# Optimal Interventions in Markets with Adverse Selection<sup>\*</sup>

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November 29, 2010

#### Abstract

We characterize cost-minimizing interventions to restore lending and investment when markets fail due to adverse selection. We solve a mechanism design problem where the strategic decision to participate in a government's program signals information that affects the financing terms of non-participating borrowers. In this environment, we find that the government cannot selectively attract good borrowers, that the efficiency of an intervention is fully determined by the market rate for non-participating borrowers, and that simple programs of debt guarantee are optimal, while equity injections or asset purchases are not. Finally, the government does not benefit from shutting down private markets.

JEL: D02, D62, D82, D86, E44, E58, G01, G2

<sup>\*</sup>We are grateful to the Editor, four outstanding referees, Philip Bond, Florian Heider, Richard Sylla, Jean Tirole and Larry White. We also thank seminar participants at the NY Fed Economic Policy conference, London School of Economics, Stanford University, UC Berkeley, UCLA, University of Maryland, Stern Micro Lunch, Athens University of Economics and Business, and Carnergie Mellon University for their helpful suggestions.

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Akerlof (1970) shows how asymmetric information can lead to a market collapse. Economic and legal institutions-auditors, underwriters, accountants, used-car dealers, etc.-typically emerge to limit adverse selection and allow markets to function, thereby rendering direct government interventions unnecessary. If a market does collapse, however-presumably following the failure of the institutions designed to prevent this collapse in the first place-a government might want to intervene. This paper asks what form these interventions should take if the goal of policy is to improve economic efficiency with minimal cost to taxpayers.

We study an economy with borrowing and investment under asymmetric information. Firms receive profitable investment opportunities but have private information about the value of their existing assets, as in Myers and Majluf (1984). Optimally chosen debt contracts can limit the mispricing of securities used to raise capital, but cannot eliminate it since expected repayments always increase with the quality of assets in place. Adverse selection occurs when good borrowers, perceiving unfairly high capital costs drop out of the market, and lenders rationally charge a high rates to the remaining ones. Lending and investment are, then, inefficiently low, and there is scope for a government intervention.

We characterize cost-minimizing interventions that improve lending and investment and propose implementations with standard financial contracts. When designing its intervention, the government takes into account that participation in its program is a signal of private information used by agents outside the program. Therefore, participation decisions affect outside options through signaling, while outside options influence the cost of the program through participation constraints. This feedback distinguishes our work from the standard mechanism-design literature, in which outside options are exogenous.

The range of design choices—from the selection of types to the size of the program and the nature of financial contracts to be offered—is broad. For instance, since good types are the ones leaving the market, it might make sense for the government to try to selectively attract these types. To do so, it might be optimal to offer securities other than those used by the market. On the one hand, if the government succeeds, interventions could be cheap or even profitable. On the other hand, if interventions are costly, it might be important to minimize the size of the program. Finally, the government might be tempted to shut down private markets and be the sole provider of funds. Our contribution is to clarify these ideas.

We derive five results. First, the government cannot selectively attract the best types, and interventions are always costly. Second, the investment level achieved by a program is determined by its impact on the borrowing rate of *non-participating* banks. Third, for a given impact on private markets, the size of the program is irrelevant. In particular, the cost to taxpayers does not depend on

how many firms participate. Fourth, it is strictly optimal to intervene with debt contracts, either by lending directly or by guaranteeing privately-issued debt. Finally, given optimally designed interventions, the government has no incentives to shut down private markets.

Our approach to uncovering the optimal mechanism is to focus on participation decisions and their signaling properties. We first derive a single crossing property: Whenever a particular type chooses to invest within the program rather than outside the program, all inferior types make the same choice. Consequently, the government cannot selectively attract the best types. However, the composition of types opting out of the program determines the outside marginal type–the highest type investing without government support–and the associated rate at which private lenders break even. By considering only the participation constraints, we then derive a lower bound for the costs of interventions. A key property of this lower bound is that it depends only on the outside market rate. Conditional on the market rate, participation in the program can vary greatly but is irrelevant because lending at the market rate is a zero net-present-value proposition.

We then return to our original design problem with participation, signaling, incentive and investment constraints. We show that direct-lending and debt-guarantee programs are optimal, for the following reasons. When the government announces its willingness to lend at a particular rate, this rate becomes the equilibrium rate in the market. All investing types are indifferent between opting in or out, and the equilibrium allocation of types between the market and the government must be such that private lenders break even at the announced rate. Investment and incentive constraints are then satisfied and, the government minimizes its expected loss since all participation constraints are binding. Other programs (e.g., equity injections) are not optimal because participation constraints cannot bind for several types at once. A corollary of our results is that there is no need to shut down private markets. Without markets, the binding participation constraints become an incentive constraint. Since they refer to the same marginal type, the cost of intervention is the same.

We finally extend our benchmark model by relaxing the assumption that investment opportunities are the same for all types. Our results continue to hold with asymmetric information about new opportunities. When we allow banks to choose the riskiness of their investments after they opt into the program, we find that moral hazard is mitigated by the endogenous response of the private interest rate, and can be eliminated by indexing the terms of the government's program to that rate.

#### Discussion of the literature

Our work is motivated by the history of financial crises. Calomiris and Gorton (1991) ana-

lyze the evolution of two competing views of banking panics. The "random withdrawal" theory (Diamond and Dybvig 1983, Bhattacharya and Gale 1987, Chari 1989) focuses on bank liabilities and coordination among depositors. The "asymmetric information" theory emphasizes asymmetric information about banks' assets. According to Calomiris and Gorton (1991) and Mishkin (1991), the historical evidence supports the idea that asymmetric information plays a critical role in banking crises. Several features of the financial-market collapse in Fall 2008 also suggest a role for asymmetric information (Heider, Hoerova, and Holthausen 2008, Duffie 2009, Gorton 2009).<sup>1</sup> Governments stepped in with large-scale interventions, but there was no consensus about exactly which programs should be offered.<sup>2</sup> Finally, there is ample evidence that participation in government programs carries a stigma (Corbett and Mitchell 2000, Mitchell 2001, Ennis and Weinberg 2009).

Our paper builds on the rich literature that studies asymmetric information, following Akerlof (1970), Spence (1974), and Stiglitz and Weiss (1981). It is useful to relate our work to the particular branch that deals with security design. Myers and Majluf (1984) argue that debt can be used to reduce mispricing when issuers have private information. Brennan and Kraus (1987) consider various financing strategies to reduce adverse selection. Technically, we build on the contribution of Nachman and Noe (1994), who clarify the conditions under which debt is optimal in a multi-type capital raising game. DeMarzo and Duffie (1999) also discuss the optimality of debt when the security design occurs before private information is learned.

Our paper is also related to the literature on government interventions to improve market outcomes. Some of the literature deals specifically with bank bailouts. Gorton and Huang (2004) argue that the government can bail out banks in distress because it can provide liquidity more effectively than private investors. Diamond and Rajan (2005) show that bank bailouts can backfire by increasing the demand for liquidity and causing further insolvency. Diamond (2001) emphasizes that governments should bail out only the banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bailouts can be designed so as not to

<sup>&</sup>lt;sup>1</sup>Heider, Hoerova, and Holthausen (2008) discuss the collapse of the interbank market. Duffie (2009) discusses the OTC and repo markets. In the OTC market, the range of acceptable forms of collateral was dramatically reduced, "leaving over 80% of collateral in the form of cash during 2008," while the "repo financing of many forms of collateralized debt obligations and speculative-rate bonds became essentially impossible." Gorton (2009) explains how the complexity of securitized assets created asymmetric information about the size and the location of risk. Investors and banks were unable to agree on prices for legacy assets or for bank equity. The classic references on financial crises (Bagehot 1873, Sprague 1910) do not discuss the role of asymmetric information explicitly.

<sup>&</sup>lt;sup>2</sup>In the U.S., the original TARP called for \$700 billion to purchase illiquid assets but was transformed into a Capital Purchase Program (CPP) to invest \$250 billion in U.S. banks. As of August 2009, \$307 billion of outstanding debt was issued by financial companies and guaranteed by the FDIC. The treasury also insured \$306 billions of Citibank's assets, and \$118 billion of Bank of America's. Soros (2009) and Stiglitz (2008) argue for equity injections; Bernanke (2009) favors asset purchases and debt guarantee; Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) view purchases and equity injection as the best alternatives; and Ausubel and Cramton (2009) argue for a careful way to 'price the assets, either implicitly or explicitly.'

distort ex-ante lending incentives. Efficient bailouts are studied by Philippon and Schnabl (2009) in the context of debt overhang, and by Farhi and Tirole (2009) in the context of collective moral hazard, while Chari and Kehoe (2009) argue that the time inconsistency problem is more severe for the government than for private agents.

Some papers study government interventions in the presence of competitive markets. Bond and Krishnamurthy (2004) study enforcement when a defaulting borrower can only be excluded from future credit markets. Bisin and Rampini (2006) argue that market access can be a substitute for government's commitment. Golosov and Tsyvinski (2007) study the crowding-out effect of government interventions in private insurance markets.<sup>3</sup> Farhi, Golosov, and Tsyvinski (2009), building on Jacklin (1987), show how liquidity requirements can improve equilibrium allocations. The critical difference is that, in our paper, government intervention affects market conditions through signaling and adverse selection.

The most closely related papers are Minelli and Modica (2009) and Tirole (2010). Minelli and Modica (2009), building on Stiglitz and Weiss (1981), model the intervention as a sequential game between the government and a monopolistic lender. Like us, Tirole (2010) emphasizes the role of endogenous outside options. Our models assume different frictions to limit the financing of new projects. Tirole (2010) assumes moral hazard in addition to adverse selection, while we follow Myers and Majluf (1984) and assume that returns of old and new projects are fungible. Some results are, nonetheless, similar. For instance, Tirole (2010) also finds that the government cannot selectively attract good types, and that it has no incentives to shut down private markets. Another difference between our work and both Minelli and Modica (2009) and Tirole (2010) is that we allow for continuous payoffs (as opposed to binary ones). This allows us to discuss security design.

We present our model in Section 1. In Section 2, we characterize its decentralized equilibria. We formally describe the mechanism-design problem in Section 3. In Section 4, we characterize lower bounds on the costs of government interventions. Those bounds can actually be achieved by simple, common interventions as we show in Section 5. Section 6 discusses extensions, and we close the paper with some final remarks in Section 7.

## 1 The Model

Our model has three dates, t = 0, 1, 2, and a continuum of banks with pre-existing 'legacy' assets. The banks start with private information regarding the quality of their legacy assets and receive the

 $<sup>^{3}</sup>$ There is also an extensive literature on how government interventions can improve risk sharing. See Acemoglu, Golosov, and Tsyvinski (2008). For excellent surveys, see Kocherlakota (2006), Golosov, Tsyvinski, and Werning (2006), and Kocherlakota (2009).

opportunity to make new loans at time 1. To avoid confusion, we refer to the new loans that banks make at time 1 as "new investments," and we use "borrowing and lending" when banks borrow from outside investors (or from other banks). The government can offer various programs at time 0, and banks can borrow and lend in a competitive market at time 1. We assume that all agents are risk-neutral and we normalize the risk-free rate to zero.

#### Initial assets and cash balance

Banks start with cash and legacy assets, and no pre-existing liabilities.<sup>4</sup> Cash is liquid and can be kept, invested or lent at time 1. Let  $c_t$  denote the cash holdings at the beginning of period t. All banks start with  $c_0$  in cash, but  $c_1$  can differ from  $c_0$  if the government injects cash in the banks at time 0. Cash holdings cannot be negative:  $c_t \ge 0$  for all t.

The book value of legacy assets, A, is known, but some assets may be impaired and the eventual payoff at time 2 is the random variable  $a \in [0, A]$ . Banks privately know their type  $\theta$ , which determines the conditional distribution of the value of legacy assets  $f_a(a|\theta)$ . Types are drawn from compact set  $\Theta \subset [\underline{\theta}, \overline{\theta}]$  with cumulative distribution  $H(\theta)$ .

#### Investment and borrowing

Banks receive investment opportunities at time 1. Investment requires the fixed amount x and delivers a random payoff v at time 2. Banks can borrow at time 1 in a competitive market. After learning its type  $\theta$ , a bank offers a contract  $(l, y^l)$  to the competitive investors, where l is the amount raised from investors at time 1, and  $y^l$  is the schedule of repayments to investors at time 2. Without government intervention, the funding gap of the banks is  $l_0 \equiv x - c_0$ . The government can reduce the funding gap with cash injections, denoted m. In this case,  $c_1 = c_0 + m$  and the bank only needs to borrow  $l = l_0 - m$ . We use the generic notation l for the amount actually raised from private investors. In period 2, the cash balance of the bank is  $c_2(i) = c_1 + l - x \cdot i$ , and its total income is

$$\tau_2(i) = c_2(i) + a + v \cdot i, \tag{1}$$

where  $i \in \{0, 1\}$  is a dummy for the decision to invest at time 1. Total bank income at time 2 depends on the realization of the two random variables a and v.

#### Assumptions

We assume in our benchmark model that all banks receive the same investment opportunities. The random payoff v is distributed on [0, V] according to the density function  $f_v(v)$ . Let  $\bar{v} \equiv E[v]$ 

<sup>&</sup>lt;sup>4</sup>This is without loss of generality if efficient renegotiation among creditors is possible. Impediments to renegotiation can create debt overhang. This issue is analyzed in Philippon and Schnabl (2009).

be the expected value of v. To make the problem interesting, we assume that new projects have positive NPV and that banks need to borrow in order to invest:  $\bar{v} > x > c_0$ . We further assume that contracts can be written only on the total income of the bank at time 2:

Assumption A1: The only observable outcome is total income  $\tau_2$  defined in equation (1).

Under Assumption 1, repayment schedules can be contingent on total income  $\tau_2$  but not on a and v separately. If a bank does not invest, it keeps  $c_2 = c_1$ , and its total income at time 2 is  $a + c_1$ . If it invests, it ends up with  $c_2 = 0$  and total income a + v.<sup>5</sup> We define this total income conditional on investment as:  $y \equiv a + v$ . The distribution of y, which is the convolution of  $f_a$  and  $f_v$ , is denoted by f. Since  $f_a$  depends on  $\theta$ , so does f. Let Y denote the support of y. We assume that  $Y \subset [0, \infty)$  is the same for all types and that  $f(y|\theta)$  satisfies the strict monotone hazard rate property:

Assumption A2: For all  $(y, \theta) \in Y \times \Theta$ ,  $f(y|\theta) > 0$ , and  $\frac{f(y|\theta)}{1 - F(y|\theta)}$  is decreasing in  $\theta$ .

Total income is used to repay the loans taken at time 1 according to a schedule  $y^l$ . When the government intervenes, the bank also might need to repay the government, according to a schedule  $y^g$ . Our last assumption is to impose a monotonicity condition on the repayment schedules.

Assumption A3: The repayment schedules  $y^l$  and  $y^g$  of private lenders and of the government are non-decreasing in  $\tau_2$ .

Let us briefly discuss the main features of our model. We introduce a binary investment technology to simplify the strategy space of banks, but it is easy to extend the model to partial investment, for instance, by having  $i \in \{0, 1/2, 1\}$ . In order to stay close to the workhorse model of Myers and Majluf (1984), we initially assume that all banks receive the same investment opportunities. In this context, A1 makes private information relevant by preventing the parties from contracting directly on v (by spinning off the new investment, for instance). As an extension, we introduce private information on v in Section 6. Assumption A2 defines a natural ranking among types regarding the quality of their legacy assets, from the worst type  $\underline{\theta}$  to the best type  $\overline{\theta}$ . The strict inequalities in A2 are not crucial, but they simplify some of the proofs. We allow for a general set of types  $\Theta$  because, while much intuition can be obtained with just two types, the implementation results are somewhat special for the two-type case, as we explain in Section 5. A3 has been standard in the literature on financial contracting since Innes (1990) and Nachman and Noe (1994). It renders optimal contracts more realistic by effectively smoothing sharp discontinuities in repayments, and it can be formally justified by the possibility of hidden trades.<sup>6</sup>

<sup>&</sup>lt;sup>5</sup>It will become clear that the government never finds it optimal to inject cash into the banks beyond  $m = l_0$ .

<sup>&</sup>lt;sup>6</sup>The justification is that if repayments were to decrease with income, the borrower could secretly add cash to

## 2 Equilibria without Interventions

Because the credit market is competitive and investors are risk-neutral, in any candidate equilibrium, the expected repayments to the lenders must be at least the size of the loan

$$E\left[y^l|\mathcal{I}\right] \ge l,\tag{2}$$

where  $\mathcal{I}$  denotes the information set of the private lenders at the time they make the loan. Under symmetric information, investment decisions would have been independent of the quality of legacy assets, and all banks would invest since  $\bar{v} \geq x$ . The symmetric-information allocation is an equilibrium under asymmetric information when banks can issue risk-free debt. By contrast, adverse selection can occur when new investments are risky and when there is significant downside risk on legacy assets.

#### Contracting game

Banks offer financial contracts to investors. A contract specifies an amount borrowed at time 1, denoted l, and a repayment schedule at time 2, denoted  $y^l$ . The contracting game is potentially complex because the kind of security a bank offers might signal its type. Under assumption A1 to A3, however, it is a standard result that all banks that invest offer the same security, and this security is a debt contract.<sup>7</sup> The intuition is that bad types want to mimic good types, while good types seek to separate from bad types. Contracts with high repayments for low income realizations are relatively more attractive for good types than for bad types. This is the core idea of Myers and Majluf (1984) in a model with two types, extended by Nachman and Noe (1994) to an arbitrary set of types.

#### Equilibria without government intervention

We have explained above that all investing banks offer the same debt contract. Our next step is to characterize the set of banks that actually invest. Let r be the (gross) interest rate at which banks borrow. We can define the expected repayment function for type  $\theta$  as:

$$\rho(\theta, rl) \equiv \int_{Y} \min(y; rl) f(y|\theta) \, dy.$$
(3)

the bank's balance sheet by borrowing from a third party, obtaining the lower repayment, immediately repaying the third party, and obtaining strictly higher returns. See Sections 3.6 and 6.6 in Tirole (2006) for further discussion.

<sup>&</sup>lt;sup>7</sup>There are several ways to obtain this result. One is to let each bank offer one contract and solve the issuance/signaling game. Nachman and Noe (1994) show that the unique equilibrium is pooling on the same debt contract as long as the distribution of payoffs can be ranked by hazard rate dominance (A2). Another way to obtain the result is to follow Myerson (1983) and let banks offer a menu of securities in a first stage (see, also, Maskin and Tirole (1992)). The inscrutability principle then ensures that no signaling occurs during the contract-proposal phase, and we can focus on one incentive-compatible menu. Standard design arguments can then be used to show that, under A1-A3, the best menu contains the same debt contract for all types that invest.

For a given  $\theta$ , the function  $\rho(\theta, rl)$  is increasing in the face value rl. Under symmetric information, the fair interest rate  $r_{\theta}^*$  on a loan l to a bank with type  $\theta$  is implicitly given by  $l \equiv \rho(\theta, r_{\theta}^*l)$ . Since  $\rho(\theta, rl)$  is increasing in  $\theta$ , the fair rate is decreasing in  $\theta$  for any given l. With private information, however, the interest rate cannot depend explicitly on  $\theta$ , and higher types end up facing an unfair rate. This is the source of adverse selection.

Without government intervention, banks need to borrow  $l = l_0 = x - c_0$ . Given a market rate r, a type  $\theta$  wants to invest if and only if  $E[a|\theta] + \bar{v} - \rho(\theta, rl_0) \ge E[a|\theta] + c_0$ . The investment condition under asymmetric information is, therefore,<sup>8</sup>

$$\bar{v} - x \ge \rho\left(\theta, rl_0\right) - l_0. \tag{4}$$

The term  $\rho(\theta, rl_0) - l_0$  measures the informational rents paid by the bank. The rents are zero when the rate is fair. The information cost is positive when  $r > r_{\theta}^*$  and negative (a subsidy) when  $r < r_{\theta}^*$ . When informational rents are too large, banks might decide not to invest.

Since the right-hand side of equation (4) is increasing in  $\theta$ , if  $\theta$  wants to invest at rate r, any type below  $\theta$  also wants to invest at that same rate. The set of investing types is, therefore,  $[\underline{\theta}, \hat{\theta}]$ , and the marginal type  $\hat{\theta}$  is defined by

$$\bar{v} - x \equiv \rho\left(\hat{\theta}, rl_0\right) - l_0. \tag{5}$$

The borrowing rate depends on the market's perception about the mix of banks that invest. Let h(.|1) describe the market's beliefs about the type of banks that borrow to invest. Investors' beliefs must be consistent with Bayes' rule:  $H(\theta|1) = H(\theta) / H(\hat{\theta})$  if  $\theta \in [\underline{\theta}, \hat{\theta}]$ , and 0 otherwise. Finally, the rate r must satisfy the zero-profit condition for private investors:

$$l_0 = \int_{\underline{\theta}}^{\hat{\theta}} \rho\left(\theta, r l_0\right) dH\left(\theta|1\right).$$
(6)

**Proposition 1** The efficient outcome is sustainable without government intervention if and only if there is a borrowing rate r such that (4) and (6) hold for  $\hat{\theta} = \bar{\theta}$ . All other equilibria have  $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ and are inefficient.

The intuition for Proposition 1 is as follows. The potential for adverse selection exists because the investment condition (4) is more likely to hold for low types than for high types.<sup>9</sup> Multiple equilibria are possible because of the endogenous response of the interest rate. Let  $r^0$  denote the

<sup>&</sup>lt;sup>8</sup>We use the conventional assumption that, when indifferent, banks choose to invest.

<sup>&</sup>lt;sup>9</sup>If the scale of investment were a choice variable, the separating equilibrium would involve good banks scaling down to signal their types. With our technological assumption, they scale down to zero. The only important point is that in both cases the equilibrium can be inefficient.

lowest interest rate that can be supported without government intervention, and let  $\hat{\theta}^0$  be the corresponding threshold. The best decentralized equilibrium  $\left(r^0, \hat{\theta}^0\right)$  depends on  $c_0$  and on the prior distribution of types.

In the remainder of the paper, we examine cases where the efficient outcome is not sustainable as a decentralized equilibrium-i.e.,  $\hat{\theta}^0 < \bar{\theta}$ . It is clear that higher cash levels increase  $\hat{\theta}^0$  and improve economic efficiency. Therefore, governments might seek to inject liquidity into the banks. Our goal is to design the most cost-effective interventions that achieve a given level of investment. We do so formally in the next sections.

## 3 Mechanism Design with a Competitive Fringe

In this section, we present the government's objective and we describe the mechanism-design problem. Without intervention, the best equilibrium is  $(r^0, \hat{\theta}^0)$  described above. The government's goal it to find the cheapest possible way to implement any given level of investment.<sup>10</sup> We denote the cost of a government program by  $\Psi$ . While the objective of the government is straightforward, the mechanism-design problem is non-standard because we assume that private markets remain open. The market rate for non-participating banks, then, depends on the mechanism the government uses because participation decisions convey information about private types. This interrelationship does not exist in standard mechanism design where outside options are independent from the mechanism.<sup>11</sup> We refer to our model as mechanism design with a "competitive fringe."

#### Government's strategy

A government program  $\mathcal{P}$  is a menu of contracts. The revelation principle applies and we can, without loss of generality, consider programs with one contract per type.<sup>12</sup> A contract specifies the cash m injected at time 1 and the schedule of payments  $y^g$  received by the government at time 2. A generic program, therefore, takes the form:  $\mathcal{P} = \{m_{\theta}, y^g_{\theta}\}_{\theta \in \Theta}$ . Under A3,  $y^g_{\theta}$  is increasing in yfor all  $\theta \in \Theta$ .<sup>13</sup>

<sup>&</sup>lt;sup>10</sup>In a general equilibrium model, one could– after the cost minimization– solve for the optimal level of investment. We do not study this second stage here. Rather, we characterize the cost-minimizing intervention for any particular level of investment the government might want to implement.

<sup>&</sup>lt;sup>11</sup>In common agency problems, the interelationship of the design problems is more complex since both principals that offer contracts have bargaining power. In our paper, the principal's (the government's) mechanism induces a competitive market's response.

 $<sup>^{12}</sup>$ The banks are all ex-ante identical, so the government offers the same menu to all. In a general setup, the government should condition on observable characteristics, such as size, or leverage, and our results apply after this conditioning.

<sup>&</sup>lt;sup>13</sup>This covers any program based on equity payoffs (common stock, preferred stock, warrants, etc.), as well as all types of direct lending and debt-guarantee programs. The case of asset purchases can be analyzed by allowing  $y_{\theta}^{g}$  to depend on a. We discuss this extension in Section 6.

#### Banks' strategy

The strategy of a bank consists of a participation decision and an investment decision as a function of its type. At time 0, after the announcement of the government's program, each bank chooses a contract in  $\mathcal{P}$  or opts out by choosing  $\mathcal{O}$ . We allow banks to randomize their participation decisions. At time 1, given its type and its realized participation decision, each bank decides whether or not to invest:

$$i: \Theta \times \{\mathcal{P} \cup \mathcal{O}\} \to \{0, 1\}.$$

The choice of a government contract in  $\mathcal{P}$  is observed by the market and induces a private lending contract  $(l_{\theta}, y_{\theta}^l)$ . If a bank of type  $\theta$  chooses a contract  $\{m_{\theta'}, y_{\theta'}^g\}$  designed for  $\theta'$ , its cash becomes  $c_1 = c_0 + m_{\theta'}$ . If it invests, it must then borrow  $l_{\theta'} = x - c_0 - m_{\theta'}$  from the private market, and its expected payoff is:

$$V\left(\theta,\theta',1\right) = \int_0^\infty \left(y - y_{\theta'}^l\left(y\right) - y_{\theta'}^g\left(y\right)\right) f\left(y|\theta\right) dy.$$

$$\tag{7}$$

If it does not invest, its expected payoff is  $V(\theta, \theta', 0) = E[a|\theta] + c_0 + m_{\theta'} - \int_0^\infty y_{\theta'}^g(y) f(y|\theta) dy$ . If the bank opts out of the government program, it has the option to borrow in the private market at an interest rate  $\tilde{r}$ . The outside option of a type  $\theta$  bank is, therefore,

$$\tilde{V}(\theta, \tilde{r}) = E\left[a|\theta\right] + \max\left\{c_0, \bar{v} - \rho\left(\theta, \tilde{r}l_0\right)\right\}.$$
(8)

#### **Competitive Fringe**

Regardless of whether a bank opts in or out, the interest rate at which it borrows must satisfy the break-even condition of competitive lenders in equation (2). The information set  $\mathcal{I}$  contains the equilibrium strategies of the banks-the participation and investment mappings-and the observed choices-the particular contract in  $\mathcal{P}$  and the decision to demand a loan of size l. The loan is  $l_0 = x - c_0$  for banks opting out, and  $l_0 - m_{\theta}$  for banks opting in and choosing the contract designed for  $\theta$ .

#### **Equilibrium Conditions:**

Fix a government intervention and market rate  $\tilde{r}$  for non-participating banks. Let  $\Theta_{\mathcal{P},1}$  denote the set of types that participate and invest, and let  $\Theta_{\mathcal{P},0}$  denote the types that participate but do not invest. Define  $\Theta_{\mathcal{P}} = \Theta_{\mathcal{P},1} \cup \Theta_{\mathcal{P},0}$ . Similarly, we can define  $\Theta_{\mathcal{O},1}$  (respectively  $\Theta_{\mathcal{O},0}$ ) to be the corresponding non-participating sets of types, and  $\Theta_{\mathcal{O}} = \Theta_{\mathcal{O},1} \cup \Theta_{\mathcal{O},0}$ . In order to have an equilibrium we must have:

• For  $i \in \{0,1\}$  and  $\theta \in \Theta_{\mathcal{P},i}, V(\theta,\theta,i) \ge \max\left(V(\theta,\theta',j), \tilde{V}(\theta,\tilde{r})\right)$ , for  $j \in \{0,1\}$  and  $\theta' \in \Theta_{\mathcal{P}}$ .

• For all  $\theta \in \Theta_{\mathcal{O}}$ ,  $\tilde{V}(\theta, \tilde{r}) \ge V(\theta, \theta', j)$  for  $j \in \{0, 1\}$  and  $\theta' \in \Theta_{\mathcal{P}}$ , and  $\theta \in \Theta_{\mathcal{O}, 1} \iff \bar{v} - x \ge \rho(\theta, \tilde{r}l_0) - l_0$ .

Private lenders must expect to break even, and their beliefs must be consistent with the equilibrium behavior of the banks. For instance, if  $H_{\mathcal{O},1}$  denotes the market's perception about the distribution of bank types that choose to invest alone, the outside rate  $\tilde{r}$  must satisfy  $l_0 = \int_{\Theta_{\mathcal{O},1}} \left[ \int_Y \min(y, \tilde{r}l_0) f(y|\theta) \, dy \right] dH_{\mathcal{O},1}(\theta)$ . Similar conditions must hold for banks that opt in and borrow  $l_{\theta} = x - c_0 - m_{\theta}$ . Finally, the resource constraint  $y^l(y) + y^g(y) \leq y$  must hold for all contracts. We now, without loss of generality, restrict our attention to interventions where the government receives junior claims-i.e., where new lenders are paid first according to  $y_{\theta}^l(y) = \min(y, r_{\theta}l_{\theta})$ .<sup>14</sup>

To sum up, the design problem is complex because the participation decision is influenced by the non-participation payoffs that depend on the market reaction, which is, in turn, endogenous to the mechanism. In Section 4, we characterize the set of feasible interventions and derive lower bounds on their costs. In Section 5, we show that these bounds are actually achieved by realistic and commonly used interventions.

### 4 Cost Minimizing Interventions

In this section, we study feasible interventions and derive lower bounds for their costs. We analyze interventions where the competitive fringe is active-i.e., where some banks invest without the government's assistance. The case where the competitive fringe is completely inactive-i.e., when  $\Theta_{\mathcal{O},1} = \emptyset$ - is discussed in Section 5.3.

#### 4.1 Feasible Interventions

Any bank opting out of the program can borrow in the competitive fringe at rate  $\tilde{r}$ . This rate defines a marginal type  $\hat{\theta}(\tilde{r})$  for which condition (4) holds with equality. The government knows that the outside option of any type below  $\hat{\theta}$  is to invest, while the outside option of any type above  $\hat{\theta}$  is to do nothing. We are going to show that in all feasible interventions, the types that invest with the help of the government are worse than  $\hat{\theta}$ . We first introduce some notation and establish an important building block of our analysis in Lemma 1.

<sup>&</sup>lt;sup>14</sup>To see this, imagine a program where the government has some senior claims  $\tilde{y}_{\theta}^{gs}$  and some junior claims  $\tilde{y}_{\theta}^{gj}$ . The optimal private debt contract is  $\tilde{y}_{\theta}^{l} = \min(y - y_{\theta}^{gs}, \tilde{r}_{\theta}l_{\theta})$ , and from the resource constraint we have  $y_{\theta}^{gs} \leq y$  and  $\tilde{y}_{\theta}^{gj} = 0$  for all  $y < \tilde{r}(l_{\theta}) l_{\theta}$ . Now consider an alternative program where all government claims are junior. The private contract is  $y_{\theta}^{l} = \min(y, r_{\theta}l_{\theta})$ . Define  $y_{\theta}^{g} = \min(y - y_{\theta}^{l}, \tilde{r}_{\theta}l_{\theta}) + (\tilde{y}_{\theta}^{gs} + \tilde{y}_{\theta}^{gj}) \cdot 1_{y > \tilde{r}_{\theta}l_{\theta}}$ . This contract gives exactly the same payoff function to the bank, so it leaves all participation, incentive, and investment constraints unchanged. Finally, since  $l_{\theta} = E\left[\tilde{y}_{\theta}^{l}\right] = E\left[y_{\theta}^{l}\right]$ , it follows that  $E\left[y_{\theta}^{g}\right] = E\left[\tilde{y}_{\theta}^{gs} + \tilde{y}_{\theta}^{gj}\right]$ , and the cost to the government is unchanged.

Banks that participate and invest receive income  $y - y_{\theta}^l - y_{\theta}^g$ . The difference between inside and outside payoffs conditional on investment in both cases is then:  $\gamma_{\theta}(y) \equiv \min(y, \tilde{r}l_0) - \min(y, r_{\theta}l_{\theta}) - y_{\theta}^g(y)$ . From the participation constraint  $V(\theta, \theta, 1) \geq \tilde{V}(\theta, \tilde{r})$  of investing types, we then obtain:

$$E[\gamma_{\theta}(y)|\theta] \ge 0 \text{ for all } \theta \in \Theta_{\mathcal{P},1}.$$
 (9)

The following Lemma establishes a Single Crossing Property that plays a central role in our analysis.

**Lemma 1** Single Crossing Property. If  $E[\gamma_{\theta}(y)|\theta] \ge 0$  for some  $\theta \in \Theta$ , then  $E[\gamma_{\theta}(y)|\theta'] \ge 0$ for all  $\theta' < \theta$ . If, in addition,  $\gamma_{\theta}(y) \ne 0$  for some  $y \in Y$ , then  $E[\gamma_{\theta}(y)|\theta'] > 0$ .

#### **Proof.** See appendix.

Lemma 1 says that if a type prefers the strategy 'opt-in-and-invest' to the strategy 'invest-alone,' then all types below it have the same preference. This result is driven by the same forces that cause the original market failure-namely, that good types expect to repay *relatively* more than bad types. An important subtlety, however, is that what matters is the difference in expected payments inside or outside the program, both of which are increasing in  $\theta$ . An additional complication is that the function  $\gamma_{\theta}(y)$  is not monotonic. It is typically positive for low values of y and then decreasing. The key is that it can switch sign only once, for some income realization  $\hat{y}$ . This explains why first-order stochastic dominance is not enough and why we need conditional stochastic dominance (or hazard rate dominance) in A2. We can then apply stochastic dominance conditional on income being more (or less) than  $\hat{y}$ , and obtain our result. The interesting point is that A2 is also the necessary and sufficient condition for debt to be optimal in the capital-raising game under asymmetric information (Nachman and Noe 1994).

Lemma 1 has several important implications. First, as long as some banks invest without the government's help, no type above  $\hat{\theta}(\tilde{r})$  invests.

#### **Proposition 2** In all equilibria where the competitive fringe is active, no type above $\hat{\theta}(\tilde{r})$ invests.

**Proof.** For non-participating types  $(\theta \in \Theta_{\mathcal{O}})$ , the Proposition follows from the definition of  $\hat{\theta}(\tilde{r})$ . For participating types, we argue by contradiction. Suppose that there is a participating type  $\theta$  strictly above  $\hat{\theta}(\tilde{r})$  that invests-that is,  $\theta \in \Theta_{\mathcal{P},1}$ . Because  $\theta > \hat{\theta}(\tilde{r})$ , we have that  $\tilde{V}(\theta, \tilde{r}) > E[a|\theta] + \bar{v} - \rho(\theta, \tilde{r}l_0)$ . Moreover, since  $\theta \in \Theta_{\mathcal{P},1}$ , we must have  $E[\gamma_{\theta}(y)|\theta] > 0$ . From Lemma 1, we then know that  $E[\gamma_{\theta'}(y)|\theta] > 0$  for all types  $\theta' < \theta$ . Therefore,  $\left[\underline{\theta}, \hat{\theta}(\tilde{r})\right] \subset \Theta_{\mathcal{P},1}$  and  $\Theta_{\mathcal{O},1} \subset \left[\hat{\theta}(\tilde{r}), \bar{\theta}\right]$ , but then the rate  $\tilde{r}$  must be below the fair rate for the worst type investing alone  $\hat{\theta}(\tilde{r})$ , which implies that (4) cannot hold with equality contradicting the definition of  $\hat{\theta}(\tilde{r})$ .

Proposition 2 shows that the best type investing is  $\hat{\theta}$ . It also implies that we can, without loss of generality, focus on programs where only types below  $\hat{\theta}$  participate in the government program. To see why, imagine that a type  $\theta' > \hat{\theta}(\tilde{r})$  participates. We know that this type does not invest and that it must get at least its outside option. But then, the government can simply have this type drop out (by charging an infinitesimal fee, for instance). This does not affect the outside market rate because  $\theta'$  does not invest. Therefore, we have the following corollary to Proposition 2.

**Corollary 1** Without loss of generality, only types below  $\hat{\theta}(\tilde{r})$  participate in the government's program:  $\Theta_{\mathcal{P}} \subset \left[\underline{\theta}, \hat{\theta}(\tilde{r})\right]$ .

Another important implication of Proposition 2 is that the government cannot design a program that attracts only good banks and induces them to invest. This suggests that all interventions that increase investment will be costly. This is indeed what we show next.

#### 4.2 Minimum Cost of Intervention

In this section, we obtain lower bounds for the cost of interventions by focusing only on the participation constraints. Consider an equilibrium with a given market rate  $\tilde{r}$ . The total value of the banking sector equals the expected value of legacy assets, cash and new investments. From the definition of  $\hat{\theta}(\tilde{r})$  and Proposition 2, it follows that banks with types above  $\hat{\theta}(\tilde{r})$  do not invest. Therefore we have:

$$W(\tilde{r}) = E[a] + c_0 + (\bar{v} - x) \int_{\underline{\theta}}^{\theta(\tilde{r})} i(\theta) \, dH(\theta), \tag{10}$$

where  $i(\theta)$  is the investment choice of type  $\theta$ . Since private lenders break even, the total payoffs of banks are equal to the total value of the banking sector plus the net transfers from the government. In other words, the expected cost  $\Psi$  is:

$$W(\tilde{r}) + \Psi = \int_{\theta \in \Theta_{\mathcal{P}}} V(\theta, \theta, i) + \int_{\theta \in \Theta_{\mathcal{O}}} \tilde{V}(\theta, \tilde{r}).$$
(11)

The participation constraints impose  $V(\theta, \theta, i) \geq \tilde{V}(\theta, \tilde{r})$  for  $i \in \{0, 1\}$ . To minimize  $\Psi$ , we maximize W by setting  $i(\theta) = 1$  for all  $\theta \in [\underline{\theta}, \hat{\theta}(\tilde{r})]$ , and minimize the right-hand side of (11), by making the participation constraints tight. Letting  $\Psi^*$  be the minimum cost, and  $W^*$  the maximized value of W, we have

$$\Psi^*(\tilde{r}) = E\left[\tilde{V}(\theta, \tilde{r})\right] - W^*(\tilde{r}).$$
(12)

From the definition of  $\gamma_{\theta}(y)$ , we know that  $V(\theta, \theta, 1) - \tilde{V}(\theta, \tilde{r}) = E[\gamma_{\theta}(y)|\theta]$  for all  $\theta \in \Theta_{\mathcal{P},1}$ . From (11) and (12), we can then write the excess cost of any program as

$$\Psi - \Psi^* = \int_{\Theta_{\mathcal{P},1}} E\left[\gamma_{\theta}(y)|\theta\right] + \int_{\Theta_{\mathcal{P},0}} \left(V\left(\theta,\theta,0\right) - \tilde{V}\left(\theta,\tilde{r}\right)\right) + W^*\left(\tilde{r}\right) - W\left(\tilde{r}\right).$$
(13)

Equation (13) says that excess costs arise from rents earned by participating types and from inefficient investment. Participation constraints imply that the first two terms are positive, and the definition of  $W^*$  implies that the third term is positive. The following Theorem characterizes the lower bound  $\Psi^*$  and the programs that can reach it.

**Theorem 1** Feasible programs are characterized by an investment cutoff  $\hat{\theta}$  and an associated market rate  $\tilde{r}$ . The cost of a feasible program cannot be less than the informational rents at rate  $\tilde{r}$ 

$$\Psi^*(\tilde{r}) = \int_{\underline{\theta}}^{\hat{\theta}(\tilde{r})} \left( l_0 - \rho\left(\theta, \tilde{r}l_0\right) \right) dH(\theta).$$
(14)

A program reaches the lower bound  $\Psi^*(\tilde{r})$  if and only if all types below  $\hat{\theta}$  invest and  $\gamma_{\theta}(.)$  is identically zero for all types above the lowest type.

**Proof.** Replacing  $W^*(\tilde{r})$  and  $\tilde{V}(\theta, \tilde{r})$  in (12) and using  $l_0 = x - c_0$ , we obtain (14). When i = 1 for all  $\theta \leq \hat{\theta}(\tilde{r})$ , we have  $W(\tilde{r}) = W^*(\tilde{r})$  by definition, and  $\Theta_{\mathcal{P},0} = \emptyset$  from Corollary 1. From equation (9), we know that  $E[\gamma_{\theta}(y)|\theta] \geq 0$  for all participating types. If we want  $\Psi = \Psi^*$  in (13), we must, therefore, have  $E[\gamma_{\theta}(y)|\theta] = 0$  for  $\theta \in \Theta_{\mathcal{P}}$ . From Lemma 1, we further know that if  $\gamma_{\theta}(y) \neq 0$  for some  $y \in Y$  and some type  $\theta \in \Theta_{\mathcal{P}}$ , then  $E[\gamma_{\theta}(y)|\theta'] > 0$  for all  $\theta < \theta'$ , implying that  $\Psi > \Psi^*$ . We must, therefore, have  $\gamma_{\theta}(y) = 0$  for all  $y \in Y$  and all  $\theta \in \Theta_{\mathcal{P},1}/{\{\underline{\theta}\}}$ .

The lower bound cannot be less than the informational rents at rate  $\tilde{r}$  and is strictly positive for all interventions that increase investment-i.e., for all interventions that implement  $\hat{\theta}(\tilde{r}) > \hat{\theta}^0$ , since  $\hat{\theta}^0$  is the highest type for which the break even constraint (6) holds. To go beyond  $\hat{\theta}^0$ , the government is forced to pay information rents. Theorem 1 tells us that the number and types of participating banks matters only through  $\tilde{r}$ . Hence, we have the following Corollary:

**Corollary 2** For a given outside rate  $\tilde{r}$ , the minimum cost does not depend on the participation in the program.

Let  $p(\theta) \in [0,1]$  be the participation rate (the probability of participation in the program) for any type  $\theta \in [\underline{\theta}, \hat{\theta}(\tilde{r})]$ . Corollary 2 tells us that the actual participation rate of various types is irrelevant so long as it leads to the same market rate. The equilibrium participation function p must be such that lenders break even on types opting out, when the rate is  $\tilde{r}$ , that is:

$$l_{0} = \int_{\underline{\theta}}^{\hat{\theta}(\tilde{r})} \left[ \int_{Y} \min\left(y, \tilde{r}l_{0}\right) f\left(y|\theta\right) dy \right] \frac{(1-p\left(\theta\right)) dH\left(\theta\right)}{\int_{\underline{\theta}}^{\hat{\theta}(\tilde{r})} (1-p\left(s\right)) dH\left(s\right)}.$$
(15)

There are many participation rates p that induce the same  $\tilde{r}$ . The minimal size program attracts the worst types, that is  $p(\theta) = 1_{\theta < \theta^p}$  for all types below some cutoff  $\theta^p$ , that is such that the break-even constraint  $l_0 = \frac{1}{H(\theta^T) - H(\theta^p)} \int_{\theta^p}^{\hat{\theta}(\tilde{r})} \rho(\theta, \tilde{r}l_0) dH(\theta)$  holds at  $\tilde{r}$ . At the other extreme, we can have  $p(\theta) \to 1$  for all  $\theta \in \left[\underline{\theta}, \hat{\theta}(\tilde{r})\right]$  and  $\Theta_{\mathcal{O},1} \to \emptyset^{.15}$  Any size between the minimal size and the limit where all investing types participate delivers the same exact cost and level of investment. So any size between  $H(\theta^p)$  and  $H\left(\hat{\theta}(\tilde{r})\right)$  can be an equilibrium.

One way to grasp the intuition for the result is the following. Start from the minimal intervention with size  $H(\theta^p)$ . Consider the types left out to invest alone, that is types between  $\theta^p$  and  $\hat{\theta}(\tilde{r})$ . Now take a random sample of these types. By definition, the expected repayment for this random sample is exactly equal to the loan. This has two implications. First, removing the sample does not change the private rate  $\tilde{r}$ . Second, adding the sample to the program does not change the expected cost for the government. Therefore any random sample can be taken and any size between  $H(\theta^p)$  and  $H(\hat{\theta}(\tilde{r}))$  can be obtained. Actually, the range of possible participation is even greater than suggested by this explanation: One can have a program where some types below  $\theta^p$  do not participate, and participation schedules can obviously be discontinuous and contain holes. All these programs would appear different, since participation would vary greatly, but their cost and investment level would be exactly the same.

The second important implication of Theorem 1 concerns the schedule of payments received by the government. The requirement that  $\gamma_{\theta}(.)$  be identically zero restricts the shape of the payoff functions that the government should offer, as explained in the following corollary.

**Corollary 3** Any feasible program that achieves the minimum cost must be such that, for all  $y \in Y$ and all  $\theta \in \Theta_{\mathcal{P},1} / \{\underline{\theta}\}$ 

$$y_{\theta}^{g}(y) = \min\left(y, \tilde{r}l_{0}\right) - \min\left(y, r_{\theta}l_{\theta}\right).$$
(16)

Theorem 1 tells us what the government can hope to achieve, but the derivation of the lower bound takes into account only the participation constraints of the banks, ignoring the investment and incentive constraints. In the next section, we show how to design interventions that reach the lower bound and establish that (16) is achieved by debt-like instruments.

<sup>&</sup>lt;sup>15</sup>For instance, start from  $p(\theta) = 1_{\theta < \theta^p}$  and let p increase uniformly for all types in  $\left[\theta^p, \overline{\theta}\right]$ .

### 5 Implementation

We now show that there exist feasible interventions that reach the lower bound  $\Psi^*$ . Recall from Section 2 that the best decentralized equilibrium is characterized by  $\hat{\theta}^0 < \bar{\theta}$ . We can, then, think of the implementation as follows. The government chooses a target for aggregate investment  $\theta^T \in (\hat{\theta}^0, \bar{\theta}]$ .<sup>16</sup> We study programs that achieve  $\hat{\theta} = \theta^T$ . Through equation (5), this is equivalent to choosing a target  $R^T$  for the market rate. For now, we take  $\hat{\theta}$  and  $R^T$  as given, and we study the minimum cost implementation. This implementation determines the structure of payoffs and implies an allocation of types to the sets  $\Theta_{\mathcal{P},1}$  and  $\Theta_{\mathcal{O},1}$ .

### 5.1 Direct lending

Corollary 3 shows that financial instruments that do not have the payoff structure of equation (16) cannot achieve the lowest cost. This suggests that the government should intervene with debt-like instruments. This is indeed what we show. We start with the simplest program: direct lending by the government.

**Proposition 3** Let  $R^T$  be the market rate such that the target type  $\theta^T$  is the marginal type. Direct lending of  $l_0$  at rate  $R^T$  uniquely implements the desired investment at the minimum cost.

**Proof.** In order to achieve the investment target  $\theta^T$ , the interest rate  $R^T$  is such that  $\hat{\theta}(R^T) = \theta^T$  in equation (5). Note that  $1 < R^T < r^0$  since  $\theta^T \in (\hat{\theta}^0, \bar{\theta}]$ . The program corresponds to  $m_{\theta} = l_0, l_{\theta} = 0$ , and  $y_{\theta}^g(y) = \min(y, R^T l_0)$  for all types. The incentive and investment constraints are clearly satisfied for all participating banks. Banks in  $\Theta_{\mathcal{O},0}$  do not want to participate without investing since  $R^T > 1$ . We cannot have  $\tilde{r} < R^T$  since all banks would then drop out and the market rate would be  $\tilde{r} = r^0 > R^T$ . We cannot have  $\tilde{r} > R$ ; otherwise,  $\Theta_{\mathcal{O},1} = \emptyset$ . Hence, we must have  $\tilde{r} = R^T$ . Then,  $\gamma_{\theta}(y) = 0$  and  $\Psi = \Psi^*$ .

The direct-lending program implements the desired level of investment and achieves the minimum cost, which are the only outcomes the government cares about. The key point is that incentives and investment constraints are satisfied, while participation constraints hold with equality for all participating types. With other programs, such as equity injections, the participation constraint binds only for the best participating type while all others receive rents.<sup>17</sup>

<sup>&</sup>lt;sup>16</sup>This is equivalent to a target  $xH(\theta^T)$  for investment spending, or a target  $(\bar{v} - x)H(\theta^T)$  for value added.

<sup>&</sup>lt;sup>17</sup>The only exception is when only the worst type participates-i.e.,  $\Theta_{\mathcal{P}} = \{\underline{\theta}\}$ . With an atom-less distribution, such a program cannot increase investment, but if  $\Pr(\theta = \underline{\theta}) > 0$ , the government might choose a program where  $\Theta_{\mathcal{P}} = \{\underline{\theta}\}$ . Then optimality requires that  $E[\gamma_{\underline{\theta}}(y)|\underline{\theta}] = 0$ , and this can be achieved in various ways. For instance, the government might offer cash m against a share  $\alpha$  of equity returns. Opting into the program reveals the type to the lenders and we must have  $E[\min(y, r_{\underline{\theta}} l_{\underline{\theta}}) |\underline{\theta}] = l_0 - m$ . The condition  $E[\gamma_{\underline{\theta}}(y)|\underline{\theta}] = 0$  simply implies  $(1 - \alpha)m - \alpha E[y|\underline{\theta}] = l_0$ 

#### 5.2 Equivalent Implementations

While the optimal payoff-relevant outcomes are unique, some details of the implementation are not. We now discuss how equivalent implementations can vary along three dimensions: extensive margin (participation), intensive margin (size of the loan), and security design (direct lending versus debt guarantees). As explained in Corollary 2, the actual participation rate of various types is irrelevant so long as it leads to the same market rate. For simplicity in the following discussion, we consider a participation function of the type  $p(\theta) = 1_{\theta < \theta^p}$ . Then  $\theta^p$  is uniquely pinned down by the break-even constraint  $l_0 = \frac{1}{H(\theta^T) - H(\theta^p)} \int_{\theta^p}^{\theta^T} \rho(\theta, R^T l_0) dH(\theta)$ .

Given a participation function consistent with  $\tilde{r} = R^T$  in the competitive fringe, the government still has several choices regarding the size of its loans. Consider a program where the government lends  $m < l_0$  at a rate  $R < R^T$ . Participating types  $[\underline{\theta}, \theta^p]$  must now borrow  $l^u = l_0 - m$  on the market at a rate r that satisfies the zero-profit condition of the lenders

$$l^{u} = l_{0} - m = \frac{1}{H\left(\theta^{p}\right)} \int_{\underline{\theta}}^{\theta^{p}} \rho\left(\theta, rl^{u}\right) dH\left(\theta\right).$$
(17)

Given  $\theta^p$ , equation (17) defines a schedule  $rl^u$  strictly decreasing in m. Finally, the face value of the government loan Rm must satisfy the condition  $\gamma = 0$ -i.e.,  $Rm + rl^u(m) = R^T l_0$ . The case analyzed in Proposition 3 corresponds to  $m = l_0$ ,  $rl^u = 0$ , and  $R = R^T$ . We cannot have R < 1; otherwise, some banks would take the cash without investing. The minimal lending program  $m^{\min}$  is defined as the unique solution to  $rl^u(m) + m - R^T l_0 = 0$ .<sup>18</sup> Any outcome that can be implemented by lending  $l_0$  at rate  $R^T$  can also be implemented by a continuum of programs with  $m \in (m^{\min}, l_0]$  and  $R \in (1, R^T]$ .<sup>19</sup>

The government also has a choice of which debt-like instrument to use. In particular, directlending and debt-guarantee programs are equivalent. Instead of lending directly, the government can guarantee new debt up to S for a fee  $\phi$  per unit of face value. Private lenders accept an interest rate of 1 on the guaranteed debt. The debt-guarantee and direct-lending programs are equivalent when  $R = (1 - \phi)^{-1}$  and  $m = (1 - \phi) S$ . In practice, central banks use direct lending, while governments seem to favor debt guarantees. A reason might be that, while equivalent in  $\overline{\rho(\underline{\theta}, \tilde{r}l_0) - l_0}$ . The parameters  $(m, \alpha)$  pin down the generosity of the program and therefore, in equilibrium, the

outside rate  $\tilde{r}$ . The more bad types that opt in, the lower is  $\tilde{r}$ , and the more costly the program becomes. <sup>18</sup>The solution exists because the function is continuous, negative at  $m = l_0$  since  $rl^u(l_0) = 0$  and  $R^T > 1$ , and positive at m = 0 since  $rl^u(l_0) \ge R^T l_0$ . The solution is unique because  $\frac{\partial rl^u}{\partial m} < -1$ . To see why, notice that (17) implies  $\frac{\partial rl^u}{\partial m} = -\frac{H(\theta^P)}{\int_{\theta}^{\theta P} \frac{\partial \rho(\theta, rl^u)}{\partial rl^u} dH(\theta)}$  and (3) implies  $\frac{\partial \rho(\theta, rl^u)}{\partial rl^u} = 1 - F(rl^u|\theta) < 1$ .

<sup>&</sup>lt;sup>19</sup>The only potential issue is unicity. The endogeneity of the borrowing rate r could lead to multiple equilibria for some distribution H. This problem can be ruled out if we allow coordination on the best feasible outcome, or if we impose enough concavity on H (log-concavity is often used in mechanism design for this purpose). Note, however, that this multiplicity does not arise from the intervention itself, since it is also present without intervention, as discussed in Section 2. It is different from the multiplicity created by menus, which occurs for any distribution, as discussed below.

market-value terms, the programs differ in accounting terms since debt guarantees are contingent liabilities and do not appear as increases in public debt.

We conclude this section with a brief discussion of menus arguing that they are dominated by simple programs. Notice that the payoff structure of equation (16) applies to all interventions. Even with menus, the government must use debt-like instruments. Proposition 3 describes an optimal implementation using one debt contract. In the Appendix, we describe interventions with menus of debt contracts. While (non-trivial) menus can implement an optimal outcome, they can never do so uniquely. Alongside the equilibrium where each type chooses the correct contract, there is always an equilibrium where all the types pool on the contract designed for the worst type. Any non-trivial menu can, thus, overshoot its target and end up costing more than intended. This cannot happen with simple programs since all participating types *already* pool on the unique contract offered by the government. In this sense, simple programs are more robust than programs with menus.

#### 5.3 Implementation without Competitive Fringe

We have assumed so far that some fraction of banks invest outside the government program. Given this assumption, we established in Proposition 2 that, in equilibrium, only types below a threshold can invest. This result remains true-and is actually easier to establish-if the competitive fringe is inactive and no bank invests outside the program. The difference between participation and non-participation payoff conditional on investing in the program is  $\bar{v} - c_0 - \int_Y \left[\min(y, r_{\theta}l_{\theta}) + y_{\theta}^g(y)\right] f(y|\theta) dy$ . The first two terms are the same for all types, whereas the term  $\int_Y \left[\min(y, r_{\theta}l_{\theta}) + y_{\theta}^g(y)\right] f(y|\theta) dy$  is increasing in  $\theta$ . This immediately implies that if  $\theta$ prefers to participate and invest rather than drop out, all types below prefer to do the same. Proposition 3 tells us how to design an optimal government intervention.

**Corollary 4** Direct lending of  $l_0$  at rate  $R^T$  (or the equivalent debt guarantee) is also optimal when  $\Theta_{\mathcal{O},1} = \emptyset$ .

When the competitive fringe is inactive and  $\Theta_{\mathcal{O},1} = \emptyset$ , the government implements exactly the same outcome with direct lending of  $l_0$  at rate  $R^T$ . The only difference is that all types below the marginal type now participate-i.e.,  $\Theta_{\mathcal{P}} = [\underline{\theta}, \theta^T]$ -while the latent market rate is some  $\tilde{r}$  strictly above  $R^T$  (and below  $r^*_{\theta}$ ). The market is then effectively irrelevant.

# **Corollary 5** The cost of implementing $\theta^T$ is $\Psi^*$ even if private markets are shut down.

The second corollary says that the competitive fringe does not impose extra costs when the government uses an optimal implementation. The intuition is that the minimum cost is pinned

down by participation constraints when private markets are active, and by incentive constraints when private markets are shut down or irrelevant. In both cases, the constraints bind at the same marginal type  $\theta^T$ . This marginal type always earns  $E\left[a|\theta^T\right] + c_0$  since the value of investment  $\bar{v} - x$  is exactly dissipated by informational rents  $\rho\left(\theta^T, R^T l_0\right) - l_0$ .

We summarize our results in the following theorem:

**Theorem 2** Direct-lending and debt-guarantee programs uniquely implement any desired investment at minimum cost, remain optimal if the markets shut down, and are more robust than programs with menus.

## 6 Extensions

In this section, we provide three important extensions to our main results. The first extension is to consider asymmetric information with respect to new investment opportunities. The second extension is to analyze asset purchases. The third extension is to consider the consequences of moral hazard in addition to adverse selection.

#### 6.1 Asymmetric information about new loans

We have assumed so far that the distribution of v is independent of the bank's type. Let us now relax this assumption, while maintaining assumptions A2 and A3. We replace A1 by:

Assumption A1':  $E[v|\underline{\theta}] > x$  and  $E[v|\theta]$  is increasing in  $\theta$ .

Our results hold under A1'. Banks continue to offer debt contracts, and expected repayments are still given by  $\rho(\theta, rl)$ . The investment condition, however, becomes  $E[v|\theta] - x > \rho(\theta, rl) - l$ for type  $\theta$ . The significant difference is that it might potentially hold for good banks and not for bad banks. This does not, however, change the nature of the decentralized equilibrium: There is still some threshold below which types invest. Proposition 1 still holds, and the results in Sections 3 to 5 are unaffected.<sup>20</sup>

<sup>&</sup>lt;sup>20</sup>Note that A1' implies that all types still have positive NPV projects. When  $E[v|\underline{\theta}] < x$ , the complication is that Proposition 1 need not hold. Essentially, adverse selection worsens and total market breakdowns are possible (equilibria where decentralized investment is zero). Interventions can become, at the same time, more desirable and more expensive (since the government finances some negative NPV projects). As long as A2 holds, however, debt contracts are still optimal, and, based on the discussion in Section 5, we conjecture that optimal interventions still involve simple debt-guarantee programs or direct lending.

#### 6.2 Asset Purchases

Our benchmark model follows Myers and Majluf (1984) in assuming that cash flows are fungible. Assumption A1 rules out contracts written directly on a. We can dispense with this assumption in two ways: One possibility is to assume asymmetric information with respect to v, as explained above.<sup>21</sup> Another possibility is to allow asset purchases, but not spin-offs. In other words, we can allow contracts that are increasing in *both* a and y.<sup>22</sup> One such contract is the purchase of Z units of face value of the legacy assets. If p is the purchase prize, the net payoffs are aZ/A - pZ. All our results hold in this setup. We can show that banks continue to offer debt contracts, and that optimal interventions still use debt-guarantee or direct-lending programs. Moreover, we can show that, among sub-optimal interventions, asset purchases do strictly worse than equity injections.

#### 6.3 Moral hazard

Our benchmark model takes investment opportunities as exogenous. In practice, however, banks can partially control the riskiness of their new loans. To understand how endogenous risk-taking affects our results, we introduce a new project with random payoff v'. This project also costs x but is riskier (in the sense of second-order stochastic dominance) and has a lower expected value: E[v'] < E[v]. To emphasize moral hazard created by government interventions, we assume that market participants can detect the choice of v', but this choice is not contractible by the government.<sup>23</sup>

**Assumption A4**: The choice of project v' is observed by private lenders but cannot be controlled by the government.

Under A4, we can derive three important results (the proofs are in the appendix). First, there is no risk-shifting without government intervention, and Proposition 1 is unchanged. The intuition is that choosing v' is doubly costly. It increases the borrowing rate because of greater objective risk (a direct effect), but it also sends a negative signal about the bank's type. This indirect effect occurs because good types dislike high rates relatively more than bad types.<sup>24</sup>

<sup>&</sup>lt;sup>21</sup>A particularly simple case to analyze is when investment simply scales up existing operations. We can capture this with  $v = \alpha a$  for some constant  $\alpha$ . In this case, we can allow contracts to be written on either a or v.

<sup>&</sup>lt;sup>22</sup>This rules out a contract on v = y - a, which is effectively a spin-off. Details can be found in an earlier version of the paper and are available upon request.

<sup>&</sup>lt;sup>23</sup>Either because the government has an inferior detection technology and does not observe v', or because this choice cannot be verified as in the incomplete contract literature. The point of A4 is to study inefficiencies *created* by government interventions. If v' is not observable by private investors, risk-shifting occurs with or without government intervention.

 $<sup>^{24}</sup>$ In fact, it is easy to see that good types would be willing sacrifice NPV in exchange for *safer* projects because such anti-risk shifting would function as a costly signaling device. Good banks would become too conservative in their lending policies during a crisis in order to signal their types. We note that this would make debt guarantees more appealing since guarantees would lean against the conservatism bias.

The second result is that government interventions are more likely to induce moral hazard when government loans are larger. Risk-taking might occur because the government lends at a rate that is independent of the project chosen. The risk of moral hazard is maximized by the program  $(l_0, R^T)$ and minimized by the program  $(m^{\min}, 1)$  described in Section 5.2.

The third point is that, even if moral hazard occurs with minimal lending, the government can solve the problem by making the intervention contingent on the rate at which the bank borrows from private lenders. The government lends  $m^{\min}$  at rate R = 1 as long as the private lenders' rate r does not exceed the rate consistent with equation (17). Otherwise, the government charges the same rate as the private lenders. Any risk-shifting deviation would bring the bank back to the case without intervention, where we have already seen that risk-shifting does not take place. Hence, deviations never occur and risk-shifting is eliminated.

**Proposition 4** The government can prevent risk-shifting by making its lending rate contingent on the rate charged by private lenders.

We conclude that moral hazard has important implications for the design of interventions. It breaks the equivalence results of Section 5.2 and makes it optimal for the government to never be the sole lender. With small-loan interventions, the government can prevent risk-shifting by observing private rates even if the government itself has no direct monitoring technology. The same results apply with debt guarantees.

## 7 Conclusion

We provide a complete characterization of optimal interventions to restore efficient lending and investment in markets where inefficiencies arise because of asymmetric information.

We show that the mere existence of a government program affects the borrowing costs of all banks, even the ones that do not participate in the program. These endogenous borrowing costs determine the outside option of participating banks and, therefore, the cost of implementing the program. We find that it is impossible to attract only better banks and that the government can attract only types below a target threshold. This threshold depends on the interest rate that nonparticipating banks face, and it pins down both the level of investment and cost of the optimal program. Our most remarkable result is that the optimal intervention can be implemented by offering a simple program of debt guarantees or of direct lending. These simple and realistic programs are optimal among all possible interventions aimed at increasing investment. If we consider moral hazard in addition to adverse selection, we find that the government can prevent risk-shifting by making the terms of the program contingent on the borrowing terms that banks face in the private markets.

On the technical side, we solve a non-standard mechanism-design problem with two informationsensitive decisions (investing or not, participating or not) and in the presence of a competitive fringe (the government does not shut down the private markets).

# Appendix

## A Proof of Lemma 1

The proof focuses on the shape of the function  $\gamma_{\theta}(y) \equiv \min(y, \tilde{r}l_0) - \min(y, r_{\theta}l_{\theta}) - y_{\theta}^g(y)$ . We first show that  $\gamma_{\theta}$  is weakly positive for low values of y, and then decreasing. Recall that  $y_{\theta}^g$  is increasing in y, and the resource constraint imposes  $y^g \leq y - \min(y, r_{\theta}l_{\theta})$ . For  $y \leq \min(\tilde{r}l_0, r_{\theta}l_{\theta})$ , we have  $y = \min(y, r_{\theta}l_{\theta})$  and, therefore,  $\gamma_{\theta} = -y^g \geq 0$ . If  $\tilde{r}l_0 < r_{\theta}l_{\theta}$ , then  $\gamma$  is decreasing for all  $y > \tilde{r}l_0$ . If  $\tilde{r}l_0 > r_{\theta}l_{\theta}$ , the relevant case in later analysis, then for  $y \in [r_{\theta}l_{\theta}, \tilde{r}l_0]$ , we have  $\gamma_{\theta} = y - r_{\theta}l_{\theta} - y_{\theta}^g(y)$ . Since  $y^g \leq y - r_{\theta}l_{\theta}$ , this means  $\gamma_{\theta} \geq 0$ . For  $y > \tilde{r}l_0$ , we have  $\gamma_{\theta}(y) = \tilde{r}l_0 - r_{\theta}l_{\theta} - y_{\theta}^g(y)$  decreasing in y since  $y_{\theta}^g$  is increasing. We conclude that, in all cases,  $\gamma_{\theta}$  is weakly positive for low values of y, and then decreasing. There are two possibilities: Either  $\gamma_{\theta}(y) \geq 0$  for all y, or there exists a  $\hat{y}$  such that  $\gamma_{\theta}(y) < 0$  for all  $y > \hat{y}$ . If  $\gamma_{\theta}(y) \geq 0$ , then the Lemma (both with both weak and with strong inequalities) follows directly from the first part of A2. In the second case,  $E[\gamma_{\theta}(y)|\theta] \geq 0$  implies that

$$\int_{0}^{\hat{y}} \gamma_{\theta}(y) f(y|\theta) \, dy \ge \int_{\hat{y}}^{\infty} -\gamma_{\theta}(y) f(y|\theta) \, dy.$$
(18)

Since  $\gamma_{\theta}(y) < 0$  for all  $y > \hat{y}$ , both sides are strictly positive. Consider  $\theta' < \theta$ .

For  $y > \hat{y}$ , we know that  $\gamma_{\theta}$  is negative and decreasing, implying that  $-\gamma_{\theta}$  is positive and increasing. The monotone hazard rate property implies conditional expectation dominance (see Nachman and Noe (1994)) that gives us  $\int_{\hat{y}}^{\infty} -\gamma_{\theta}(y) \frac{f(y|\theta)}{1-F(\hat{y}|\theta)} dy > \int_{\hat{y}}^{\infty} -\gamma_{\theta}(y) \frac{f(y|\theta')}{1-F(\hat{y}|\theta')} dy$  or

$$\int_{\hat{y}}^{\infty} -\gamma_{\theta}(y) f\left(y|\theta\right) dy > \frac{1 - F\left(\hat{y}|\theta\right)}{1 - F\left(\hat{y}|\theta'\right)} \int_{\hat{y}}^{\infty} -\gamma_{\theta}(y) f\left(y|\theta'\right) dy.$$
(19)

The monotone hazard rate property implies that  $\frac{1-F(y|\theta)}{1-F(y|\theta')}$  is increasing in y, which, together with the definition of MHR, implies that  $f(y|\theta) \leq f(y|\theta') \frac{1-F(\hat{y}|\theta)}{1-F(\hat{y}|\theta')}$ . Then, for all  $y < \hat{y}$ , we have

$$\int_{0}^{\hat{y}} \gamma_{\theta}(y) f\left(y|\theta\right) dy < \frac{1 - F\left(\hat{y}|\theta\right)}{1 - F\left(\hat{y}|\theta'\right)} \int_{0}^{\hat{y}} \gamma_{\theta}(y) f\left(y|\theta'\right) dy.$$

$$\tag{20}$$

Combining (19) with (20), we finally obtain:

$$\frac{1-F\left(\hat{y}|\theta\right)}{1-F\left(\hat{y}|\theta'\right)}\int_{0}^{\hat{y}}\gamma_{\theta}(y)f\left(y|\theta'\right)dy > \frac{1-F\left(\hat{y}|\theta\right)}{1-F\left(\hat{y}|\theta'\right)}\int_{\hat{y}}^{\infty}-\gamma_{\theta}\left(y\right)f\left(y|\theta'\right)dy$$

and, therefore,  $E\left[\gamma_{\theta}(y)|\theta'\right] = \int_{0}^{\infty} \gamma_{\theta}(y) f\left(y|\theta'\right) dy > 0.$ 

### **B** Menus

Suppose that the government offers a menu of contracts  $(m_{\theta}, R_{\theta})$  where  $m_{\theta}$  is the loan and  $R_{\theta}$  is the interest rate. Type  $\theta$  then borrows  $l_{\theta} = l_0 - m_{\theta}$  from the market, and since the choice of the contract reveals the type, the zero profit condition type by type implies:

$$\rho\left(\theta, r_{\theta} l_{\theta}\right) = l_0 - m_{\theta}.$$
(21)

In order for the menu to be optimal, we need  $\gamma_{\theta}(y) = 0$ , or, equivalently,

$$R_{\theta}m_{\theta} + r_{\theta}l_{\theta} = R^T l_0. \tag{22}$$

The menu is feasible if and only if (21) and (22) are satisfied and  $R_{\theta} \geq 1$  for all  $\theta$ . There are obviously several ways to design a menu. Since  $\frac{\partial \rho(\theta, r_{\theta} l_{\theta})}{\partial r_{\theta} l_{\theta}} = 1 - F(r_{\theta} l_{\theta} | \theta)$ , the menu must solve the differential system

$$\frac{\partial \rho}{\partial \theta} d\theta + (1 - F(r_{\theta} l_{\theta} | \theta)) d(r_{\theta} l_{\theta}) + dm_{\theta} = 0,$$
  
$$d(R_{\theta} m_{\theta}) + d(r_{\theta} l_{\theta}) = 0.$$

For concreteness, we study  $\Theta_{\mathcal{P}} = [\underline{\theta}, \theta^p]$  and  $R_{\theta} = 1$  for all types in  $\Theta_{\mathcal{P}}$ . Then, the schedule is pinned down by the differential equation

$$F(r_{\theta}l_{\theta}|\theta) dm_{\theta} = -\frac{\partial \rho}{\partial \theta} d\theta \text{ for all } \theta \in \Theta_{\mathcal{P}},$$

and the initial condition  $\rho\left(\theta^{p}, R^{T}l_{0} - m_{\theta^{p}}\right) = l_{0} - m_{\theta^{p}}$ . Government loans decrease with  $\theta$  and compensate the types for revealing their private information. This menu is clearly feasible and reaches the lower bound for cost  $\Psi^{*}$ .

Menus, however, are susceptible to multiple equilibria. To see why, imagine that all types in  $\Theta_{\mathcal{P}}$  pool on the contract designed for the worst type  $\underline{\theta}$ . Let  $\bar{r}$  be the corresponding break-even rate. Clearly, we must have  $\bar{r} < r_{\underline{\theta}}$ . Therefore,  $R_{\underline{\theta}}m_{\underline{\theta}} + \bar{r}l_{\underline{\theta}} < R_{\underline{\theta}}m_{\underline{\theta}} + r_{\underline{\theta}}l_{\underline{\theta}} = R^T l_0$ . This deviation is profitable for all types and is incompatible with  $\theta^T$  remaining the marginal type. The equilibrium will then be one of a unique contract of lending  $m_{\underline{\theta}}$  at rate  $R_{\underline{\theta}}$ , but with a strictly higher marginal type  $\hat{\theta} > \theta^T$  and a strictly higher cost.

## C Moral Hazard

We prove three claims: (i) there is no risk-shifting without interventions; (ii) smaller government loans create less moral hazard; and (iii) there exists a simple contingent program that removes moral hazard.

For (i), consider an equilibrium with risk-shifting. Let r be the borrowing rate for v and r' > r for v'. Type  $\theta$  chooses to risk-shift if and only if

$$\bar{v} - \rho\left(\theta, rl_0\right) > \bar{v}' - \rho'\left(\theta, r'l_0\right).$$

It is clear that if particular type  $\theta$  wants to risk-shift, a worst type would also want to risk-shift.<sup>25</sup> Hence, the set of risk-shifting types is  $\Theta' = [\underline{\theta}, \theta']$  for some  $\theta'$ . We first show that  $\Theta' = \emptyset$  when there is no intervention.

**Lemma 2** The decentralized equilibrium without intervention is unaffected by the availability of project v'.

**Proof.** Consider the highest type  $\theta'$  that chooses v'. For type  $\theta'$  we have  $\rho(\theta', r'l_0) > l_0$  since  $\theta'$  pools with lower types. This type can strictly benefit by choosing v because v is safer and because it would pool with better types.

For (ii), suppose that the government lends  $l_0$  at  $R^T$ . Then the condition for risk-shifting to occur is

$$\rho\left(\theta, R^{T} l_{0}\right) - \rho'\left(\theta, R^{T} l_{0}\right) > \bar{v} - \bar{v}'.$$

$$\tag{23}$$

The bank faces no penalty for risk-shifting, apart from the NPV loss. If condition (23) holds for  $\underline{\theta}$ , then some risk-shifting does occur in the program with large loans. When  $m < l_0$ , on the other hand, we need to look for a cutoff  $\theta'$  such that

$$l_0 - m = E\left[\rho'\left(\theta, Rm + r'l^u\right) \mid \theta \in \left[\underline{\theta}, \theta'\right]\right],$$

<sup>&</sup>lt;sup>25</sup>This always holds when banks can offer menus of borrowing and projects and the inscrutability principle holds as in Myerson (1983). If there is signalling at the proposal stage we need a refinement to rule out unreasonable equilibria where risk-taking happens to be a good signal.

and

$$l_0 - m = E\left[\rho\left(\theta, Rm + rl^u\right) \mid \theta \in \left[\theta', \theta^p\right]\right].$$

The condition for risk-shifting is the indifference of the marginal type:  $\rho(\theta', Rm + rl^u) - \rho'(\theta, Rm + r'l^u) = \bar{v} - \bar{v}'$ . This is clearly stronger than condition (23) because of the risk-sensitive rate and the adverse signalling effect. The government needs to maximize the amount borrowed on the market to minimize risk-shifting incentives. Within the class of interventions studied in the paper, it does so by lending  $m = m^{\min}$  defined below equation (17).

For (iii), consider the implementation with lending  $m^{\min}$  at rate R = 1. Define  $r^*$  as the solution to

$$l_0 - m = E\left[\rho\left(\theta, m^{\min} + r^* l^u\right) \mid \theta \in [\underline{\theta}, \theta^p]\right].$$

Suppose that the government offers to lend  $m^{\min}$  at rate R = 1 as long as  $r \leq r^*$ , and at a rate equal to the private rate for this bank (R = r) otherwise. If a bank risk-shifts, its interest rate will strictly exceed  $r^*$  and it will be punished by losing the government subsidy. From the above discussion, it is clear that no bank will ever risk-shift. QED.

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