Revealed preference analysis of noncooperative household consumption*

Laurens Cherchye[†], Thomas Demuynck[‡] and Bram De Rock[§]

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Abstract

We develop a revealed preference approach to analyze non-unitary household consumption behavior that is not cooperative (or Pareto efficient). We derive global and necessary and sufficient conditions for data consistency with the model. Interestingly, contrary to existing results for the differential approach, these revealed preference conditions for the noncooperative model are independent from (or non-nested with) the conditions for the cooperative model. We show that the conditions can be verified by means of relatively straightforward mixed integer programming (MIP) methods, which is particularly attractive in view of empirical analysis. Our framework extends to tests for separate spheres and joint contribution to public goods. An application to data drawn from the Russia Longitudinal Monitoring Survey (RLMS) demonstrates the empirical relevance of the noncooperative consumption model. To the best of our knowledge, this is the first empirical application of the noncooperative consumption model.

JEL Classification: D11, D12, D13, C14.

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†CentER, Tilburg University, and Center for Economic Studies, University of Leuven. E. Sabbelaan 53, B-8500 Kortrijk, Belgium. E-mail: L.Cherchye@uvt.nl and laurens.cherchye@kuleuven-kortrijk.be. Laurens Cherchye gratefully acknowledges financial support from the Research Fund K.U.Leuven through the grant STRT1/08/004.

‡Center for Economic Studies, University of Leuven, and Sherppa, University of Ghent. E. Sabbelaan 53, B-8500 Kortrijk, Belgium. E-mail: thomas.demuynck@kuleuven-kortrijk.be. Thomas Demuynck gratefully acknowledges the Fund for Scientific Research - Flanders (FWO-Vlaanderen) for his postdoctoral fellowship.

§ECARES-ECORE, Université Libre de Bruxelles. Avenue F. D. Roosevelt 50, CP 114, B-1050 Brussels, Belgium. E-mail: bderock@ulb.ac.be. Bram De Rock gratefully acknowledges the European Research Council (ERC) for his Starting Grant.

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1 Introduction

We present a nonparametric revealed preference characterization of the non-unitary household consumption model that is identified by noncooperative behavior. This characterization allows us to develop a practical method for analyzing noncooperative household consumption. In addition, it enables us to derive some interesting theoretical results, such as independence (or non-nestedness) between the cooperative model and the noncooperative model of household consumption. We use our method to analyze household consumption data taken from the Russia Longitudinal Monitoring Survey (RLMS). To the best of our knowledge, this is the first empirical application of the noncooperative household consumption model. This introductory section motivates our main research questions, and relates them to the existing literature.

Non-unitary household consumption. There is a growing consensus that multi-person household consumption behavior should no longer be treated as if it the household were a single decision maker that optimizes a household utility function subject to the household budget constraint. Indeed, this so-called unitary model of household consumption imposes empirically testable restrictions on the household demand function (e.g. Slutsky symmetry) that are frequently rejected when confronted with consumption or labor supply data of multi-person households. See, for example, Lundberg (1988), Thomas (1990), Fortin and Lacroix (1997), Browning and Chiappori (1998), Chiappori, Fortin and Lacroix (2002), Duflo (2003) and Cherchye and Vermeulen (2008).

Because of these empirical problems of the unitary model, an emerging literature explicitly acknowledges that households are composed of distinct individuals who are endowed with their own preferences, and that household consumption decisions are determined by an underlying intrahousehold decision mechanism.[1] We refer to this approach as the non-unitary approach to household consumption. Typically, non-unitary consumption models allow for privately consumed goods as well as publicly consumed goods within the household. In addition, following Apps and Rees (1988) and Chiappori (1988, 1992), the usual assumption is that household allocations are Pareto efficient; in the household consumption literature, Pareto efficiency corresponds to the so-called cooperative within-household solution of the intrahousehold

[1] See, for example, Lundberg and Pollak (2007) and Donni (2008) for recent surveys of alternative household consumption models that have been suggested in the literature.

allocation problem.[2] However, the Pareto efficiency assumption has been questioned for the publicly consumed goods. Most notably, it has been argued that the informational requirement and the resulting cost of implementing cooperation may often be unrealistic. See, for example, Lechene and Preston (2005, 2010) and Browning, Chiappori and Lechene (2010).

In this paper we develop a methodology that allows for analyzing noncooperative household consumption behavior. Specifically, we will model noncooperative behavior as corresponding to a Nash equilibrium allocation within the household. It will be interesting to compare our characterization of this noncooperative within-household allocation with the one of the cooperative (or Pareto efficient) allocation. At this point, noting that we see at least two reasons why it is important to know whether or not household consumption behavior is cooperative. First, from a welfarist perspective, it gives an idea of the welfare improvement that is possible within a certain household. If it is possible to link intrahousehold (non)cooperation to household characteristics, it may be possible to use this knowledge for welfare enhancement measures that correct the efficiency loss originating from household behavior that is noncooperative. Second, the issue has also important implications for the structure of optimal taxation and policies that target to alter the intrahousehold income distribution. See, for example, Blundell, Chiappori and Meghir (2005) for a discussion on such targeting issues in a non-unitary setting. In this respect, different (cooperative or noncooperative) consumption models may lead to other intrahousehold allocations.

Cooperative and noncooperative household consumption models: literature review. By now, the modeling of the cooperative case is quite complete. Browning and Chiappori (1998) provide a differential characterization of the cooperative model.[3] The specific feature of this differential approach is that it focuses on properties of functions (e.g. cost, indirect utility and demand functions). In this respect, it is well-known that the unitary condition of Slutsky symmetry no longer holds as a (necessary and sufficient) condition for consistency with the non-unitary cooperative model. Browning and Chiappori have shown that a necessary condition for the cooperative model is that the (pseudo) Slutsky matrix is the sum of a symmetric negative semidefinite matrix and a deviation matrix of rank smaller than 1 (in the case of two household members). As shown by Chiappori and Ekeland (2006), this condition, together with homogeneity and adding up, is also locally sufficient for consistency with the cooperative model. Cherchye, De Rock and Vermeulen (2007,

[2] Following Chiappori (1988, 1992), the consumption literature often refers to the cooperative model as the 'collective' model of household behavior.

[3] The term 'differential' refers to the fact that the characterization is obtained by integrating and/or differentiating the functional specifications of the fundamentals of the model (e.g. the individual preferences of the group members).

2010a,b) complement these differential results by presenting a revealed preference characterization of the same cooperative model. In the tradition of Afriat (1967) and Varian (1982),[4] this revealed preference characterization does not pertain to properties of functions representing household consumption behavior (e.g. cost, indirect utility or demand functions). It implies a necessary and sufficient condition for a finite set of household consumption data to be consistent with the model. For the publicly consumed quantities, this condition requires the existence of suitable Lindahl prices such that each individual in the household satisfies the Generalized Axiom of Revealed Preference (GARP; see Section 2.1) when using these individual Lindahl prices to evaluate the public goods.

In contrast to the cooperative model, the noncooperative model assumes that each individual within the household maximizes her/his own utility given the consumption of the other household members. In this case, the household consumption decision is determined by the Nash equilibrium solution. See, among others, Ulph (1988), Lundberg and Pollak (1993), Chen and Wolley (2001), Lechene and Preston (2005, 2010) and Browning, Chiappori and Lechene (2010). Lechene and Preston (2010) derived a differential characterization of the model with voluntary contributions for the publicly consumed goods. They find that a testable implication of this noncooperative model is that the (pseudo) Slutsky matrix must be the sum of a symmetric negative semidefinite matrix and a deviation matrix with rank smaller than $(K + 1)$ where K is the number of public goods (again in the case of two household members). Three remarks are important in view of our following exposition. First, at present it is not known whether this necessary condition is also (locally) sufficient. Second, this noncooperative condition is nested with the (differential) cooperative condition mentioned above: data consistency with the cooperative condition always implies data consistency with the noncooperative condition, but not vice versa. Finally, a complementary revealed preference characterization of the noncooperative household consumption model is nonexistent in the literature.[5]

This study. In contrast to the existing literature, we focus on the revealed preference characterization of noncooperative household consumption behavior. The revealed preference approach has a number of attractive features. First of all, our characterization is global, while a differential characterization is typically local in nature.- We get a global condition that enables checking consistency of a given data set with a particular consumption model; in the spirit of Varian (1982), we refer to this as 'testing' data consistency with the model under study.[6] Second, we are able to

[4]See also Samuelson (1938), Houthakker (1950) and Diewert (1973) for seminal contributions on the revealed preference approach to analyzing consumption behavior.

[5]However, see Sprumont (2000) for a revealed preference characterization of the noncooperative Nash solution in a choice-theoretic framework à la Richter (1966).

[6]As is standard in the revealed preference literature, the type of tests that we consider here are 'sharp' tests; either a data set satisfies the data consistency conditions or it does not.

verify this condition while keeping its inherent nonparametric nature, i.e. the associated tests do not require an a priori (typically non-verifiable) parametric specification of the intrahousehold decision process (e.g. individual preferences). By contrast, the differential approach (until present) usually maintains additional assumptions concerning the functional form for the demand function (and thus individual preferences) when verifying the abovementioned rank conditions of the (pseudo) Slutsky matrix (e.g. Browning and Chiappori (1998) start from a quadratic almost ideal demand system in their empirical analysis). More specifically, we show that our nonparametric tests imply a relatively straightforward mixed integer programming (MIP) problem, which combines linear constraints with binary integer variables. This MIP formulation is particularly attractive from a practical point of view: for a given data set, it allows for testing data consistency with a specific consumption model by applying standard MIP solution techniques.

Two further features imply notable differences with the differential results described above. First, the testable revealed preference condition is not only necessary but also sufficient for data consistency with the noncooperative consumption model. Second, we will show that the condition for the noncooperative model is not nested with the condition for the cooperative model: data consistency with the (global) condition for full cooperation is neither necessary nor sufficient for data consistency with the (global) condition for noncooperation. This makes it interesting to compare the empirical validity of different models. In fact, we can meaningfully verify data consistency with a given model (and compare different models) even if there are only a few observations and without restriction on the number of privately consumed goods (see Section 3.2 and Section 4).

We demonstrate the practical usefulness of our approach through an empirical application to data taken from the RLMS. As indicated above, as far as we know, this is the first application of the noncooperative household consumption model to a real-life data set. Interestingly, this application demonstrates the empirical relevance of our theoretical insights on independence (or non-nestedness) of, on the one hand, the revealed preference condition for the cooperative model and, on the other hand, the revealed preference condition for the noncooperative model. In addition, for the data under study we obtain that the cooperative model and the noncooperative model have about the same power in terms of detecting random behavior. As such, our application motivates considering the noncooperative model in addition to the (more common) cooperative model in empirical analysis of household consumption behavior.

The rest of this study is organized as follows. To set the stage, Section 2 considers the rational individual model and briefly recaptures the revealed preference characterization of the cooperative model. Subsequently, Section 3 provides the characterization of the noncooperative model and shows the independence result. Section 4 introduces the MIP approach for empirical verification of the noncooperative condition, and presents our empirical application. Section 5 summarizes and formulates a number of concluding remarks.

2 Setting the stage

In this section, we introduce our notation and a number of preliminary concepts that will be instrumental for our following exposition. First, we consider the revealed preference characterization of the rational individual model. Next, we introduce the household consumption problem in a non-unitary framework. Subsequently, we consider the cooperative solution to this problem. This cooperative model will serve as a useful benchmark for the noncooperative model that we discuss in the next section.

2.1 The rational individual

Consider an individual with a utility function U. Throughout, we will assume that utility functions U are continuous, concave, non-satiated and non-decreasing in their arguments. Let $T = \{1, \ldots, |T|\}$ be a set of observations. Given a (strictly positive) price vector p_t and income Y_t ($t \in T$), we assume that the rational individual chooses the consumption bundle q in her/his budget set that maximizes her/his utility. In particular, the rational individual solves the following optimization problem (**OP-I**):[7]

$$q_t \in \arg\max_{q} U(q) \;\; s.t. \;\; \langle p_t, q \rangle \leq Y_t$$

A data set $S = \{p_t, q_t\}_{t \in T}$ consists of a collection of observed strictly positive price vectors p_t and a collection of observed positive demand vectors q_t. In this paper, we distinguish between the concepts of consistency, which is a property of an individual observation $\{p_t, q_t\}$, and the notion of rationalizability, which is a property of the entire data set, $\{p_t, q_t\}_{t \in T}$. We use the following definition of individual-consistency.

Definition 1 (individual-consistency) *An observation* $\{p_t, q_t\}$ *is* individual-consistent *for utility function* U*, if* q_t *solves* ***OP-I*** *given the function* U *and income* $Y_t = \langle p_t, q_t \rangle$.

If an observation $\{p_t, q_t\}$ is not individual-consistent for U, then U cannot be the true underlying utility function. The concept of individual-rationalizability requires that there exists at least one (common) utility function for which every observation $t \in T$ is individual-consistent.

[7]Observe that, under our maintained assumptions for the utility function U, we have that in equilibrium the budget restriction should hold with equality. The same applies to all other problems in this paper.

Definition 2 (individual-rationalizability) *Consider a data set $S = \{p_t, q_t\}_{t \in T}$. The set S is* individual-rationalizable *if there exist a utility function U such that, for all $t \in T$, the observation $\{p_t, q_t\}$ is individual-consistent for the utility function U.*

Varian (1982) established that the set S is individual-rationalizable if and only if it satisfies the Generalized Axiom of Revealed Preference (GARP).

Definition 3 (GARP) *Consider a data set $S = \{p_t, q_t\}_{t \in T}$. The set S satisfies* GARP *if there exists a binary relation R such that the following holds. If $\langle p_t, q_t \rangle \geq \langle p_t, q_v \rangle$ then $q_t R q_v$. Next, $q_t R q_v$ if $q_t R q_s$, $q_s R q_l$, ..., $q_z R q_v$ for some sequence s, l, ..., z. Finally, if $q_t R q_v$ then $\langle p_v, q_v \rangle \leq \langle p_v, q_t \rangle$.*

In words, R captures the revealed preference relation in the data set S. We have $q_t R q_v$ if q_t is directly revealed preferred to q_v (i.e. $\langle p_t, q_t \rangle \geq \langle p_t, q_v \rangle$) or indirectly revealed preferred to q_v (i.e. there exists a sequence s, l, ..., z such that $q_t R q_s$, $q_s R q_l$, ..., $q_z R q_v$). Finally, if $q_t R q_v$, then we must have $\langle p_v, q_v \rangle \leq \langle p_v, q_t \rangle$, i.e. q_v cannot be more expensive than any revealed preferred q_t.

The following theorem is probably the single most important result in revealed preference theory (see Varian, 1982, based on Afriat, 1967).

Theorem 1 *Consider a data set $S = \{p_t, q_t\}_{t \in T}$. The following conditions are equivalent:*

1. *There exists a utility function U that individual-rationalizes S.*
2. *S satisfies* GARP.
3. *For all $t \in T$, there exist a positive number U_t and a strictly positive number λ_t such that, for all $t, v \in T$,*

$$U_t - U_v \leq \lambda_v \langle p_v, q_t - q_v \rangle .$$

This result has two important implications. First, data consistency with GARP is necessary and sufficient for individual-rationalizability of the data; see condition 2. Next, condition 3 provides an equivalent characterization in terms of the so-called Afriat inequalities, which allow an explicit construction of the utility levels associated with each observation t (i.e. utility level U_t for observed q_t). In our following discussion of household consumption models, we will mainly concentrate on the GARP characterization of rational individual behavior. As we will show, this focus on GARP enables us to formulate testable implications of consumption models in mixed integer programming (MIP) terms (see Section 4). However, in principle our GARP-based characterization of consumption models can equivalently be expressed in terms of Afriat inequalities (by building on Theorem 1; see also the proof of Theorem 3).

2.2 Non-unitary household consumption

To keep our exposition simple, we focus on 2-person (A and B) households in what follows. However, extensions to households with more than 2 members are fairly straightforward. Individuals have to decide over the consumption of a bundle of $|J|$ private goods ($J = \{1, \ldots, |J|\}$) and a bundle of $|K|$ public goods ($K = \{1, \ldots, |K|\}$).

As indicated above, the non-unitary approach to household consumption explicitly recognizes that different household members have own (rational) preferences, which are represented by individual utility functions. In what follows, because we account for private and public consumption in the household, the utility of the individuals A and B is given by the functions $U^A(q^A, Q)$ and $U^B(q^B, Q)$, with q^A and q^B the private consumption bundles of A and B, and Q the public consumption bundle.[8]

In our non-unitary framework, the empirical analysis of household consumption starts from a data set $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. For every observation $t \in T$, the vectors $Q_t \in \mathbb{R}_+^{|K|}$ and q_t $(= q_t^A + q_t^B)$ in $\mathbb{R}_+^{|J|}$ represent the household bundles of public and private goods demanded at t; and we write $q_{t,j}$, and $Q_{t,k}$ for the demanded quantity of private good j or public good k at t ($j \in J$, $k \in K$). Thus, using $p_t \in \mathbb{R}_{++}^{|J|}$ for the (strictly positive) price vector of the private goods, $P_t \in \mathbb{R}_{++}^{|K|}$ for the (strictly positive) price vector of the public goods and Y_t $(\equiv \langle p_t, q_t \rangle + \langle P_t, Q_t \rangle)$ for household income, the household faces the following budget constraint at t:

$$\langle p_t, q \rangle + \langle P_t, Q \rangle \leq Y_t.$$

We take the first private good as the numeraire. As such, we may normalize the prices in any data set S such that $p_{t,1} = 1$ for all $t \in T$. To facilitate our exposition, we will maintain two additional assumptions in this and the next section. First, we will assume that the empirical analyst only observes the aggregate private demands q_t, and not the individual bundles q_t^A and q_t^B. However, it is easy to extend our analysis and results to include information on q_t^A and q_t^B, i.e. the private consumption of the individuals A and B is (partly) observed. See our empirical application in Section 4 for a specific example. Next, we will assume that all components of the aggregate demands Q_t are strictly positive and that the numeraire is desirable, i.e. $q_{t,1}^A$ and $q_{t,1}^B$ are always assumed to be strict positive. Again, we could easily relax this assumption by introducing some additional notation, but this would only complicate the discussion while not really adding any new insights. In fact, our empirical application in Section 4 will consider data sets with some components of Q_t equal

[8] Throughout, we will abstract from externalities associated with privately consumed quantities. Importantly, however, our setting can actually account for such externalities. Specifically, if an individual is the exclusive consumer of a particular private good, then we can account for externalities for this good by formally treating it as a public good.

to zero; our basic theoretical insights developed below apply with equal strength to this setting.

2.3 The cooperative benchmark

Let us then consider the revealed preference characterization of the cooperative model. We recall that this model assumes that for each $t \in T$ the household consumption decision coincides with a Pareto optimal allocation. Thus, we can define the following concept of cooperative-consistency.

Definition 4 (cooperative-consistency) *An observation* $\{p_t, P_t, q_t, Q_t\}$ *is* cooperative-consistent *for utility functions* U^A *and* U^B *if there exist bundles* q_t^A *and* q_t^B *in* $\mathbb{R}_+^{|J|}$ *that sum to* q_t *such that, for all bundles* $q^A, q^B \in \mathbb{R}_+^{|J|}$ *and* $Q \in \mathbb{R}_+^{|K|}$ *that satisfy* $\langle p_t, q^A + q^B \rangle + \langle P_t, Q \rangle \leq Y_t$ $(\equiv \langle p_t, q_t \rangle + \langle P_t, Q_t \rangle)$*, the following holds (for* $M, L \in \{A, B\}$ *with* $M \neq L$*):*

$$U^M(q^M, Q) > U^M(q_t^M, Q_t) \text{ implies } U^L(q^L, Q) < U^L(q_t^L, Q_t).$$

The interpretation of cooperative-consistency is analogous to the one of individual-consistency. Specifically, assume that an observation $\{p_t, P_t, q_t, Q_t\}$, $t \in T$, is not cooperative-consistent for U^A and U^B. Then, any household whose members hold these utility functions cannot have chosen $\{q_t, Q_t\}$ if the household decision outcome is to be Pareto efficient.

Next, we can define the following concept of cooperative-rationalizability.

Definition 5 (cooperative-rationalizability) *Consider a data set* $S = \{p_t,\ P_t,\ q_t,\ Q_t\}_{t \in T}$*. The set* S *is* cooperative-rationalizable *if there exist utility functions* U^A *and* U^B *such that, for all* $t \in T$*,* $\{p_t, P_t, q_t, Q_t\}$ *is cooperative-consistent for the utility functions* U^A *and* U^B*.*

The intuition is readily similar to before: if a data set S is not cooperative-rationalizable then it cannot have been generated by a household choosing Pareto-efficient allocations.

The next result gives the revealed preference condition associated with cooperative-rationalizability (see also Cherchye, De Rock and Vermeulen 2007, 2010b).

Theorem 2 *Consider a data set* $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$*. The following conditions are equivalent:*

1. *The data set* S *is cooperative-rationalizable.*

2. *For all $t \in T$, there exist public price vectors $\tilde{P}_t^A, \tilde{P}_t^B \in \mathbb{R}_{++}^{|K|}$ and private quantity vectors $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ such that:*

$$q_t^A + q_t^B = q_t, \tag{C.1}$$

$$\tilde{P}_t^A + \tilde{P}_t^B = P_t, \text{ and} \tag{C.2}$$

$$\{p_t, \tilde{P}_t^A, q_t^A, Q_t\}_{t\in T} \text{ and } \{p_t, \tilde{P}_t^B, q_t^B, Q_t\}_{t\in T} \text{ satisfy GARP}. \tag{C.3}$$

Condition C.3 implies that cooperative-rationalizability implies a GARP condition (i.e. individual-rationalizability) at the level of individuals A and B. The specificity of the cooperative model is that these GARP conditions use the public price vectors $\tilde{P}_t^A$ and $\tilde{P}_t^B$ for evaluating the publicly consumed quantities. More formally, we have that $\tilde{P}_{t,k}^A = U_{Q_{t,k}}^A / U_{q_{t,1}}^A$ and $\tilde{P}_{t,k}^B = U_{Q_{t,k}}^B / U_{q_{t,1}}^B$, with $U_{Q_{t,k}}^M$ and $U_{q_{t,k}^M}^M$ the marginal utilities of U^M for the k-th public good and the numeraire private good at the cooperative household outcome.[9] In words, the vectors $\tilde{P}_t^A$ and $\tilde{P}_t^B$ represent the individuals' marginal willingness to pay (MWTP) for the bundle of public goods. Condition C.2 then implies that these MWTP vectors can be interpreted as Lindahl prices, because they must sum to the market price vector P_t. Indeed, any Pareto optimal allocation $\{q_t^A, q_t^B, Q_t\}$ must satisfy the Lindahl-Bowen-Samuelson conditions, i.e.

$$\frac{U_{Q_{t,k}}^A}{U_{q_{t,1}}^A} + \frac{U_{Q_{t,k}}^B}{U_{q_{t,1}}^B} \equiv \tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B = P_{t,k} \text{ for all } k \leq |K|;$$

this effectively corresponds to condition C.2 in Theorem 2.

One important concluding remark is on order. We recall that a data set S only contains information on Y_t and not on the income shares $Y_t^A = \langle p_t, q_t^A \rangle + \left\langle \tilde{P}_t^A, Q_t \right\rangle$ and $Y_t^B = \langle p_t, q_t^B \rangle + \left\langle \tilde{P}_t^B, Q_t \right\rangle$. However, in principle it is possible to empirically identify Y_t^A and Y_t^B (i.e. the income shares) if the set S is cooperative-rationalizable, i.e. we can identify the individual income shares that underlie the observed cooperative consumption behavior. We refer to Cherchye, De Rock and Vermeulen (2010b) for a detailed discussion of this identifiability result that starts from the revealed preference characterization in Theorem 2.[10] In what follows, we will see that this identifiability result does not hold in general for consumption models that are non-cooperative.

[9] If U^M is not differentiable, we may take the suitable subdifferentials that satisfy the first order conditions.

[10] Chiappori and Ekeland (2009) provide related identifiability results that start from a differential characterization of the cooperative model.

3 The noncooperative model

As indicated above, the noncooperative model assumes that each observed household allocation corresponds to a Nash equilibrium solution with voluntary contributions for the publicly consumed goods. This section provides the revealed preference characterization of this noncooperative model. Specifically, we provide a necessary and sufficient condition for data consistency with the model. In addition, we show independence (or non-nestedness) between this condition and the condition for the cooperative model that we discussed above.

3.1 Revealed preference characterization

In view of our further exposition, we use the notation Q^A and Q^B to represent the contributions to the public goods by A and B, so that $Q^A+Q^B = Q$. The fact that we explicitly distinguish between A and B's contribution to the public consumption may seem a bit unconventional. However, this distinction will be useful to characterize the noncooperative model (e.g. Lechene and Preston, 2005 and 2010, make a similar distinction).

Assume prices p_t and P_t for the private and public goods, and some household income Y_t. Then, an allocation $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ is a household Nash equilibrium if it solves (**OP-N**):

$$\{q_t^M, Q_t^M\} \in \arg \max_{\{q^M, Q^M\}} U^M(q^M, Q) \ s.t. \ \langle p_t, q^M + q_t^L\rangle + \langle P_t, Q\rangle \leq Y_t, Q^M + Q_t^L = Q,$$

where we use $M, L \in \{A, B\}$ with $M \neq L$.

Given this, we can define when a certain observation is noncooperative-consistent

Definition 6 (noncooperative-consistency) *An observation $\{p_t, P_t, q_t, Q_t\}$ is* noncooperative-consistent *for utility functions U^A and U^B if there exist bundles q_t^A and $q_t^B \in \mathbb{R}_+^{|J|}$ that sum to q_t and bundles Q_t^A and $Q_t^B \in \mathbb{R}_+^{|K|}$ that sum to Q_t such that the allocation $\{q_t^A, q_t^B, Q_t^A, Q_t^B\}$ simultaneously solves **OP-N** for $M = A$, $L = B$ and for $M = B$, $L = A$ under prices p_t, P_t and income Y_t $(\equiv \langle p_t, q_t\rangle + \langle P_t, Q_t\rangle)$.*

The interpretation of noncooperative-consistency is readily similar to the one of individual- and cooperative-consistency. Using this concept, we can define noncooperative-rationalizability.

Definition 7 (noncooperative-rationalizability) *Consider a data set $S = \{p_t, P_t, q_t, Q_t\}_{t\in T}$. The set S is* noncooperative-rationalizable *if there exist utility functions U^A and U^B such that, for all $t \in T$, $\{p_t, P_t, q_t, Q_t\}$, is noncooperative-consistent for the utility functions U^A and U^B.*

We then obtain the following revealed preference characterization of noncooperative-rationalizability.

Theorem 3 *Consider a data set* $S = \{p_t, P_t, q_t, Q_t\}_{t \in T}$. *The following conditions are equivalent:*

1. *The data set* S *is noncooperative-rationalizable.*

2. *For all* $t \in T$ *and* $k \in K$, *there exist public price vectors* $\tilde{P}_t^A, \tilde{P}_t^B \in \mathbb{R}_{++}^{|K|}$ *and private quantity vectors* $q_t^A, q_t^B \in \mathbb{R}_+^{|J|}$ *such that*

$$q_t^A + q_t^B = q_t, \tag{NC.1}$$

$$\max\left\{\tilde{P}_{t,k}^A, \tilde{P}_{t,k}^B\right\} = P_{t,k}, \textit{and} \tag{NC.2}$$

$$\{p_t, \tilde{P}_t^A, q_t^A, Q_t\}_{t \in T} \textit{ and } \{p_t, \tilde{P}_t^B, q_t^B, Q_t\}_{t \in T} \textit{ satisfy } \text{GARP}. \tag{NC.3}$$

Moreover, it follows that

$$\tilde{P}_{t,k}^A < P_{t,k} \textit{ if and only if } Q_{t,k}^A = 0 \textit{ and } Q_{t,k}^B = Q_{t,k}, \textit{ and} \tag{NC.4}$$

$$\tilde{P}_{t,k}^B < P_{t,k} \textit{ if and only if } Q_{t,k}^B = 0 \textit{ and } Q_{t,k}^A = Q_{t,k}. \tag{NC.5}$$

The conditions NC.1-NC.3 replace C.1-C.3 in Theorem 2. The main difference is condition NC.2 in Theorem 3. Like before, the public price vectors $\tilde{P}_t^A$ and $\tilde{P}_t^B$ represent the individuals' MWTP for the public goods (in casu at the noncooperative household equilibrium). Then, for each public good k (with $Q_{t,k} > 0$), the first order equilibrium conditions for problem **OP-N** effectively imply NC.2. To interpret this equality requirement as an equilibrium condition, let us consider the two possible inequality situations. First, if $\tilde{P}_{t,k}^A > P_{t,k}$ then the MWTP of A for one additional unit of k (i.e. $\tilde{P}_{t,k}^A$) is larger than the price A has to pay for it (i.e. $P_{t,k}$). Hence, A will increase her/his contribution to good k. A directly similar interpretation applies to the situation $\tilde{P}_{t,k}^B > P_{t,k}$. And, thus, $\max\{\tilde{P}_{t,k}^A; \tilde{P}_{t,k}^B\} > P_{t,k}$ implies a disequilibrium. Similarly, if $\max\{\tilde{P}_{t,k}^A; \tilde{P}_{t,k}^B\} < P_{t,k}$ then either A or B (whoever contributes positively to good k) will want to decrease her/his contribution to k. Again, this implies a disequilibrium situation.

Next, the conditions NC.4 and NC.5 follow from the fact that, if $\tilde{P}_{t,k}^A < P_{t,k}$ ($\tilde{P}_{t,k}^B < P_{t,k}$), then A (B) will sell back any positive amount of the public good k. This implies $Q_{t,k}^A = 0$ ($Q_{t,k}^B = 0$) and, thus, $Q_{t,k}^B = Q_{t,k}$ ($Q_{t,k}^A = Q_{t,k}$). Note that we can have $\tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B > P_{t,k}$, which contrasts with the cooperative case (see condition C.2). In fact, this difference between $\tilde{P}_{t,k}^A + \tilde{P}_{t,k}^B$ and $P_{t,k}$ indicates an efficiency loss in the consumption of public goods caused by Pareto inefficient (or noncooperative) behavior.

Three further remarks are in order. First, if we had imposed the additional assumption that for all $t \in T$ and $k \in K$ the contributions $Q^A_{t,k}$ and $Q^B_{t,k}$ are everywhere strictly positive, then we would have derived a simpler characterization of noncooperative behavior. Specifically, it can be verified that condition NC.3 in Theorem 3 would have reduced to requiring bundles q^A_t and q^B_t that sum to q_t such that the sets $\{p_t, P_t, q^A_t, Q_t\}_{t\in T}$ and $\{p_t, P_t, q^B_t, Q_t\}_{t\in T}$ both satisfy GARP. However, the assumption that all $Q^A_{t,k}$ and $Q^B_{t,k}$ are positive is problematic. Specifically, Browning, Chiappori and Lechene (2010) have shown that generically (i.e. in all but a particular set of cases) the number of public goods to which both individuals contribute is less than or equal to one. This suggests only assuming that $Q^A_{t,k}$ and $Q^B_{t,k}$ are non-negative, which effectively obtains the characterization in Theorem 3.[11]

The second remark pertains to our earlier argument that, in principle, under cooperative-rationalizability the (unobserved) within-household income distribution (i.e. the sharing rule) can be identified from the observed set S. This identifiability result does not generally hold under noncooperative-rationalizability. Specifically, it directly follows from the budget constraint in **OP-N** that the income shares of the two individuals are given by:

$$\langle p_t, q^A_t \rangle + \langle P_t, Q^A_t \rangle = Y^A_t \text{ and } \langle p_t, q^B_t \rangle + \langle P_t, Q^B_t \rangle = Y^B_t. \tag{1}$$

Given this, conditions NC.4 and NC.5 imply that Y^A_t and Y^B_t are uniquely identified only if for all k and t we have $\tilde{P}^A_{t,k} < P_{t,k}$ (so that $Q^A_{t,k} = 0$ and $Q^B_{t,k} = Q_{t,k}$) or $\tilde{P}^B_{t,k} < P_{t,k}$ (so that $Q^B_{t,k} = 0$ and $Q^A_{t,k} = Q_{t,k}$). This last situation conforms to the so-called separate spheres Nash equilibrium concept; see Lundberg and Pollak (1993) and Browning, Chiappori and Lechene (2010). On the other hand, as soon as there is one public good k to which both individuals contribute for some t (i.e. $\tilde{P}^A_{t,k} = \tilde{P}^B_{t,k} = P_k$), it is impossible to exactly recover the income shares Y^A_t and Y^B_t that underlie the observed noncooperative behavior. Specifically, in this case $Q^A_{t,k}$ and $Q^B_{t,k}$ can take any value (under the sole condition $Q^A_{t,k} + Q^B_{t,k} = Q_{t,k}$) and, thus, the expenditures on good k cannot be assigned to the individual household members. Interestingly, this result complies with the so-called local income pooling result, which also applies to situations where both individuals contribute to the same public good in a noncooperative setting; see Kemp (1984), Bergstrom, Blume and Varian (1986) and Browning, Chiappori and Lechene (2010). However, even though we cannot identify Y^A_t and Y^B_t in such a situation, it is still possible to recover upper and lower bounds on values for Y^A_t and Y^B_t that are consistent with a noncooperative-rationalization of the given data set. These bounds then account for the total (non-assignable) expenditures on the jointly contributed public goods.

[11] However, see Lechene and Preston (2005) for some example settings where the assumption of strictly positive $Q^A_{t,k}$ and $Q^B_{t,k}$ is always satisfied.

Finally, it is worth indicating that the revealed preference characterization in Theorem 3 also provides a useful starting point for testing alternative assumptions regarding the specific nature of the noncooperative behavior that is under study. For example, one can investigate whether noncooperative-rationalizable behavior is consistent with the separate spheres Nash equilibrium concept discussed above. Or, if this is not the case, one can test alternative assumptions regarding the (number of) jointly contributed public goods. It follows from our above discussion that such assumptions essentially imply constraints that are linear in the public price vectors, which must then be added to the rationalizability conditions NC.1-NC.3. For example, if we assume that member A (B) contributes to good k at observation t then we impose $\tilde{P}_{t,k}^{A} = P_{t,k}$ ($\tilde{P}_{t,k}^{B} = P_{t,k}$). The linear nature of these constraints is attractive in view of practical applications. Specifically, it makes that we can easily include them in the mixed integer programming (MIP) formulation of conditions NC.1-NC.3 that we will introduce in the Section 4.1. As we will discuss, this MIP formulation makes it possible to verify noncooperative-rationalizability in practice.

3.2 Independence

We can show that the revealed preference condition for noncooperative behavior is independent of the revealed preference condition for cooperative behavior: a data set that satisfies the cooperative condition does not necessarily satisfy the noncooperative condition, and vice versa. In particular, the two examples in Appendix 2 show that there is neither any inclusion nor any exclusion relation between the collection of data sets that satisfy the condition in Theorem 2 and the collection of data sets that satisfy the condition in Theorem 3.

This independence/non-nestedness conclusion is important for at least two reasons. Firstly, this result stands in sharp contrast with the findings in the differential approach to modeling non-unitary consumption behavior. As discussed in the Introduction, the rationalizability condition for the noncooperative model derived in that approach are generally nested with the rationalizability condition for the cooperative model: if a given data set passes the condition for cooperative rationalizability, then it must also pass the test for noncooperative rationalizability, but not vice versa. Secondly, our empirical application in Section 4 will show that this independence is not a theoretical curiosity but also has empirical relevance. This application does effectively include data that are cooperative-rationalizable but not noncooperative-rationalizable, and (different) data that are noncooperative-rationalizable but not cooperative-rationalizable.

Apart from independence, the examples in Appendix 2 demonstrate two further features of our revealed preference conditions that are important in view of empirical applications. Firstly, they show that we can meaningfully test data consistency with specific household consumption models (and compare the empirical validity of different models) even if only a few observations are available. Secondly, because all consumption is public in both examples, such empirical analysis in principle does not require privately consumed goods. In fact, this last feature implies an additional difference with the existing differential characterization of the noncooperative model: this differential characterization typically requires (much) more privately consumed goods than publicly consumed goods in order to obtain empirically testable restrictions; see Lechene and Preston (2005, 2010).

We see two possible explanations for these differences between the differential approach and the revealed preference approach that we follow here. A first explanation is that our revealed preference characterization is global, while the existing differential characterization is local in nature. Next, our condition for the noncooperative model is both necessary and sufficient, while the existing differential condition of the model is necessary but not sufficient (see our discussion in the Introduction). In fact, our results suggest that there may exist a sufficient differential condition for the noncooperative model that is not nested with the existing necessary and sufficient condition for the cooperative model.

4 Empirical application

We apply our method to data drawn from the Russia Longitudinal Monitoring Survey (RLMS). Cherchye, De Rock and Vermeulen (2009, 2010b) studied the same data set. These authors focused on consistency of these data with the cooperative model of household consumption. We extend these earlier studies by providing complementary results pertaining to the noncooperative model. In doing so, we also generalize the MIP methodology introduced by these authors (for the cooperative case) to apply to noncooperative household behavior.

Our following analysis will concentrate on consistency testing, and will particularly illustrate the empirical relevance of the independence result articulated above (see Section 3.2). If household behavior is found consistent with a particular (cooperative or noncooperative) model, then subsequent analysis can focus on recovering/identifying the specificities of the decision model that underlies the (rationalizable) observed consumption behavior. For brevity, we do not consider recovery issues in this application. However, we will return to recovery (based on our MIP methodology) in the concluding section.

4.1 Verification

To be able to verify the GARP conditions in Theorems 2 and 3, we reformulate these conditions in mixed integer programming (MIP) terms. To obtain this MIP formulation, we define the binary variables $x_{t,v}^M \in \{0,1\}$, with $x_{t,v}^M = 1$ interpreted as $\left(q_t^M, Q_t\right)\ R^M\ \left(q_v^M, Q_v\right)$ (where $\left(q_t^M, Q_t\right)\ R^M\ \left(q_v^M, Q_v\right)$, $M = A, B$, has a straightforwardly similar meaning as $q_t R q_v$ in Section 2). Then, a data set S satisfies the necessary and sufficient condition for noncooperative-rationalizability in Theorem 3 if and only if the following MIP problem is feasible:

For all $t, v \in T$ and all $k \in K$, there exist strictly positive vectors $\tilde{P}_t^A, \tilde{P}_t^B$ and binary variables $z_{t,k}$, $x_{t,v}^M \in \{0,1\}$ such that (for $s, t, v \in T$, $k \in K$, $M = A, B$):[12]

$$\tilde{P}_t^M \leq P_t, \tag{2}$$

$$P_{t,k} - \tilde{P}_{t,k}^A \leq z_{t,k} C_t, \tag{3}$$

$$P_{t,k} - \tilde{P}_{t,k}^B \leq (1 - z_{t,k}) C_t, \tag{4}$$

$$q_t^A + q_t^B = q_t, \tag{5}$$

$$\langle p_t, q_t^M - q_v^M \rangle + \left\langle \tilde{P}_t^M, Q_t - Q_v \right\rangle < x_{t,v}^M C_t, \tag{6}$$

$$x_{t,s}^M + x_{s,v}^M \leq 1 + x_{t,v}^M, \tag{7}$$

$$(1 - x_{t,v}^M) C_v \geq \langle p_v, q_v^M - q_t^M \rangle + \left\langle \tilde{P}_v^M, Q_v - Q_t \right\rangle, \tag{8}$$

with $C_t > P_{t,k}$ and $C_t > Y_t$ for all t and k.

The interpretation is as follows. Constraint (5) imposes that the private consumption bundles q_t^A and q_t^B sum to the observed aggregate quantities q_t, as required by condition NC.1. Further, constraints (2)-(4) comply with condition NC.2 in Theorem 3. The constraint (2) imposes the given upper bound restriction for $\tilde{P}_t^A$ and $\tilde{P}_t^B$. Next, (3) imposes $P_{t,k} \leq \tilde{P}_{t,k}^A$ if $z_{t,k} = 0$, while (4) imposes $P_{t,k} \leq \tilde{P}_{t,k}^B$ if $z_{t,k} = 1$. Because $z_{t,k} \in \{0,1\}$, this implies $\max\{\tilde{P}_{t,k}^A; \tilde{P}_{t,k}^B\} = P_{t,k}$ and thus condition NC.2 is satisfied. Finally, constraints (6)-(8) correspond to the GARP conditions for each individual M ($= A$ or B) (condition NC.3 in Theorem 3). The constraint (6) states that $\langle p_t, q_t^M - q_v^M \rangle + \left\langle \tilde{P}_t^M, Q_t - Q_v \right\rangle \geq 0$ implies $x_{t,v}^M = 1$ (or $\left(q_t^M, Q_t\right)$ $R^M\ \left(q_v^M, Q_v\right)$). Next, constraint (7) imposes transitivity of the individual revealed preference relations R^M: if $x_{t,s}^M = 1$ (i.e. $\left(q_t^M, Q_t\right)\ R^M\ \left(q_s^M, Q_s\right)$) and $x_{s,v}^M = 1$ (i.e. $\left(q_s^M, Q_s\right)\ R^M\ \left(q_v^M, Q_v\right)$) then $x_{t,v}^M = 1$ (i.e. $\left(q_t^M, Q_t\right)\ R^M\ \left(q_v^M, Q_v\right)$)). And (8) requires $\langle p_v, q_v^M - q_t^M \rangle + \left\langle \tilde{P}_v^M, Q_v - Q_t \right\rangle \leq 0$ if $x_{t,v}^M = 1$ (i.e. $\left(q_t^M, Q_t\right)\ R^M\ \left(q_v^M, Q_v\right)$). Clearly, all constraints are linear, which implies that the above program can be solved by standard MIP methods for a given data set S.

[12] The strict inequality $\langle p_t, q_t^M - q_v^M \rangle + \left\langle \tilde{P}_t^M, Q_t - Q_v \right\rangle < x_{t,v}^M C_t$ is difficult to use in IP analysis. Therefore, in practice we can replace it with $\langle p_t, q_t^M - q_v^M \rangle + \left\langle \tilde{P}_t^M, Q_t - Q_v \right\rangle + \epsilon \leq x_{t,v}^M C_t$ for ϵ (> 0) arbitrarily small.

4.2 Data

We refer to Cherchye, De Rock and Vermeulen (2009, 2010b) for a detailed discussion of the RLMS data that we use. These authors also provide more specific information on the assignability procedure that we present below. For compactness, we restrict ourselves to a brief summary here.

Our sample consists of 148 adult couples, with both (female and male) household members employed. We consider each of the 148 households separately, which avoids (often debatable) preference homogeneity assumptions across male or female members of different households. This illustrates the use of our method for a panel data set. However, it is worth emphasizing that revealed preference methods such as ours are equally applicable to (repeated) cross-section data sets. In this respect, we refer to Blundell, Browning and Crawford (2003, 2008) for some recent methodological advances.

Our data set covers the period from 1994 to 2003. We have consumption data for each year except for the years 1997 and 1999, so that we end up with 8 ($= |T|$) observations (prices and quantities) per household. We consider bundles consisting of 21 ($= |J| + |K|$) nondurable goods: (1) food outside the home, (2) clothing, (3) car fuel, (4) wood fuel, (5) gas fuel, (6) luxury goods, (7) services, (8) housing rent, (9) bread, (10) potatoes, (11) vegetables, (12) fruit, (13) meat, (14) dairy products, (15) fat, (16) sugar, (17) eggs, (18) fish, (19) other food items, (20) alcohol and (21) tobacco. We assume that wood fuel, gas fuel and housing rent are public ($|K| = 3$), while the other goods are private ($|J| = 18$).

Our application will show the possibility of including specific information on q_t^A and q_t^B, i.e. we can assign private consumption to individuals A and B. Formally, this means that assignable quantities q_t^{aM} ($M = A, B$) act as lower bounds for the quantities q_t^M, i.e.

$$q_t^M \geq q_t^{aM}.$$

Essentially, the procedure starts from a base scenario for the distribution of the privately consumed quantities across the two household members. Because assignable quantity information is not directly available from the RLMS data set, this base scenario uses the observed consumption of male and female singles (or one-person households).[13] In subsequent steps, we consider less and less assignability, i.e. we account for (ever larger) deviations from the base scenario distribution. Formally, using q_t^{bM} for the private quantities of member M that correspond to the hypothesized

[13] For example, it is observed that the average budget share of alcohol for male singles is (about) 5 times the corresponding budget share for female singles. Given this, in the base scenario the male consumes 5/6 of all alcohol bought by the household and the female consumes 1/6. See Cherchye, De Rock and Vermeulen (2010b) for a more detailed discussion of the base scenario that we use here.

base scenario, we define

$$q_t^{aM} = \kappa q_t^{bM},$$

with $0 \leq \kappa \leq 1$. The parameter κ captures the extent to which we allow for deviations from the base scenario distribution. For example, $\kappa = 1$ implies $q_t^{aM} = q_t^{bM}$, while $\kappa < 1$ implies $q_t^{aM} < q_t^{bM}$. Generally, lower κ values imply less stringent restrictions for the private quantities. Varying the value of κ will allow us to compare the cooperative model and the noncooperative model under different degrees of assignability.

4.3 Results

Table 1 presents pass rates for the cooperative model and the noncooperative model under different degrees of assignability (captured by κ). The table reveals that pass rates increase if κ decreases. This is not surprising given that lower κ values comply with less assignable information for the privately consumed quantities. For one household, we need $\kappa = 0.60$ for a rationalization in terms of the cooperative model as well as the noncooperative model. If we look at the aggregate pass rates in Table 1, we do not find much difference between the cooperative model and the noncooperative model. To some extent, this provides empirical support for both types of non-unitary models (conditional on the base scenario that is assumed). Our data set does not allow us to empirically distinguish between the two models for most households.

Still, even though the two models provide a rather good overall fit of observed household behavior, there are some notable differences for specific households. For example, for $\kappa = 0.90$ the noncooperative model rationalizes the behavior of two more households than the cooperative model, while for $\kappa = 0.80$ we observe a better fit of the cooperative model. Table 2 provides more detailed results pertaining to individual households. It reports on (i) the number of households that are noncooperative-rationalizable but not cooperative-rationalizable and (ii) the number of households that are cooperative-rationalizable but not noncooperative-rationalizable. The table suggest that the adequate behavioral model varies with the household under consideration: for some households the cooperative model provides a better fit of observed behavior than the noncooperative model, while the opposite holds for other households. Generally, this motivates the empirical relevance of considering the noncooperative model of household behavior in addition to the (more common) cooperative model.

As a further base of comparison, we have also calculated power results for the different model specifications. For each household and each κ, we compute a power measure that quantifies the probability of detecting random behavior. Random behavior is then modeled using a bootstrap method: for each observation, with given prices and income, we define quantities by randomly drawing budget shares (for the 21 goods) from the set of 1184 (= 148 x 8) observed household choices. Thus, our power assessment gives information on the expected distribution of violations under random choice, while incorporating information on the households' actual choices.[14]

Table 1 reports on the distribution of the power measure defined over the 148 households under study. These results are based on Monte Carlo-type simulations that include 1000 iterations. We find that the power varies a lot across households and models: while it is reasonably high for some households (see in particular the maximum and 3rd quartile values for higher κ), it is also very low for other households (see the minimum and 1st quartile values). Generally, these results suggest that assignable quantity information can be particularly helpful to enhance the power of tests for non-unitary models (with or without cooperation). Next, we recall that our analysis uses only 8 observations per household. Obviously, power can only improve when more observations become available.

In the context of the present study, it seems particularly interesting to compare the power of the cooperative and noncooperative models.[15] For the data under consideration, we observe that the power distribution for the noncooperative model is situated somewhat below the one for the cooperative model for each value of κ. However, the difference is very small; we can safely conclude that the power distributions are generally close to each other. In our opinion, this provides additional motivation for considering the noncooperative consumption model in addition to the cooperative model.

[Table 1 about here]

[Table 2 about here]

As a final exercise, we illustrate the possibility to test separate spheres and alternative assumptions regarding the number of jointly contributed public goods (in casu with joint contributions for one, two or three goods). As explained above, such assumptions imply linear restrictions that are easily included in the MIP formulation of our noncooperative rationalizability conditions. Table 3 presents our results. For the scenarios with joint contributions to less than three public goods, we also indicate the identity of the public goods (with good 1 = wood fuel, good 2 = gas fuel

[14]See Bronars (1987) and Andreoni and Harbaugh (2006) for general discussions on alternative procedures to evaluate power in the context of revealed preference tests such as ours.

[15]To compute the power results in Table 1, we have used the same distribution of randomly drawn budget shares to evaluate the cooperative and noncooperative models. Obviously, this is needed to meaningfully compare the power of the two types of models.

and good 3 = housing rent). For compactness, we only provide results for $\kappa = 1.00$; the pattern of results for other values of κ is largely similar to the one in Table 1. A similar qualification holds for the power results associated with the different models, which are also not reported here. Of course, power is slightly higher under additional restrictions.[16]

A first observation from Table 3 is that the assumptions of separate spheres and jointly contributed public goods do matter for our tests of the noncooperative model. We do find this noncooperative model no longer rationalizes the behavior of some households under these additional assumptions; the more restrictive models are generally associated with a lower pass rate. Also, it seems that the identity of the jointly contributed public good may play an important role. For example, in our application the pass rate decreases substantially if we assume that household members provide joint contributions to public good 3. Next, for the given data set we cannot reject that some households have joint contributions to two or three public goods. We note that this need not necessarily contradict the theoretical result of Browning, Chiappori and Lechene (2010), which states that (generically) the number of public goods to which both individuals contribute is at most one. Specifically, we have only 8 observations per household in this study. It may well be that we would empirically reject joint contributions to more than one public good for every household if we had more than 8 observations.

[Table 3 about here]

5 Concluding discussion

We have presented a revealed preference methodology for analyzing noncooperative household consumption behavior. We started from a global characterization of the noncooperative consumption model, which complements the existing local differential characterization. Our approach allows for an empirical analysis of such behavior while avoiding (typically nonverifiable) parametric structure for the household decision process. Such analysis can make use of MIP techniques, and is thus easy-to-implement. Interestingly, our framework extends to tests for separate spheres and joint contribution to public goods. Our application to RLMS data suggests the empirical relevance of considering the noncooperative consumption model in addition to the (more common) model that assumes cooperative behavior.

To focus our discussion, we have concentrated on the characterization of consumption models with and without cooperation, and testing consistency of observed behavior with alternative model specifications. If observed behavior is consistent

[16] Specific pass rates and power results for various values of κ are available upon request.

with a particular model (i.e. can be rationalized), then a natural next question pertains to recovering/identifying the decision model that underlies the (rationalizable) observed consumption behavior. Such recovery can start from the MIP methodology presented in this paper. In this respect, we can refer to Cherchye, De Rock and Vermeulen (2010b), who consider these questions for the cooperative model; their analysis is directly extended to the noncooperative model discussed here. Their basic argument is that nonparametric revealed preference recovery on the basis of an MIP characterization of rational behavior boils down to defining feasible sets characterized by the MIP constraints.

We see at least two interesting applications of recovery. First, recovery can focus on the individuals' MWTP for the publicly consumed goods. As indicated above, lack of intrahousehold cooperation implies that the sum of these individual MWTP deviates from the observed prices for the publicly consumed goods. The MIP method can be used for quantifying this discrepancy between MWTP and observed prices (as a measure for the efficiency loss caused by noncooperation) in empirical applications. Next, one can try to recover the income distribution that is associated with rationalizable behavior while accounting for noncooperation. As a matter of fact, the literature on cooperative household consumption behavior has paid considerable attention to analyzing the intrahousehold distribution underlying observed cooperative-rationalizable behavior. See, for example, Browning, Bourguignon, Chiappori and Lechene (1994), Blundell, Chiappori and Meghir (2005), Browning, Chiappori and Lewbel (2006) and Lewbel and Pendakur (2008), who focus on various welfare-related questions associated with sharing rule recovery. The methodology presented in this paper allows for analyzing similar questions for the noncooperative model.[17]

As for empirical applications, we remark that our methodology can also be used to analyze multi-person group behavior. Indeed, a lot of situations involve groups of individuals spending a joint budget; e.g. decisions of committees, clubs, villages and other local organizations, or firms with multiple decision makers. Chiappori and Ekeland (2006, 2009) suggest the cooperative (Pareto efficient) model as a natural benchmark for assessing the collective rationality of such group decisions. Our methodology allows for assessing group decisions that do not meet this benchmark. In this respect, an interesting avenue for follow-up research consists of analyzing group consumption behavior on the basis of data gathered by means of a laboratory experiment. In fact, it has been argued that the nonparametric revealed preference

[17]However, we recall our discussion (at the end of Section 3.1) on identifiability problems for noncooperative household consumption behavior when both individuals contribute the same public goods. In this case, it is only possible to recover upper and lower bounds on the individual income shares that account for the total (non-assignable) expenditures on these jointly contributed public goods.

methodology is particularly useful in combination with such experimental data. See, for example, Sippel (1997), Harbaugh, Krause and Berry (2001) and Andreoni and Miller (2002) for earlier applications that experimentally analyze individually rational behavior. Interestingly, such an experimental set-up easily allows for gathering (individual-specific and group-specific) information that can help us to better understand the mechanics underlying group consumption behavior (e.g. the within-group income distribution and the within-group (non)cooperation).

Finally, as indicated above, our own application to RLMS data motivates the empirical analysis of models that account for limited intrahousehold cooperation. In fact, this can also suggest a need for 'semicooperative' household consumption models, which allow for 'partial' cooperation between household members. Because these semicooperative models are situated between the fully cooperative model and the fully noncooperative model, they may often provide a more realistic description of household consumption behavior. d'Aspremont and Dos Santos Ferreira (2009) provided a first study of such a semicooperative model. Focusing on the local differential characterization of semicooperative behavior, they also derive a corresponding rank condition on the (pseudo) Slutsky matrix. We see the development of a revealed preference approach to analyze semicooperative behavior as an interesting avenue for follow-up research.

Appendix 1: proof of Theorem 3

1⇒2. Pick any $t \in T$ and consider **OP-N**. Let $U^M_{q_t^M}$ and $U^M_{Q_t}$ $(M = A, B)$ be the subgradients for the function U^M at bundle (q_t^M, Q_t), and λ_t^A and λ_t^B the Lagrange multipliers for the budget constraints in **OP-N**. The first order conditions for **OP-N** are:

$$\begin{aligned}
U^A_{q_t^A} &\leq \lambda_t^A p_t, \\
U^B_{q_t^B} &\leq \lambda_t^B p_t, \\
U^A_{Q_t} &\leq \lambda_t^A P_t, \\
U^B_{Q_t} &\leq \lambda_t^B P_t.
\end{aligned}$$

The inequalities are replaced by equalities in case the quantities of the goods under consideration are strictly positive. Next, concavity of the utility functions U^A and U^B implies for all $t, v \in T$:

$$\begin{aligned}
U^A(q_t^A, Q_t) - U^A(q_v^A, Q_v) &\leq \left\langle U^A_{q_v^A}, q_t^A - q_v^A \right\rangle + \left\langle U^A_{Q_v}, Q_t - Q_v \right\rangle, \\
U^B(q_t^B, Q_t) - U^B(q_v^B, Q_v) &\leq \left\langle U^B_{q_v^B}, q_t^B - q_v^B \right\rangle + \left\langle U^B_{Q_v}, Q_t - Q_v \right\rangle.
\end{aligned}$$

For all $t \in T$, define $U^A_{Q_t}/\lambda^A_t = \tilde{P}^A_t$ and $U^B_{Q_t}/\lambda^B_t = \tilde{P}^B_t$, $U^A(q^A_t, Q_T) = U^A_t$ and $U^B(q^B_t, Q_t) = U^B_t$. This gives:

$$U^A_t - U^A_v \leq \lambda^A_v \left(\langle p_v, q^A_t - q^A_v \rangle + \left\langle \tilde{P}^A_v, Q_t - Q_v \right\rangle \right),$$
$$U^B_t - U^B_v \leq \lambda^B_v \left(\langle p_v, q^B_t - q^B_v \rangle + \left\langle \tilde{P}^B_v, Q_t - Q_v \right\rangle \right).$$

Using Theorem 1, we know that these two conditions are equivalent to the conditions that $\{p_t, \tilde{P}^A_t, q^A_t, Q_t\}_{t \in T}$ and $\{p_t, \tilde{P}^B_t, q^B_t, Q_t\}_{t \in T}$ satisfy GARP. This obtains NC.3.

Also, if $\tilde{P}^A_{t,k} < P_{t,k}$, we know that $Q^A_{t,k} = 0$ and, thus, $Q^B_{t,k} = Q_{t,k} > 0$. Then, the first order condition for $k \in K$ in **OP-N** must be binding, so that $\tilde{P}^B_{t,k} = P_{t,k}$. This obtains the first part of NC.2. Reversing the roles of A and B shows the other part of NC.2. Similarly, one can verify NC.4 and NC.5.

2⇒1. From the GARP conditions and Theorem 1 we know that there exist positive numbers U^A_t, U^B_t and strictly positive numbers λ^A_t and λ^B_t such that:

$$U^A_t - U^A_v \leq \lambda^A_v \left(\langle p_v, q^A_t - q^A_v \rangle + \left\langle \tilde{P}^A_v, Q_t - Q_v \right\rangle \right),$$
$$U^B_t - U^B_v \leq \lambda^B_v \left(\langle p_v, q^B_t - q^B_v \rangle + \left\langle \tilde{P}^B_v, Q_t - Q_v \right\rangle \right).$$

Define the functions U^A and U^B such that:

$$U^A(q^A, Q) = \min_{v \in T} \left\{ U^A_v + \lambda^A_v \left(\langle p_v, q^A - q^A_v \rangle + \left\langle \tilde{P}^A_v, Q - Q_v \right\rangle \right) \right\},$$
$$U^B(q^B, Q) = \min_{v \in T} \left\{ U^B_v + \lambda^B_v \left(\langle p_v, q^B - q^B_v \rangle + \left\langle \tilde{P}^B_v, Q - Q_v \right\rangle \right) \right\}.$$

Notice that U^A and U^B are continuous, concave, strictly monotone and that for all $t \in T$, $U^A(q^A_t, Q_t) = U^A_t$ and $U^B(q^B_t, Q_t) = U^B_t$. See, for example, Varian (1982).

We need to show that the functions U^A and U^B provide a noncooperative-rationalization of the data set. For brevity, we only provide the argument for U^A, but a straightforwardly analogous reasoning applies to U^B. For all $t \in T$, define Q^A_t and Q^B_t so that if $\tilde{P}^A_{t,k} < P_{t,k}$ then $Q^A_{t,k} = 0$ and $Q^B_{t,k} = Q_{t,k}$, and if $\tilde{P}^B_{t,k} < P_{t,k}$ then $Q^B_{t,k} = 0$ and $Q^A_{t,k} = Q_{t,k}$ (see NC.4 and NC.5). (If $\tilde{P}^A_{t,k} = P_{t,k}$ and $\tilde{P}^B_{t,k} = P_{t,k}$, then we can randomly allocate $Q_{t,k}$ between $Q^A_{t,k}$ and $Q^B_{t,k}$.) Next, consider $t \in T$ and a bundle (q^A, Q^A) with $Q = Q^A + Q^B_t$ such that

$$\langle p_t, q^A \rangle + \langle P_t, Q^A \rangle \leq \langle p_t, q^A_t \rangle + \langle P_t, Q^A_t \rangle.$$

Then, we have to prove $U^A(q^A, Q) \leq U^A(q^A_t, Q_t)$. To obtain this result, we first note that by construction $\left\langle \tilde{P}^A_t, Q^A_t \right\rangle = \langle P_t, Q^A_t \rangle$. Thus, because $\tilde{P}^A_t \leq P_t$ (which implies $\left\langle \tilde{P}^A_t, Q^A \right\rangle \leq \langle P_t, Q^A \rangle$), we get $\left\langle \tilde{P}^A_t, Q^A - Q^A_t \right\rangle \leq \langle P_t, Q^A - Q^A_t \rangle$. Using this, we then obtain

$$
\begin{aligned}
& U^A(q^A, Q) \\
= \;& \min_{v \in T} \left\{ U_v^A + \lambda_v^A \left(\langle p_v, q^A - q_v^A \rangle + \left\langle \tilde{P}_v^A, Q - Q_v \right\rangle \right) \right\} \\
\leq \;& U_t^A + \lambda_t^A \left(\langle p_t, q^A - q_t^A \rangle + \left\langle \tilde{P}_t^A, Q - Q_t \right\rangle \right) \\
= \;& U_t^A + \lambda_t^A \left(\langle p_t, q^A - q_t^A \rangle + \left\langle \tilde{P}_t^A, Q^A - Q_t^A \right\rangle \right) \\
\leq \;& U_t^A + \lambda_t^A \left(\langle p_t, q^A - q_t^A \rangle + \langle P_t, Q^A - Q_t^A \rangle \right) \\
\leq \;& U_t^A
\end{aligned}
$$

This provides the wanted result, i.e. $\{q_t^A, Q_t^A\}$ solves **OP-N**.

Appendix 2: independence - examples

Throughout, we will use ε to represent a strictly positive but sufficiently small number.

Example 1: cooperative-rationalizable but not noncooperative-rationalizable

We first construct a data set that is cooperative-rationalizable but not noncooperative-rationalizable. The data set contains 3 observations ($T = \{t, v, w\}$) and 3 public goods ($K = \{1, 2, 3\}$). More specifically, the set S contains the following information:

$$
Q_t = \begin{pmatrix} 1 \\ \varepsilon \\ \varepsilon \end{pmatrix}, \qquad Q_v = \begin{pmatrix} \varepsilon \\ 1 \\ \varepsilon \end{pmatrix}, \qquad Q_w = \begin{pmatrix} \varepsilon \\ \varepsilon \\ 1 \end{pmatrix} \text{ and}
$$

$$
P_t = \begin{pmatrix} 7 \\ 4 \\ 4 \end{pmatrix}, \qquad P_v = \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix}, \qquad P_w = \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}.
$$

To show cooperative-rationalizability, we consider the following specification:

$$\tilde{P}_t^A = \begin{pmatrix} 7-\varepsilon^2 \\ 4-\varepsilon \\ 4-\varepsilon \end{pmatrix}, \qquad \tilde{P}_v^A = \begin{pmatrix} 4-\varepsilon \\ 3.5 \\ \varepsilon \end{pmatrix}, \qquad \tilde{P}_w^A = \begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon^2 \end{pmatrix} \text{ and}$$

$$\tilde{P}_t^B = \begin{pmatrix} \varepsilon^2 \\ \varepsilon \\ \varepsilon \end{pmatrix}, \qquad \tilde{P}_v^B = \begin{pmatrix} \varepsilon \\ 3.5 \\ 4-\varepsilon \end{pmatrix}, \qquad \tilde{P}_w^B = \begin{pmatrix} 4-\varepsilon \\ 4-\varepsilon \\ 7-\varepsilon^2 \end{pmatrix}.$$

This specification clearly meets the condition $\tilde{P}_s^A + \tilde{P}_s^B = P_t$ $(s \in T)$. By computing for both members all inner vector-products, $\tilde{P}_s^M Q_u (s, u \in T, M = A, B)$, it is straightforward to verify that $\{\tilde{P}_t^A, Q_t\}_{t \in T}$ and $\{\tilde{P}_t^B, Q_t\}_{t \in T}$ both satisfy GARP. As such the data set meets the necessary and sufficient conditions for cooperative-rationalizability in Theorem 2.

We still need to prove that the data set S is not noncooperative-rationalizable. Recall that we must have $\max\left\{\tilde{P}_{s,k}^A, \tilde{P}_{s,k}^B\right\} = P_{s,k}$ for all $s \in T$ and $k \in K$. Thus, we have to specify $\tilde{P}_{t,1}^A = 7$ or $\tilde{P}_{t,1}^B = 7$. Without loss of generality, we assume $\tilde{P}_{t,1}^A = 7$. Then, given that ε is small enough, it directly follows that $Q_t R^A Q_v$ and $Q_t R^A Q_w$. Similarly, for observation v, there must be an individual M $(= A$ or $B)$ so that $\tilde{P}_{v,2}^M = 7$. Because the set $\{\tilde{P}_t^A, Q_t\}_{t \in T}$ has to satisfy GARP (and $Q_t R^A Q_v$), we have to choose $M = B$ and thus $Q_v R^B Q_t$ and $Q_v R^B Q_w$. Finally, we must specify M $(= A$ or $B)$ so that $\tilde{P}_{w,3}^M = 7$. Any choice of M makes that GARP is violated either by the set $\{\tilde{P}_t^A, Q_t\}_{t \in T}$ (because $Q_t R^A Q_w$) or by the set $\{\tilde{P}_t^B, Q_t\}_{t \in T}$ (because $Q_v R^B Q_w$). We conclude that the given data set does not meet the necessary and sufficient conditions for noncooperative-rationalizability in Theorem 3.

Example 2: noncooperative-rationalizable but not cooperative-rationalizable

We next construct a data set that is noncooperative-rationalizable but not cooperative-rationalizable. This data set contains 4 observations $(T = \{t, v, w, z\})$ and 4 public goods ($K = \{1, 2, 3, 4\}$):

$$Q_t = \begin{pmatrix} 100 \\ 11 \\ \varepsilon \\ 20 \end{pmatrix}, \; Q_v = \begin{pmatrix} 20 \\ \varepsilon \\ 11 \\ 100 \end{pmatrix}, \; Q_w = \begin{pmatrix} 5 \\ 10 \\ 10 \\ 5 \end{pmatrix}, \; Q_z = \begin{pmatrix} 10 \\ 4 \\ 4 \\ 10 \end{pmatrix} \text{ and}$$

$$P_t = \begin{pmatrix} 1 \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix}, \; P_v = \begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ 1 \end{pmatrix}, \; P_w = \begin{pmatrix} \varepsilon \\ 1 \\ 1 \\ \varepsilon \end{pmatrix}, \; P_z = \begin{pmatrix} 1 \\ \varepsilon \\ 1 \\ 1 \end{pmatrix}.$$

We first demonstrate that this data set is noncooperative-rationalizable. To see this, we consider the following specification:

$$\tilde{P}_t^A = \begin{pmatrix} 1 \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix}, \; \tilde{P}_v^A = \begin{pmatrix} \varepsilon^3 \\ \varepsilon \\ \varepsilon^3 \\ \varepsilon^3 \end{pmatrix}, \; \tilde{P}_w^A = \begin{pmatrix} \varepsilon \\ 1 \\ \varepsilon \\ \varepsilon \end{pmatrix}, \; \tilde{P}_z^A = \begin{pmatrix} 1 \\ \varepsilon \\ 1 \\ \varepsilon \end{pmatrix} \text{ and}$$

$$\tilde{P}_t^B = \begin{pmatrix} \varepsilon^3 \\ \varepsilon^3 \\ \varepsilon \\ \varepsilon^3 \end{pmatrix}, \; \tilde{P}_v^B = \begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ 1 \end{pmatrix}, \; \tilde{P}_w^B = \begin{pmatrix} \varepsilon \\ \varepsilon \\ 1 \\ \varepsilon \end{pmatrix}, \; \tilde{P}_z^B = \begin{pmatrix} \varepsilon \\ \varepsilon \\ 1 \\ 1 \end{pmatrix}.$$

This specification clearly meets the condition $\max\left\{\tilde{P}_{s,k}^A, \tilde{P}_{s,k}^B\right\} = P_{s,k}$ ($s \in T$ and $k \in K$). Again, it is straightforward to verify that the sets $\{\tilde{P}_t^A, Q_t\}_{t \in T}$ and $\{\tilde{P}_t^B, Q_t\}_{t \in T}$ both satisfy GARP. Therefore, we conclude that the given data set meets the necessary and sufficient conditions for noncooperative-rationalizability in Theorem 3.

Next, it can be verified that the given data set does not pass the condition for consistency with the cooperative model that is given in Proposition 2 of Cherchye, De Rock and Vermeulen (2007); the reasoning is similar to the one in their Example 1. For brevity, we do not include the argument here, but it can be obtained upon request. We thus conclude that the given data set violates the necessary and sufficient condition in Theorem 2.

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Table 1: Pass rates and power; cooperative and noncooperative models

Cooperative model						
Value of κ	Pass (on total of on 148 households)	Power (probability of detecting random behavior)				
		minimum	1st quartile	median	3rd quartile	maximum
1.00	137	0.000	0.063	0.087	0.123	0.230
0.90	143	0.000	0.029	0.040	0.056	0.110
0.80	147	0.000	0.011	0.018	0.025	0.056
0.70	147	0.000	0.003	0.007	0.011	0.039
0.60	148	0.000	0.000	0.002	0.005	0.018
Noncooperative model						
Value of κ	Pass (on total of on 148 households)	Power (probability of detecting random behavior)				
		minimum	1st quartile	median	3rd quartile	maximum
1.00	136	0.000	0.062	0.087	0.115	0.215
0.90	145	0.000	0.022	0.034	0.045	0.093
0.80	146	0.000	0.009	0.015	0.022	0.048
0.70	147	0.000	0.003	0.006	0.010	0.036
0.60	148	0.000	0.000	0.002	0.004	0.011

Table 2: Independence; cooperative and noncooperative models

Rationalizability: not cooperative but noncooperative		
Value of κ	Number: cooperative	... but noncooperative
1.00	11	1
0.90	5	3
0.80	1	0
0.70	1	0
0.60	0	0
Rationalizability: not noncooperative but cooperative		
Value of κ	Number: not noncooperative ...	... but cooperative
1.00	12	2
0.90	3	1
0.80	2	1
0.70	1	0
0.60	0	0

Table 3: Additional assumptions; separate spheres and number of jointly contributed public goods (identity of jointly contributed goods between brackets)

Pass (on total of 148 households; $\kappa = 1.00$)	
Seperate spheres model	3 jointly contributed public goods
136	108
2 jointly contributed public goods	1 jointly contributed public good
136 (for any 2 public goods)	136 (for any public good)
131 (for public goods 1 and 2)	135 (for public good 1)
114 (for public goods 1 and 3)	134 (for public good 2)
113 (for public goods 2 and 3)	116 (for public good 3)