for cohorts 1953 and 1955. Treatment and comparison are defined upon the IV. *Policy treatment* is determined by access to at least one year of secondary education, with age left at school (H-educ) versus no high school (L-educ). Standard deviations reported between brackets. Sample size for the trimmed sample are reported in the final row (*Estimation*). This sample is used to implement the IV QTE estimator.

QTE remains significant exclusively for the sub-sample delimited by Circumstance 2.

To simulate a policy change, we associate the estimated marginal effects to the individuals in the *target group* according to their positions on the target group income distribution. This procedure is valid since the target group coincides with the *compliers*, i.e. those who have changed their schooling behavior by effect of the Berthoin reform. The density of the target group, as well as other educational groups, across *population* income quantiles is reported in panel (a) of figure 2. Most of the individuals targeted by the simulation are concentrated at the bottom of the distributions and receive a zero treatment from an educational expansion. The same figure shows that a large majority of those treated with positive QTE seem to belong to type *Circumstance 2*. The estimated QTE is identified for the group of marginal students, and the full list of QTE estimates is reported in figure 3(a).

5.2.6 Equalization of Opportunity

The overall effect of a policy granting higher access to secondary education has a very narrow and often non significant impact on future earnings profiles, as shown in figure 3(b). There seems to be, however, no significant amelioration on earnings profiles associated to different circumstances (figure 4).

The outcome of the test for opportunity equalization in the ex ante perspective, based on the Algorithm 1, is illustrated in table 3. We test six comparisons between distinct pairs of circumstances, reported by row, both before policy and after policy simulation. In columns (1) and (2) we report, for each policy and for each pair of circumstances,

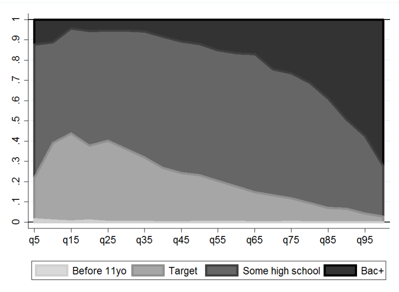
Table 2: Quantile treatment effects, IV estimator

Independent variable: Earnings	Overall	Conditional			
		Circ. 1	Circ. 2	Circ. 3	Circ. 4
	(1)	(2)	(3)	(4)	(5)
Treatment Q5%	49.5	-0.8	48.7	24.7	53.7
	(76.9)	(248.9)	(86.5)	(133.6)	(254.3)
Treatment Q10%	57.5	-47.2	53.4	76.2	1.5
	(61.7)	(245.3)	(63.4)	(139.0)	(243.1)
Treatment Q25%	90.6	-93.8	76.2	104.2	135.1
	(59.1)	(674.5)	(59.0)	(147.8)	(437.1)
Treatment Q50%	142.3**	45.7	126.4**	157.2	7.7
	(58.8)	(720.8)	(62.4)	(162.9)	(333.0)
Treatment Q75%	167.7*	-187.3	179.9	155.8*	-152.4
	(88.3)	(697.5)	(100.3)	(188.1)	(542.8)
Treatment Q90%	167.7	-759.6	228.7	228.7	-457.3
	(165.5)	(1,978.8)	(174.3)	(321.4)	(1,035.8)
Treatment Q95%	157.4	-1,021.4	167.7	213.4	-643.8
	(306.9)	(1,409.6)	(278.5)	(640.9)	(1,145.9)
Controls (reported at Q50%)					
$(cob - 1953)^2$	11.0	-48.0	4.8	29.4	-18.0
	(29.7)	(319.3)	(30.2)	(81.2)	(243.0)
$(cob - 1953)^4$	-0.8	2.2	-0.4	-2.7	0.2
	(2.8)	(29.2)	(2.8)	(8.1)	(25.3)
Circumstance 1	-0.0				
	(179.0)				
Circumstance 3	52.6				
	(45.0)				
Circumstance 4	116.9				
	(144.4)				
Survey year (FE)	yes	yes	yes	yes	yes
Sample size	17,779	981	11,351	3,720	1,727
Compliers (%)	18.1	12.7	21.2	15.8	11.4

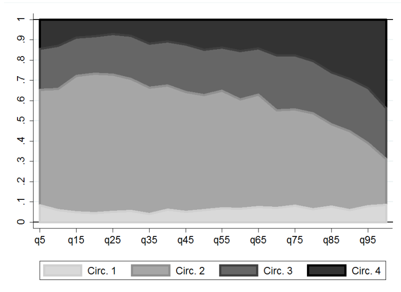
* $p < .10$, ** $p < .05$, *** $p < .01$ (one-tailed).

Source: Labor Force Survey 1990, 1993, 1996, 1999, 2004, 2006, 2008 and 2010.

Notes: Trimmed sample reduced to French male earners where circumstances have been recorded, cohorts 1950 to 1955 and trimmed to the observations with at most high school degree. The dependent variable measures earnings in 1999, once year effect has been eliminated. Robust standard errors are reported in parenthesis.

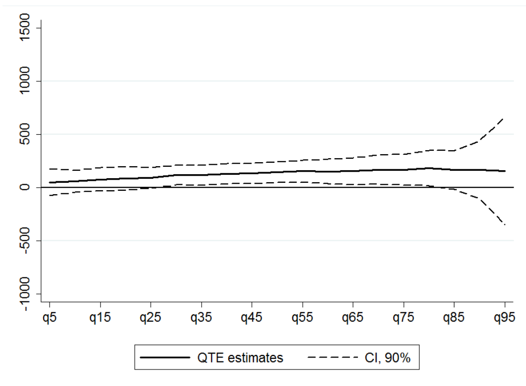


(a) By educational achievement

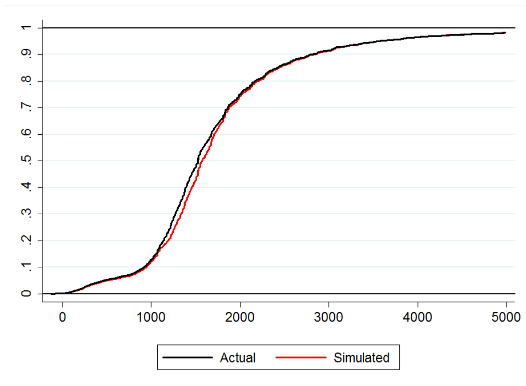


(b) By circumstances

Figure 2: Composition of the population occupying each of the 5% tranches of earnings quantiles, where groups are defined by educational achievement (a) and circumstances (b)
Notes: Scores have been calculated from a multinomial logit model, assigning to each 5% share of population arranged by increasing income, the probability of belonging to each of the groups (these probability add up to 1 for every 5% revenue tranche). In panel (a), the *target* group refers to students between age 11 to 15 who are in junior-high school (*College*). Circumstances are defined according to the father socioeconomic status. qX represent a 5% share of the population between quantile $qX\%$ and $qX\%$-5% in the overall earnings distribution.



(a) QTE, by 5% quantiles



(b) *cdf*s before/after treatment

Figure 3: QTE of the impact of access to secondary education on earnings.
Notes: Estimates based on the cohorts 1950 to 1955 of French male earners (trimmed sample). Cohorts 1953, 1954 and 1955 define the IV, participation to the higher education system is the policy treatment variable. In panel (a), quantile treatment effects are computed at 5% income intervals (IV estimator), the CI at 90% is computed with robust standard errors. Controls: cohort trends, year of survey, a quartic polynomial of the gap between year 1953 and last year spent in school, and circumstance dummies. Empirical *cdf*s in panel (b) are obtained for detrended earnings data (actual) an by providing policy treatment by quantile of earnings for the *marginal students* (simulated).

the direction and the minimal degree of ISD that cannot be rejected by the data at a 5% confidence level. For instance, one has to read the first dominance relation in (1) as $Circ.1 \succ_{ISD1} Circ.2$ (but not the inverse) under $\pi = 0$.

For any pair of circumstances, the direction of the advantage as measured by ISD is unaffected by policy implementation. Circumstance 1 provides an unambiguously higher advantage compared to Circumstance 2, according to ISD1. This result reflects possibly

Table 3: Equalization of Opportunity test: Ordinal and Distance criteria for high school expansion policies.

Circ. c vs Circ. c'	Before policy	After policy	Difference-in-differences	
	$(\pi = 0)$	$(\pi = 1)$	$\Delta_W(F_0^c, F_0^{c'}) - \Delta_W(F_1^c, F_1^{c'})$	Gap dominance k (tested model)*
	(1)	(2)	(3)	(4)
Circ. 2 vs Circ. 1	$\prec_{ISD1}$	$\prec_{ISD1}$	$\geq 0\ \forall W \in \mathcal{R}^1$	> 0 for $k = 1$ (12-12)
Circ. 3 vs Circ. 1	$\succ_{ISD3}$	$\succ_{ISD3}$	$= 0\ \forall W \in \mathcal{R}^3$	$= 0$ for $k = 1$ (31-31)
Circ. 4 vs Circ. 1	$\succ_{ISD1}$	$\succ_{ISD1}$	$= 0\ \forall W \in \mathcal{R}^1$	$= 0$ for $k = 1$ (41-41)
Circ. 3 vs Circ. 2	$\succ_{ISD1}$	$\succ_{ISD1}$	$\geq 0\ \forall W \in \mathcal{R}^1$	> 0 for $k = 1$ (32-32)
Circ. 4 vs Circ. 2	$\succ_{ISD1}$	$\succ_{ISD1}$	$\geq 0\ \forall W \in \mathcal{R}^1$	> 0 for $k = 1$ (42-42)
Circ. 4 vs Circ. 3	$\succ_{ISD1}$	$\succ_{ISD1}$	$= 0\ \forall W \in \mathcal{R}^1$	$= 0$ for $k = 1$ (43-43)

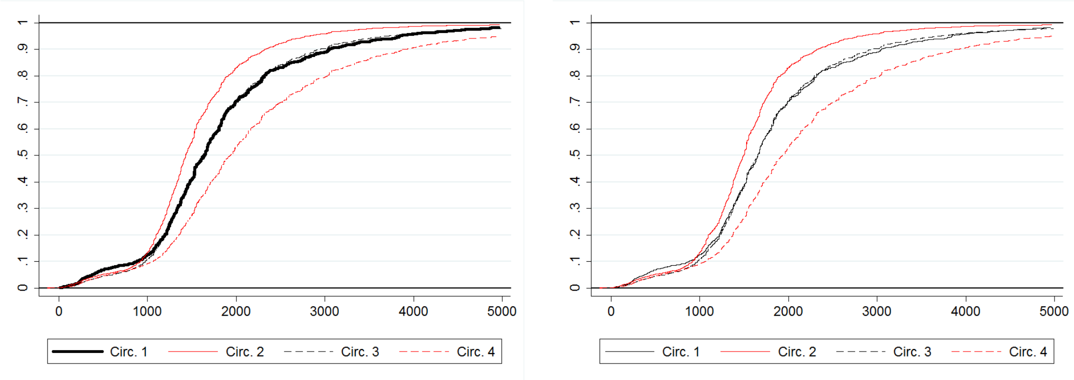
	Opportunity amelioration
	Δ Policy impact
Overall	After $(\pi = 1) \succ_{ISD1}$ Before $(\pi = 0)$
Circumstance 1	After $(\pi = 1) \sim_{ISD1}$ Before $(\pi = 0)$
Circumstance 2	After $(\pi = 1) \succ_{ISD1}$ Before $(\pi = 0)$
Circumstance 3	After $(\pi = 1) \sim_{ISD1}$ Before $(\pi = 0)$
Circumstance 4	After $(\pi = 1) \succ_{ISD1}$ Before $(\pi = 0)$

Source: Estimates from Labor Force Survey 1990, 1993, 1996, 1999, 2004, 2006, 2008 and 2010.

Notes: Earnings distribution corrected by the age effect. Sample reduced to male French earners where circumstances have been recorded, cohorts 1950 to 1955. Circumstances defined by father status: *Circ. 1*: foreign born; *Circ. 2*: French farmer or manual worker; *Circ. 3*: French artisan or non-manual worker; *Circ. 4*: French professional.

EzOP tested at 5% significance level on a selected sample of twenty quantiles. Both inverse stochastic equality and dominance null hypothesis have been separately tested for any degree $k = 1$ up to 5. Only the minimal degree of dominance $\kappa(c, c', \pi)$ is reported. The notation $c \succ_{ISD\kappa} c'$ means that the earnings distribution of circumstance c ISD at order κ the earnings distribution of circumstance c'. The distance test is defined over the class $\kappa(c, c', \pi = 1)$. Direction of the Gap curves dominance is reported, along with information on the direction of distance for the model verifying gap dominance. ISDk for $k = 1, 2$ is estimated as in Beach and Davidson (1983), while for $k \geq 3$ tests are constructed by using the asymptotic estimators of the vector of conditional Gini SWF.

*: For model $(ij - kh)$ we tested the gaps curve of circumstances i vs j in $\pi = 0$ minus the gaps curve of k vs h in $\pi = 1$, exclusively for configurations $k = i$ and $h = j$ or $k = j$ and $h = i$.



(a) Actual data ($\pi = 0$) (b) Simulated data ($\pi = 1$)

Figure 4: Empirical *cdf*s are obtained for earnings data (a) and by treating the target group of 11 to 15 years old students (b).

a substantial heterogeneity in family background for the group of people with non French fathers. The comparison between Circumstance 1 and Circumstance 3 cannot be verified according to ISD1. For the two circumstances, it is necessary to test dominance up to the order three, which is verified both before and after policy intervention. It is nevertheless possible to rank unambiguously the Circumstances 2, 3 and 4 (French father, different socioeconomic classes) according to ISD1 both before and after policy simulation. This result shows that the policy has no impact in reducing agreement over the direction of the disadvantage, nor on changing the direction of disadvantage itself.

In column (3) of table 3 we report the result for the distance comparison. The results for the gap curve dominance tests are reported in column (4) of table 3.[19] Gap curve dominance relations are tested at 5% significance level. The tested model, reported in brackets, gives the order of differentiation of circumstances' earnings distributions under each policy regime, which allows to conclude in favor of dominance in gap curves.[20] Otherwise, alternative models for gap dominance always reject the null hypothesis of equality or dominance even at orders of inverse stochastic dominance higher than one.

We find evidence that the gap curve dominance at the first order cannot be rejected at the 5% confidence level for the pairs of circumstances $\{1, 2\}$, $\{3, 2\}$ and $\{4, 2\}$. The gap curve dominance tests are coherent with the direction of advantage measured by ISD under both policy regimes, although for many comparisons the change in distance is statistically zero (that is, the gap curve coincides with the zero line). This result is coherent with the fact that the simulated policy has no sizable impact on the earnings distribution of Circumstances 1 and 3. The distance between Circumstance 2 and the Circumstances 3 and 4 is reduced by effect of policy simulation, while the distance between Circumstances 1 and 4 remains unaffected. This result is consistent with the fact that an expansion of the secondary education system provides benefits for students coming from more disadvantaged

[19]For a given model, we report the minimal order at which it is not possible to reject, with a confidence of 5%, that the gap curve generated by that model is either statistically equal to zero, or it always lie above the zero line for all the considered quantiles.

[20]For instance, the model associated to circumstances Circ. 2 and Circ. 1 is $(12 - 12)$, which means that to find dominance in Gap curves at order one it is necessary to take the difference of the earnings distribution of Circumstance 1 minus the earnings distribution of Circumstance 2 both under policy $\pi = 0$ and policy $\pi = 1$.

backgrounds. The policy does not have a statistical impact on the distribution associated to Circumstance 1. As a result, the gap between Circumstance 1 and Circumstance 2 decreases at order one by effect of the policy.

We conclude that under the assumption of the rank dependent model for preferences, the ex ante ezOP criterion is validated by the data, although there is no apparent change in consensus due to policy simulation. Thus, we conclude that the policy aimed at increasing participation in the high school system equalizes opportunities in the sense of ex post ezOP.

5.2.7 A comparison with other educational policies

The expansion of high school access produces very low average treatment effects on students earnings, and it has limited redistributive effects on their earnings distribution. However, there is evidence that the policy equalizes opportunities among groups of students defined by family background circumstances. Andreoli (2012) analyzes the impact of an expansion of access to university on the earnings French workers, using the quasi-natural experiment induced by the events of the *May 1968* on the relaxation of university admission rules (see Maurin and McNally 2008). The average impact on future earnings of such events are sizable, as are the overall distributional effects. However, the simulated expansion in university admission seems to disequalize opportunities: the ex ante equalization criterion is rejected and therefore ezOP must be rejected as well. This is so because the groups that are more advantage at the beginning are the ones who benefit more from policy implementation.

We speculate that the increase in accessibility to the educational system is more effective in equalizing opportunities if the adequate reforms take place early in the students careers. This result suggest that equalization of opportunity objectives do not contrast efficiency motivations in public provision of educational services. There is growing evidence (Cunha, Heckman and Lochner 2006, Cunha and Heckman 2007) that it is cheaper and more efficient for the society to compensate disadvantaged individuals/groups early in their educational career rather than to provide late intervention measures. Our results show that the gains from early intervention mostly affect those in the center of the distribution, while leaving unchanged the tails of the earnings distribution. However, we find that this allocation of gains from policy along the earnings curve promotes opportunity equalization. We leave for future investigations the assessment of the opportunity equalizing impact of policies that compensate disadvantaged students at the beginning of their educational career, such as kindergarten expansion policies, *vis à vis* a more traditional cost-benefit analysis or opportunity amelioration comparisons.

6 Conclusions

In this article we propose an innovative criterion for evaluation of public polices, that builds on the notion of equality of opportunity in Lefranc et al. (2009). The opportunity equalization criterion entails a difference-in-differences type of comparison between distribution functions conditional on effort levels.

In a first stage, differences are taken across distributions *within* each policy regime separately, in order to exploit the direction and distribution of the economic advantage among pairs of types outcomes distributions. This is done by imposing sequential restrictions on a class of evaluation preferences until agreement is reached in assessing the disadvantaged circumstance and it is implemented making use of stochastic orderings.

In a second stage, we compare differences across circumstances *between* policy regimes. We propose an innovative model based on comparisons of the changes in the ethic distance between pairs of distributions, which incorporates unanimity in the evaluation of the fall in the illegitimate advantage enjoyed by one circumstance with respect to other due to policy implementation. We study identification procedures and implementation issued, showing the equivalence of distance comparisons with gap curves dominance. Inference procedures are also provided.

Finally, our contribution is in the empirical literature. We evaluate two alternative simulated policies. Both policies are supposed to widen access to the educational system, although they take place in different periods of the students educational career. A policy that widens access to the educational system early in life seems to have a very mild impact on future students' earnings, although these effects are distributed in such a way that opportunities are equalized in the sense of the ezOP criterion. We let for further investigations the impact of other types of policies, such as kindergarten expansion, that take place very early in the educational career of individuals, if not before. Research in this field would provide additional information on hidden benefits of such policies that are often overlooked by traditional cost-benefit methods for policy evaluation.

References

Aaberge, R. (2006). Asymptotic distribution theory of empirical rank-dependent measures of inequality, *in* V. Nair (ed.), *Advances in Statistical Modeling and Inference – Essays in Honor of Kjell A. Doksum*, World Scientific, chapter 5.

Aaberge, R. (2009). Ranking intersecting lorenz curves, *Social Choice and Welfare* **33**: 235–259.

Abadie, A., Angrist, J. and Imbens, G. (2002). Instrumental variables estimates of the effect of subsidized training on the quantiles of trainee earnings, *Econometrica* **70**(1): 91–117.

Almås, I., Cappelen, A. W., Lind, J. T., Sørensen, E. . and Tungodden, B. (2011). Measuring unfair (in)equality, *Journal of Public Economics* **95**(7-8): 488–499.

Andreoli, F. (2012). *On Dissimilarity and Opportunity Equalization*, PhD thesis, Université de Cergy-Pontoise.

Andreoli, F. (2013). Inference for inverse stochastic dominance, *Technical report*, University of Verona.

Angrist, J. D. and Krueger, A. B. (1991). Does compulsory school attendance affect schooling and earnings?, *The Quarterly Journal of Economics* **106**(4): pp. 979–1014.

Atkinson, T. B. (1970). On the measurement of inequality, *Journal of Economic Theory* **2**: 244–263.

Beach, C. M. and Davidson, R. (1983). Distribution-free statistical inference with Lorenz curves and income shares, *The Review of Economic Studies* **50**(4): 723–735.

Björklund, A. and Salvanes, K. G. (2011). *Education and Family Background: Mechanisms and Policies*, Vol. 3 of *Handbook of the Economics of Education*, Elsevier, chapter 3, pp. 201–247.

Bourguignon, F., Ferreira, F. H. G. and Menendez, M. (2007). Inequality of opportunity in Brasil, *Review of Income and Wealth* **53**(4): 585–618.

Braga, M., Checchi, D. and Meschi, E. (2011). Institutional reforms and educational attainment in Europe: A long run perspective, *IZA Discussion Papers 6190*, Institute for the Study of Labor (IZA).

Card, D. (1993). Using geographic variation in college proximity to estimate the return to schooling, *NBER Working Papers 4483*, National Bureau of Economic Research, Inc.

Card, D. (2001). Estimating the return to schooling: Progress on some persistent econometric problems, *Econometrica* **69**(5): 1127–60.

Chakravarty, S. R. and Dutta, B. (1987). A note on measures of distance between imcome distributions, *Journal of Economic Theory* **41**(1): 185 – 188.

Checchi, D. and Peragine, V. (2010). Inequality of opportunity in Italy, *Journal of Economic Inequality* **8**: 429–450.

Cunha, F. and Heckman, J. (2007). The technology of skill formation, *American Economic Review* **97**(2): 31–47.

Cunha, F., Heckman, J. J. and Lochner, L. (2006). *Interpreting the Evidence on Life Cycle Skill Formation*, Vol. 1 of *Handbook of the Economics of Education*, Elsevier, chapter 12, pp. 697–812.

Dworkin, R. (1981). What is equality? Part 1: Equality of welfare, *Philosophy & Public Affairs* **10**(3): pp. 185–246.

Ebert, U. (1984). Measures of distance between income distributions, *Journal of Economic Theory* **32**(2): 266 – 274.

Ferreira, F. H. G. and Gignoux, J. (2011). The measurement of inequality of opportunity: Theory and an application to latin america, *Review of Income and Wealth* **57**(4): 622–657.

Fleurbaey, M. (2008). *Fairness, Responsability and Welfare*, Orxford University Press.

Fleurbaey, M. and Peragine, V. (2013). Ex ante versus ex post equality of opportunity, *Economica* **80**(317): 118–130.

Gastwirth, J. L. (1971). A general definition of the Lorenz curve, *Econometrica* **39**(6): 1037–1039.

Grenet, J. (2012). Is it enough to increase compulsory schooling to raise earnings? Evidence from French and British compulsory schooling laws, *Scandinavian Journal or Economics* **forthcoming**.

Hanushek, E. A. and Woessmann, L. (2011). *The Economics of International Differences in Educational Achievement*, Vol. 3 of *Handbook of the Economics of Education*, Elsevier, chapter 2, pp. 89–200.

Kodde, D. A. and Palm, F. C. (1986). Wald criteria for jointly testing equality and inequality restrictions, *Econometrica* **54**(5): pp. 1243–1248.

Lefranc, A., Pistolesi, N. and Trannoy, A. (2008). Inequality of opportunities vs. inequality of outcomes: Are western societies all alike?, *Review of Income and Wealth* **54**(4): 513–546.

Lefranc, A., Pistolesi, N. and Trannoy, A. (2009). Equality of opportunity and luck: Definitions and testable conditions, with an application to income in France, *Journal of Public Economics* **93**(11-12): 1189 – 1207.

Maccheroni, F., Muliere, P. and Zoli, C. (2005). Inverse stochastic orders and generalized Gini functionals, *METRON* **63**(3): 529–559.

Magdalou, B. and Nock, R. (2011). Income distributions and decomposable divergence measures, *Journal of Economic Theory* **146**(6): 2440 – 2454.

Maurin, E. and McNally, S. (2008). Vive la révolution! Long-term educational returns of 1968 to the Angry Students, *Journal of Labor Economics* **26**(1): pp. 1–33.

Meghir, C. and Palme, M. (2005). Educational reform, ability, and family background, *American Economic Review* **95**(1): 414–424.

Muliere, P. and Scarsini, M. (1989). A note on stochastic dominance and inequality measures, *Journal of Economic Theory* **49**(2): 314 – 323.

O'Neill, D., Sweetman, O. and Van De Gaer, D. (2000). Equiality of opportunity and kernel density estimation: An application to intergenerational mobility, *Advances in Econometrics : A research Annual* **14**: 259–274.

Peragine, V. (2002). Opportunity egalitarianism and income inequality, *Mathematical Social Sciences* **44**(1): 45 – 64.

Peragine, V. (2004). Ranking income distributions according to equality of opportunity, *Journal of Economic Inequality* **2**: 11–30.

Ramos, X. and Van de gaer, D. (2012). Empirical approaches to inequality of opportunity: Principles, measures, and evidence, *Technical report*, ECINEQ WP 2012 – 259.

Roemer, J. (1998). *Equality of Opportunity*, Harvard University Press, Cambridge.

Roemer, J. E., Aaberge, R., Colombino, U., Fritzell, J., Jenkins, S. P., Lefranc, A., Marx, I., Page, M., Pommer, E., Ruiz-Castillo, J., Segundo, M. J. S., Tranaes, T., Trannoy, A., Wagner, G. G. and Zubiri, I. (2003). To what extent do fiscal regimes equalize opportunities for income acquisition among citizens?, *Journal of Public Economics* **87**(3–4): 539 – 565.

Shorrocks, A. F. (1982). On the distance between income distributions, *Econometrica* **50**(5): pp. 1337–1339.

Shorrocks, A. F. (1983). Ranking income distributions, *Economica* **50**(197): pp. 3–17.

Van de gaer, D. (1993). *Equality of Opportunity and Investment in Human Capital*, PhD thesis, Katholieke Universiteit Leuven.

Weymark, J. A. (2003). Generalized gini indices of equality of opportunity, *Journal of Economic Inequality* **1**: 5–24. 10.1023/A:1023923807503.

Yaari, M. E. (1987). The dual theory of choice under risk, *Econometrica* **55**(1): pp. 95–115.

Zheng, B. (2002). Testing Lorenz curves with non-simple random samples, *Econometrica* **70**(3): 1235–1243.

Zoli, C. (2002). Inverse stochastic dominance, inequality measurement and Gini indices, *Journal of Economics* **9**: 119–161.

A Definitions and proofs

A.1 Notions of stochastic dominance

Following Gastwirth (1971), the integral function

$$GL_\pi(p|c,e) \;=\; \int_0^p F_\pi^{-1}(t|c,e)\,dt$$

defines the *Generalized Lorenz curve* (GL) of the distribution $F_\pi(y|c,e)$. The integral condition of order k constructed from the GL curve, $\Lambda_\pi^k(p|c,e)$, can be defined recursively as illustrated in the main text.

Muliere and Scarsini (1989) introduced the *inverse stochastic dominance* partial order $\succcurlyeq_{ISDk}$ as a criterion to rank pairs of distributions, inducing agreement among all preferences in $\mathcal{R}^k$ over the preferred distribution. Dominance can be empirically assessed by comparing $\Lambda_\pi^k(p|c,e)$ distributions. Furthermore, Maccheroni, Muliere and Zoli (2005) show that if $F_\pi(y|c,e) \succcurlyeq_{ISDk} F_\pi(y|c',e)$ then $F_\pi(y|c,e) \succcurlyeq_{ISDl} F_\pi(y|c',e)$, for all $l > k$. It follows that GL dominance is sufficient for any other inverse dominance comparison.[21]

To make sure that the equalization test we devise is well defined and always testable when $\mathcal{P} = \mathcal{R}^k$, we show that there always exists a degree k at which any pair of distributions can be compared according to ISD. The dominating distribution is the one that grants higher incomes to the poorest quantiles.

Proposition 7 *For any pair of distributions with bounded support, with inverse cumulative distribution functions denoted by $F^{-1}(.)$ and $F'^{-1}(.)$ satisfying:*

$\exists p_\beta > 0 | \forall p \in [0, p_\beta)$ $F^{-1}(p) \geq F'^{-1}(p)$ *and the strict inequality holds on a positive mass interval $[p_\beta - \epsilon, p_\beta)$ with $\epsilon > 0$,*
we have:

$\exists \kappa \in \mathbb{R}_+$ *and finite such that* $F \succ_{ISDk} F'$ $\forall k \in \mathbb{N}_+$ *such that* $k > \kappa$.

Proof. The proof consists in showing that if $F(y)$ inverse stochastically dominates $F'(y)$ at the first order for some positive percentiles between 0 and $p_\beta > 0$, then we have a sufficient condition for the two distribution to be comparable at a finite degree of integration k^*.

Define $\Delta F^{-1}(p) := F^{-1}(p) - F'^{-1}(p)$ and $\Delta\Lambda^k(p) := \Lambda^k(p) - \Lambda'^k(p)$ at any $p \in [0,1]$. Integrate by part up to $k-2$ times the function $\Delta\Lambda^k(p)$ to obtain the following:

$$\begin{aligned}
\Delta\Lambda^k(p) &= \int_0^p \Delta\Lambda^{k-1}(t)dt = -\int_0^p t\cdot\Delta\Lambda^{k-2}(t)dt + \Big[t\Delta\Lambda^{k-1}(t)\Big]_0^p \\
&= \int_0^p (p-t)\Delta\Lambda^{k-2}(t)dt \\
&= \int_0^p \frac{1}{2}(p-t)^2\Delta\Lambda^{k-3}(t)dt + \left[\frac{1}{2}(p-t)^2\Lambda^{k-2}(t)\right]_0^p \\
&= \int_0^p \frac{1}{(k-2)!}(p-t)^{k-2}\Delta F^{-1}(t)dt \qquad (3)
\end{aligned}$$

[21] It is well known (e.g. Muliere and Scarsini 1989) that first and second order inverse stochastic dominance are equivalent to direct first and second order stochastic dominance, which is in turn equivalent to generalized Lorenz dominance for incomes distributions with different means (Shorrocks 1983). Atkinson (1970) showed the logical relation between GL dominance with fixed means and an the utilitarian social welfare function, later generalized to all S-concave social welfare functions and to income distributions with different means.

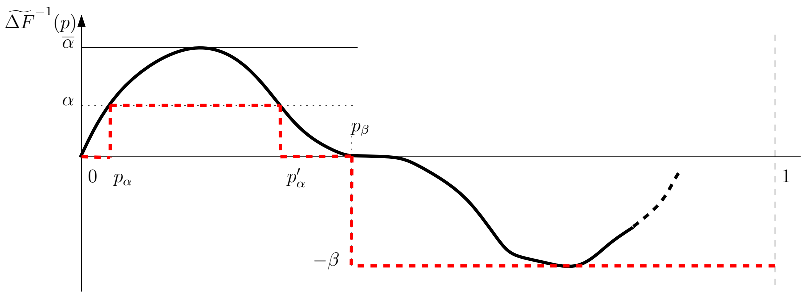


Figure 5: Proof of Proposition 7. The curves $\Delta F^{-1}(p)$ (solid black) and $\widetilde{\Delta F}^{-1}(p)$ (dashed red)

To see the result in (3) it is sufficient to note that $\Lambda^k(0) = 0$ and therefore $\Delta\Lambda^k(0) = 0$ for any k, and that $\Delta\Lambda^2(p) = \int_0^p \Delta F^{-1}(t)dt$.

The sufficient conditions of the proposition states that $\Delta F^{-1}(p) \geq 0$ for all $p \in [0, p_\beta)$ and there exists a p such that the strong inequality holds. As long as we use continuous or at most left inverse cumulative distribution functions, we make sure that the function $\Delta F^{-1}(p)$ is well behaved on the whole percentile domain. Moreover, the function takes only finite values even in $p = 1$ or $p = 0$. As a consequence the value p_β exists.

Moreover, consider the two bounds values $\overline{\alpha} := \sup\{\Delta F^{-1}(p) : p \in [0, p_\beta)\} > 0$ and $-\beta := \inf\{\Delta F^{-1}(p) : p \in [p_\beta, 1]\} < 0$, that corresponds respectively to the largest positive and negative horizontal distance between two distributions. They both exist finite, provided that the sufficient conditions given above are satisfied. [22] The curve of ΔF^{-1} is marked with a solid lines on the graph in figure 5, along with the corresponding values of α and $-\beta$.

Let $0 < \alpha \leq \overline{\alpha}$ such that it is possible to define *at least* two points $p_\alpha, p'_\alpha \in [0, p_\beta)$, such that for $p_\alpha \leq p \leq p'_\alpha$, $\Delta F^{-1}(p) > 0$ holds. Consequently, we define the new differences curve $\widetilde{\Delta F}^{-1}(p)$ in the following way:

$$\widetilde{\Delta F}^{-1}(p) := \begin{cases} 0 & \text{if } p \in [0, p_\alpha) \\ \alpha & \text{if } p \in [p_\alpha, p'_\alpha] \\ 0 & \text{if } p \in (p'_\alpha, p_\beta) \\ -\beta & \text{if } p \in [p_\beta, 1] \end{cases}$$

The curve is represented by the dashed line in figure 5. It is not difficult to see that α and $-\beta$ are defined by the distribution functions, while it always hold that $\widetilde{\Delta F}^{-1}(p) \leq \Delta F^{-1}(p)$ for all p. As a consequence, also the value of the integrals of $\widetilde{\Delta F}^{-1}(p)$ lie always below the value of the integral of $\Delta F^{-1}(p)$ calculated in p. The function reduces the positive domain of the difference $\Delta F^{-1}(p)$ for percentiles in the lower side of the domain, while it magnify the negative effect of the difference for the percentiles in the remaining side of the domain. Therefore, making use of (3), if it is possible to find a value of $\widetilde{k}^*$ such that $\forall k > \widetilde{k}^*$:

$$\int_0^p \frac{1}{(k-2)!}(p-t)^{k-2}\widetilde{\Delta F}^{-1}(t)dt \geq 0 \quad \forall p \in [0, 1], \tag{4}$$

[22] If the conditions do not hold we have either that type c' dominates type c or type c dominates on the first order type c'

then there must exists also a value k^* satisfying our proposition (that is inverse stochastic dominance at a finite order is always granted).

Not that in the interval $[0, p_\alpha)$ and (p'_α, p_β) the expression (4) is always zero. Moreover, (4) is always strictly positive on the interval domain $[p_\alpha, p'_\alpha]$. It remains to check the condition for any $p \geq p_\beta$.

$$\begin{aligned}\int_0^p \frac{(p-t)^{k-2}}{(k-2)!} \widetilde{\Delta F}^{-1}(t)dt &= \frac{1}{(k-2)!}\left[\int_{p_\alpha}^{p'_\alpha}(p-t)^{k-2}\alpha dt + \int_{p'_\alpha}^{p_\beta}(p-t)^{k-2}(-\beta)dt\right] \\ &= \frac{\left\{\alpha\left[(p-p_\alpha)^{k-1}-(p-p'_\alpha)^{k-1}\right]-\beta(p-p_\beta)^{k-1}\right\}}{(k-2)!} \geq 0 \ \forall p \geq p_\beta.\end{aligned}$$

To check the solution it suffice that there exists a $\widetilde{k}^*$ such that:

$$\frac{(p-p_\alpha)^{k-1}-(p-p'_\alpha)^{k-1}}{(p-p_\beta)^{k-1}} \ \geq \ \frac{\beta}{\alpha}, \quad \forall p \geq p_\beta. \tag{5}$$

By construction of $\widetilde{\Delta F}^{-1}(p)$, if the condition holds for $p = 1$, then it must hold for all $p < 1$, because the differential takes only negative values for $p \geq p_\beta$. Note that the numerator and denominator of the left hand side of (5) are positive, but the ratio is not said to be greater than one. Nevertheless, one can always pick up a value of $\alpha < \overline{\alpha}$ such that $(p - p'_\alpha) \approx (p - p_\beta)$ and (5) is therefore satisfied if and only if the following holds:

$$\left(\frac{1-p_\alpha}{1-p_\beta}\right)^{k-1} \ \geq \ 1+\frac{\beta}{\alpha}. \tag{6}$$

Both sides of (6) are positives and greater than one. Thus, by taking logs on the left and right side, it is easy to show that the integral condition in (3) is satisfied if and only if the integration order $\widetilde{k}^*$ is large enough to verify:

$$\widetilde{k}^* \ \geq \ 1+\frac{\ln(1+\beta/\alpha)}{\ln(1-p_\alpha)-\ln(1-p_\beta)}.$$

Note that $\widetilde{k}^*$ is positive and greater than one and it always exists *finite* for any $0 < p_\alpha < p_\beta < 1$ and for $\alpha, \beta > 0$. Therefore the value k^* exists as well, which concludes the proof. ■

A.2 Proof of Proposition 1

Proof. By contradiction.

Assume $\exists \tilde{p} \in]0, 1[$ such that $|\Gamma(F_1, F'_1, \tilde{p})| > |\Gamma(F_0, F'_0, \tilde{p})|$.

For $\pi = 0, 1$, $\Gamma(F_\pi, F'_p i, p)$ is left continuous since F_π and F'_π are left continuous. Hence, $\exists \epsilon > 0$ such that $\forall p \in [\tilde{p} - 2\epsilon, \tilde{p}]$, $|\Gamma(F_1, F'_1, p)| > |\Gamma(F_0, F'_0, p)|$ and $\text{sign}(\Gamma(F_\pi, F'_\pi, p)) = \text{sign}(\Gamma(F_\pi, F'_\pi, \tilde{p}))$

Consider the individual preferences $\tilde{W}$ given by the triangular weighting scheme over the interval $[\tilde{p} - 2\epsilon, \tilde{p}]$: $\forall p \in [0, 1]$, $\tilde{w}(p) = [(\epsilon - |p - (\tilde{p} - \epsilon)|)/\epsilon^2].\mathbb{1}_{p\in[\tilde{p}-2\epsilon,\tilde{p}]}$, where $\mathbb{1}$ denotes the indicator function.

For preferences $\tilde{W}$, the economic distance in social state π is given by :
$|\Delta_{\tilde{W}}(F, F')| = \int_{\tilde{p}-2\epsilon}^{\tilde{p}} \tilde{w}(p)|\Gamma(F, F', p)|dp$.

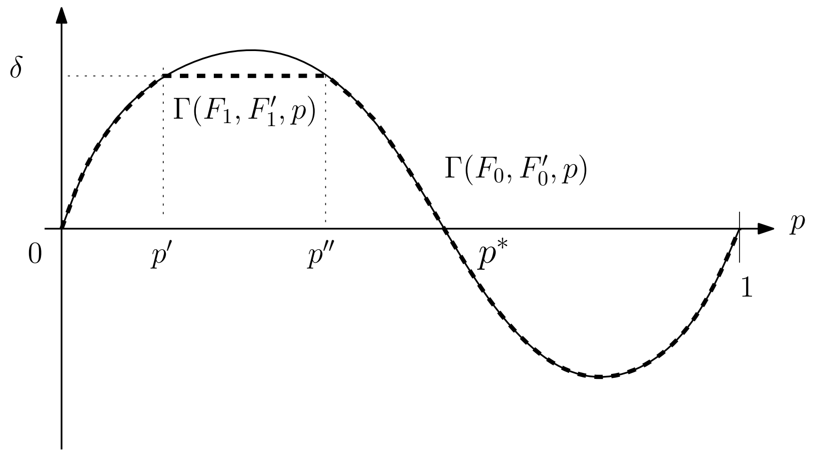


Figure 6: The curves $G(F_0, F_0', p)$ (solid line) and the perturbation generating $G(F_1, F_1', p)$ (dashed line).

Henceforth $|\Delta_{\tilde{W}}|(F_1, F_1')| > |\Delta_{\tilde{W}}(F_0, F_0')|$ which violates ezOP. ■

The reciprocal is not true. Figure 6 provides a counter-example. The plain line gives the gap curve under $\pi = 0$. At value $\bar{p}$, the curve crosses the horizontal axis. Hence, under $\pi = 0$, type c receives higher outcomes than type c' in the bottom of the distribution but lower outcomes in the top. Define the areas $A = \int_0^{\bar{p}} \Gamma(F_0, F_0', p)dp$ and $B = -\int_{\bar{p}}^1 \Gamma(F_0, F_0', p)dp > 0$. Now consider the weighting scheme that gives weight $\alpha \geq 0$ for percentiles below $\bar{p}$ and $\beta \geq$ above.[23] The economic distance for this weighting scheme is $|\alpha A - \beta B|$. For α close enough to zero, type c' is preferred to type c and the distance is given by $\beta B - \alpha A$. Under $\pi = 1$, the gap curve is given by the dashed line, which is similar to the plain line except that the advantage of type c has been reduced by a small cumulative amount ϵ in the bottom part of the distribution, so that $\int_0^{\bar{p}} \Gamma(F_1, F_1', p)dp = A - \epsilon < A$. Gap curve dominance is obviously satisfied. At the same time, individuals who initially preferred the distribution of type c' to that of type c will agree that the economic distance between type c and c' has increased and ezOP is thus violated.

A.3 Proof of Proposition 2

Proof. For the *sufficiency* part, assume that $F_0 \succcurlyeq_{ISD1} F_0'$ and $F_1 \succcurlyeq_{ISD1} F_1'$. As a consequence:

$$\forall W \in \mathcal{R}, \forall \pi \int_0^1 w(p)F_\pi^{-1}(p)dp > \int_0^1 w(p)F_\pi'^{-1}(p)dp.$$

Consequently, for all $W \in \mathcal{R}$, we can write:

$$\Delta_W(F_\pi, F_\pi') = \int_0^1 w(p)\Gamma(F_\pi, F_\pi', p)dp.$$

Hence, we have:

$$\Delta_W(F_0, F_0') - \Delta_W(F_1, F_1') = \int_0^1 w(p)[\Gamma(F_0, F_0', p) - \Gamma(F_1, F_1', p)]dp. \tag{7}$$

[23]This weighting scheme is given by $w(p) = \alpha + (\beta - \alpha).\mathbb{1}_{p>\bar{p}}$, with $\alpha, \beta \geq 0$ and $\alpha\bar{p} + \beta(1 - \bar{p}) = 1$.

If $[\Gamma(F_0, F_0', p) - \Gamma(F_1, F_1', p)] \geq 0$ for all p, since the weights $w(p)$ are non-negative, the integrand in equation (7) is positive for all p and the integral is positive.

For the *necessity* part, note that if $[\Gamma(F_0, F_0', p) - \Gamma(F_1, F_1', p)]$ is negative in the neighborhood of a quantile p_0, we can find a weight profile $w(p)$ that is arbitrarily small outside this neighborhood and it makes the integral negative. ■

A.4 Proof of Proposition 3

Proof. We use the same type of proof argument as in Aaberge (2009). As a consequence of the dominance hypothesis, we have:

$$\forall W \in \mathcal{R}^2, \forall \pi \int_0^1 w(p) F_\pi^{-1}(p) dp > \int_0^1 w(p) F_\pi'^{-1}(p) dp.$$

Consequently, for all $W \in \mathcal{R}^2$, we can write:

$$\Delta_W(F_\pi, F_\pi') = \int_0^1 w(p) \Gamma(F_\pi, F_\pi', p) dp.$$

Hence, $\forall W \in \mathcal{R}^2$ we have:

$$\Delta_W(F_0, F_0') - \Delta_W(F_1, F_1') = \int_0^1 w(p) [\Gamma(F_0, F_0', p) - \Gamma(F_1, F_1', p)] dp. \tag{8}$$

It is possible to integrate (8) by parts once,

$$\begin{aligned}\Delta_W(F_0, F_0') - \Delta_W(F_1, F_1') &= w(1) \int_0^1 \left[\Gamma(F_0, F_0', p) - \Gamma(F_1, F_1', p)\right] \\ &+ \int_0^1 (-1) w'(p) \int_0^p \left[\Gamma(F_0, F_0', t) - \Gamma(F_1, F_1', t)\right] dt dp\end{aligned}$$

By $W \in \mathcal{R}^2$ then $w(1) = 0$ and the first term disappears. By $w'(p) \leq 0$ for all p makes $\int_0^p \left[\Gamma(F_0, F_0', t) - \Gamma(F_1, F_1', t)\right] dt$ sufficient for (8). Moreover, Lemma 1 in Aaberge (2009) gives the necessary part. ■

A.5 Proof of Proposition 4

Proof. We use the same type of proof argument as in Aaberge (2009). As a consequence of the dominance hypothesis, we have:

$$\forall W \in \mathcal{R}^k, \forall \pi \int_0^1 w(p) F_\pi^{-1}(p) dp > \int_0^1 w(p) F_\pi'^{-1}(p) dp.$$

Consequently, for all $W \in \mathcal{R}^k$, we can write:

$$\Delta_W(F_\pi, F_\pi') = \int_0^1 w(p) \Gamma(F_\pi, F_\pi', p) dp.$$

Hence, $\forall W \in \mathcal{R}^k$ we have:

$$\Delta_W(F_0, F'_0) - \Delta_W(F_1, F'_1) = \int_0^1 w(p)[\Gamma(F_0, F'_0, p) - \Gamma(F_1, F'_1, p)]dp. \tag{9}$$

It is possible to integrate (9) by parts k times,

$$\begin{aligned}\Delta_W(F_0, F'_0) - \Delta_W(F_1, F'_1) \;\; = \;\; & w(1)\int_0^1 \left[\Gamma(F_0, F'_0, p) - \Gamma(F_1, F'_1, p)\right] \\ & + \sum_{j=1}^{i} (-1)^j \frac{d^j w(1)}{dp^j}\left[\Gamma(\Lambda_0^k, \Lambda_0'^k, 1) - \Gamma(\Lambda_1^k, \Lambda_1'^k, 1)\right] \\ & + (-1)^i \int_0^1 \frac{d^i w(p)}{dp^i}\left[\Gamma(\Lambda_0^k, \Lambda_0'^k, p) - \Gamma(\Lambda_1^k, \Lambda_1'^k, p)\right] dp.\end{aligned}$$

By $W \in \mathcal{R}^k$ then $w(1) = 0$ and $\frac{d^j w(1)}{dp^j} = 0$ for all $j \leq i$ and the first term disappears. Thus follows that the conditions for $W \in \mathcal{R}^k$ makes $\left[\Gamma(\Lambda_0^k, \Lambda_0'^k, 1) - \Gamma(\Lambda_1^k, \Lambda_1'^k, 1)\right] \geq 0$ sufficient for (9). Moreover, Lemma 1 in Aaberge (2009) gives the necessary part. ■

B Statistical inference for gap curve dominance

B.1 Setting and null hypothesis

Consider a sample $y_1, y_2, \ldots, y_n$ of n draws from a random variable Y with distribution F. Let assume for simplicity that $y_1 \leq y_2 \leq \ldots \leq y_n$, so that y_i refers to the observation in position i in the ranking. The empirical distribution for the sample is denoted $\widehat{F}(y) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}(y_i \leq y)$ while the empirical quantile function is denoted $\widehat{F}^{-1}(p) = \inf\{y : \widehat{F}(y) \geq p\}$. If $\widehat{F}$ is a consistent estimator for F, then $\widehat{\Lambda}^k$ is a consistent estimator for Λ^k.

The empirical counterparts of the distributions F_π and F'_π, corresponding to circumstances c and c', are denoted $\widehat{F}_\pi$ and $\widehat{F'}_\pi$ respectively, where in general $n_{c,\pi} \neq n_{c',\pi}$. Andreoli (2013) discusses the use of different inference procedures for assessing ISD relations at order k among distributions F_π and F'_π. His results can be used here to test for dominance in Gap curves.[24]

Whenever $F_\pi \succ_{ISDk} F_\pi$ for all π, the gap curves differences are well defined and gap curves dominance and equality null hypothesis can be stated by setting conditions on the realizations of $\Lambda_\pi^k(p)$ and $\Lambda_\pi'^k(p)$ in every state π.

$$\begin{array}{lll} H_0^k : & \Lambda_0^k(p) - \Lambda_0'^k(p) \geq \Lambda_1^k(p) - \Lambda_1'^k(p) & \text{for all } p \in [0,1]; \\ H_1^k : & \Lambda_0^k(p) - \Lambda_0'^k(p) < \Lambda_1^k(p) - \Lambda_1'^k(p) & \text{for some } p \in [0,1]. \end{array}$$

The random process $\Lambda^k(p)$ is, in general, continuous. Andreoli (2013) derived results for a discrete process, assuming that one can only estimate the sample counterpart $\widehat{\Lambda}^k(p)$ of $\Lambda^k(p)$ for a finite number m of abscissae $p \in \{p_1, \ldots, p_m\}$. The estimates give the column vector of coordinates:

$$\widehat{\mathbf{\Lambda}}^k = \left(\widehat{\Lambda}^k(p_1), \ldots, \widehat{\Lambda}^k(p_m)\right)^t,$$

[24] See also Beach and Davidson (1983) and Zheng (2002) for estimators of quantile functions and generalized Lorenz functions coordinates, and Aaberge (2006)

with $\mathbf{\Lambda}^k$ being the corresponding vector in the population. Within the discrete setting it has been shown that:

$$\widehat{\mathbf{\Lambda}}^k \text{ is asymptotically distributed as } \mathcal{N}\left(\mathbf{\Lambda}^k,\ \frac{\mathbf{\Sigma}^k}{n}\right), \tag{10}$$

where we use $\frac{\widehat{\mathbf{\Sigma}}^k}{n}$ as the estimator of the asymptotic $m \times m$ covariance matrix of $\widehat{\mathbf{\Lambda}}^k$ (the most robust estimator is based on the *influence functions* decomposition of $\widehat{\Lambda}^k$). As a consequence of asymptotic normality, test statistics for ISDk and gap curve dominance relations have well known distributional properties.

B.2 Application to Gap curves dominance

We estimate dominance conditions for discrete processes, summarized by vectors of coordinates $\mathbf{\Lambda}_\pi^k$ and $\mathbf{\Lambda}_\pi'^k$, corresponding respectively to the population distributions F_π and F_π' in both $\pi = 0$ and $\pi = 1$. We define $\mathbf{\Lambda}_\Gamma^k$ the $4m \times 1$ vector obtained by staking the vectors $\mathbf{\Lambda}_0^k$, $\mathbf{\Lambda}_0'^k$, $\mathbf{\Lambda}_1^k$ and $\mathbf{\Lambda}_1'^k$ in this precise order. The sample estimates are collected in the $4m \times 1$ vector $\widehat{\mathbf{\Lambda}}_\Gamma^k$, and we use $n = n_{c,0} + n_{c',0} + n_{c,1} + n_{c',1}$ to denote the overall sample size, gathering together all observations in the sub-samples delimited by circumstances c and c' under $\pi = 0$ and $\pi = 1$, while $r_{c,\pi} = n_{c,\pi}/n$ is the relative size of each sub-sample.

The hypothesis of gap curve dominance can be reformulated as a sequence of m linear constraints on the vector $\mathbf{\Lambda}_\Gamma^k$. Let $\mathbf{R}_\Gamma = (\mathbf{R}, -\mathbf{R})$ be the $m \times 4m$ *difference-in-differences* matrix, where $\mathbf{R} = (\mathbf{I}, -\mathbf{I})$ and $\mathbf{I}$ is an identity matrix of size m. Define the parametric vector $\boldsymbol{\gamma}_k = \mathbf{R}_\Gamma \mathbf{\Lambda}_\Gamma^k$.

We make two (non-testable) assumptions: (i) F_π and F_π' are independent processes for all π; (ii) the independence extends also across policy regimes. This latter assumption is verified when the sampling scheme is based upon randomized assignment to treatment and control groups. Under the two assumptions of independence and using the result in (10), it holds that:

$$\sqrt{n}\,\widehat{\boldsymbol{\gamma}}_k = \sqrt{n}\,\mathbf{R}_\Gamma \widehat{\mathbf{\Lambda}}_\Gamma^k \text{ is asymptotically distributed as } \mathcal{N}\left(\sqrt{n}\,\mathbf{R}_\Gamma \mathbf{\Lambda}_\Gamma^k,\ \mathbf{\Phi}\right), \tag{11}$$

where $\widehat{\boldsymbol{\gamma}}_k$ denotes the sample estimate of $\boldsymbol{\gamma}_k$, and

$$\mathbf{\Phi} = \mathbf{R}_\Gamma \operatorname{diag}\left(\frac{\mathbf{\Sigma}_{c,0}^k}{r_{c,0}}, \frac{\mathbf{\Sigma}_{c',0}^k}{r_{c',0}}, \frac{\mathbf{\Sigma}_{c,1}^k}{r_{c,1}}, \frac{\mathbf{\Sigma}_{c',1}^k}{r_{c',1}}\right) \mathbf{R}_\Gamma^t.$$

The empirical estimator of the asymptotic variance, $\widehat{\mathbf{\Phi}}$, is obtained by plugging $\widehat{\mathbf{\Sigma}}_{c,\pi}^k$ in the previous formula. As in the case of ISDk testing, the empirical covariance estimator can be obtained by using the empirical counterpart of the covariance matrices proposed by Andreoli (2013). We discuss separately the cases of equality and dominance in the gap curve, as well as the correct test statistics and asymptotic distributions.

B.3 Testing equality in gap curves

The null and alternative hypothesis for equality in gap curves coordinates associated to the set of abscissae $\{p_1, \ldots, p_m\}$ are:

$$H_0^k : \boldsymbol{\gamma}_k = \mathbf{0} \qquad H_1^k : \boldsymbol{\gamma}_k \neq \mathbf{0}.$$

Under the null hypothesis, it is possible to resort to a Wald test static T_1^k:

$$T_1^k := n \,\widehat{\boldsymbol{\gamma}}_k^t \,\widehat{\boldsymbol{\Phi}}^{-1} \widehat{\boldsymbol{\gamma}}_k.$$

Given the convergence results in (11), the asymptotic distribution of the test T_1^k is χ_m^2. The p-value tabulation follows the usual rules.

B.4 Testing dominance in gap curves

The null and alternative hypothesis for dominance in gap curves can be reformulated as a sequence of positivity constraints on the vector $\boldsymbol{\gamma}_k$:

$$H_0^k : \boldsymbol{\gamma}_k \in \mathbb{R}_+^m \qquad H_1^k : \boldsymbol{\gamma}_k \not\in \mathbb{R}_+^m$$

The Wald test statistics with inequality constraints has been developed by Kodde and Palm (1986). For this set of hypothesis, the test statistics T_2^k is defined as:

$$T_2^k = \min_{\boldsymbol{\gamma}_k \in \mathbb{R}_+^m} \left\{ n \left(\widehat{\boldsymbol{\gamma}}_k - \boldsymbol{\gamma}_k\right)^t \widehat{\boldsymbol{\Phi}}^{-1} \left(\widehat{\boldsymbol{\gamma}}_k - \boldsymbol{\gamma}_k\right) \right\}.$$

Kodde and Palm (1986) have shown that the statistic T_2^k is asymptotically distributed as a mixture of χ^2 distributions, provided that (11) holds:

$$T_2^k \sim \overline{\chi}^2 = \sum_{j=0}^{m} w\left(m, m-j, \widehat{\boldsymbol{\Phi}}\right) \Pr\left(\chi_j^2 \geq c\right),$$

with $w\left(m, m-j, \widehat{\boldsymbol{\Phi}}\right)$ the probability that $m-j$ elements of $\boldsymbol{\gamma}_k$ are strictly positive.[25]

To test the reverse dominance order, that is $\Gamma(\Lambda_1^k, \Lambda_1'^k, p) \geq \Gamma(\Lambda_0^k, \Lambda_0'^k, p)$ for all $p \in [0,1]$, it is sufficient to replace $-\widehat{\boldsymbol{\gamma}}_k$ and $-\boldsymbol{\gamma}_k$ for their positive counterparts.

[25]To estimate $w\left(m, m-j, \widehat{\boldsymbol{\Phi}}\right)$, we draw 10,000 multivariate normal vectors with covariance matrix $\widehat{\boldsymbol{\Phi}}$, provided it is positive definite. Then, we compute the proportion of vectors with $m-j$ positive entries.

C Empirical analysis: additional tables

Table 4: Descriptive statistics: covariates, by treatment group (IV)

	Treatment (IV=1)		Comparison (IV=0)	
	(1)		(2)	
Individual characteristics:				
Wage, monthly, in Euro	1,676.578	[2,876.4]	1,737.303	[3,246.8]
Prizes	0.511	[0.5]	0.525	[0.5]
Weekly working hours	40.120	[9.4]	40.338	[9.7]
Self employed	0.022	[0.1]	0.026	[0.2]
Employed in the public sector	0.244	[0.4]	0.251	[0.4]
Education, years	12.116	[3.3]	11.903	[3.6]
Age, in years (above 15)	43.984	[6.5]	46.165	[6.0]
Marriage status	0.758	[0.4]	0.790	[0.4]
Number of children below 18	1.034	[1.1]	0.907	[1.1]
Socioeconomic conditions of the father:				
Father without french nationality	0.066	[0.2]	0.060	[0.2]
Circumstance 2	0.539	[0.5]	0.533	[0.5]
Farmers	0.113	[0.3]	0.119	[0.3]
Manual worker	0.456	[0.5]	0.443	[0.5]
Circumstance 3	0.220	[0.4]	0.242	[0.4]
Artisans	0.101	[0.3]	0.109	[0.3]
Non manual workers	0.140	[0.3]	0.151	[0.4]
Circumstance 4	0.174	[0.4]	0.165	[0.4]
H-grade prof.	0.075	[0.3]	0.075	[0.3]
L-grade prof.	0.115	[0.3]	0.104	[0.3]
Age of leaving education	18.116	[3.3]	17.903	[3.6]
$(cob - 1953)^2$	1.667	[1.7]	4.559	[3.3]
$(cob - 1953)^3$	3.002	[3.6]	-11.658	[10.9]
$(cob - 1953)^4$	5.672	[7.3]	31.634	[34.5]
Trimmed proportion of sample size	0.672	[0.5]	0.676	[0.5]
Groups interested by policy intervention:				
Receives policy treatment	0.540	[0.5]	0.432	[0.5]
Δ *policy treatment*		0.108*** (.006)		
Marginal students (target)	0.160	[0.4]	0.268	[0.4]
Sample size	13,364		12,516	

Source: Labor Force Survey 1990, 1993, 1996, 1999, 2004, 2006, 2008 and 2010.

Notes: Sample reduced to French male earners where circumstances have been recorded, cohorts 1950 to 1955. IV is a dummy for cohorts 1953 to 1955. Treatment and comparison groups are defined upon the IV. Standard deviations in brackets. Differences in covariates between control and treatment groups are not significant at 5%. Variable *cob* identifies the cohort of birth. Trimmed sample size refers to the sub-sample of those who at most have an high school diploma. The group receiving policy treatment is given by those who completed primary education but did not qualify above this level. Marginal students are defined as the target group used to simulate policy intervention. *** indicates significance at 1%

Table 5: Earnings distributions, by cohorts before and after introduction of the policy for selected quantiles.

Quantiles	Overall (1)	Target (2)	Circ. 1 (3)	Circ. 2 (4)	Circ. 3 (5)	Circ. 4 (6)
	Before policy implementation					
Q5%	499.1	618.2	394.0	474.8	606.3	569.1
Q10%	944.6	883.6	883.6	914.7	975.1	1,066.5
Q25%	1,226.7	1,097.0	1,269.4	1,173.3	1,275.2	1,448.3
Q50%	1,534.3	1,305.6	1,638.4	1,427.6	1,620.6	1,934.4
Q75%	2,011.7	1,529.1	2,164.2	1,808.7	2,134.3	2,748.7
Q90%	2,825.0	1,840.1	3,049.0	2,316.6	2,935.4	3,876.1
Q95%	3,535.4	2,147.0	3,841.1	2,779.7	3,665.4	4,976.7
Mean	1,825.7	1,378.5	1,940.3	1,597.6	1,875.4	2,431.4
	[3,026.9]	[2,102.1]	[3,868.8]	[2,378.3]	[2,270.0]	[4,785.9]
Gini	0.303	0.204	0.330	0.256	0.287	0.352
	(0.006)	(0.015)	(0.022)	(0.008)	(0.010)	(0.018)
Sample size	26,421	5,585	1,682	14,134	6,103	4,502
	After policy implementation					
Q5%	499.1	618.2	394.0	474.8	606.3	569.1
Q10%	944.6	883.6	883.6	914.7	975.1	1,066.5
Q25%	1,264.3	1,097.0	1,290.2	1,219.6	1,310.5	1,473.3
Q50%	1,574.9	1,447.9	1,656.3	1,493.4	1,656.3	1,934.4
Q75%	2,011.7	1,676.1	2,164.2	1,808.7	2,134.3	2,748.7
Q90%	2,825.0	1,840.1	3,049.0	2,316.6	2,935.4	3,876.1
Q95%	3,535.4	2,147.0	3,841.1	2,779.7	3,665.4	4,976.7
Mean	1,842.5	1,458.3	1,950.0	1,621.0	1,888.0	2,436.3
	[3,024.7]	[2,102.7]	[3,867.5]	[2,376.5]	[2,267.6]	[4,784.9]
Gini	0.299	0.197	0.326	0.251	0.284	0.351
	(0.006)	(0.014)	(0.022)	(0.008)	(0.010)	(0.018)
Sample size	26,421	5,585	1,682	14,134	6,103	4,502

Source: Estimates from Labor Force Survey 1990, 1993, 1996, 1999, 2004, 2006, 2008 and 2010.
Notes: Earnings quantiles for earnings distribution detrended by the age effect. Sample reduced to French male earners where circumstances have been recorded, cohorts 1950 to 1955. Earnings after policy implementation are obtained by assigning quantile treatment effects estimated by model (1) in table 2 to the target group. Standard deviations reported in brackets. Gini index are reported for each subgroup's earnings distribution. Standard errors in parentheses are calculated by bootstrapping 100 replications of the Gini index.