Payroll Taxation and the structure of qualifications and wages in a segmented frictional labor market with intra-firm bargaining

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VERY PRELIMINARY VERSION

Abstract

The present paper investigate the incidence of payroll taxation - and more generally labor incomme taxation - in a search and mathching model. The model considered a production function with different type of workers, allowing to understand the interactions between segented labor markets. Furthermore, the equilibrium is reach through a double process of intra firm wage bargaining ex post and labor demand ex ante. The model is derived annalitically for linear tax function differentiated for worker type, and nummerically for non linear tax functions. Some standart results are found, such as the wages, unemployement and incidence increasing with respect to bargaining power; or the payroll tax burden falling mainly on workers. Moreover, it is shown that overshifting of payroll taxes on to net wages may happen. It is also shown that not only the direct effect is stronger but the induced effects on other worker types' employement and wages are also stronger when the bargaining power is stronger. In addition, marginal incidence decreases with respect to the payroll tax level and is therefore significantly lower than overall incidence, which may induce an underestimation of overall incidence by empirical analyses. This also induces a marginaly decreasing effect of payroll tax abatement on labor costs.

Keywords: Search and matching; segemented labor market; intra-firm bargaining; tax incidence

JEL: H22; J31; J38.

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1 Introduction

The consequences of taxation on the labor market is a central issue of applied public economics and more broadly of the understanding of impact of public policies. The need for kowledge on that subject has been strenghted by the economics crisis. Little wage moderation occured in main developped countries - including the united states despite its quite liberal and competitive labor market - and the question of labor costs and its consequences on structure of jobs and unemployement is a main concern for government, particularly european ones. France for exemple set a niew payroll tax rebate of 4% of the payroll bill for 2013 then 6% for years after 2014.

More generally, a large number of governements use the fiscal tool not only to levy resources but also conversely to subsidy labor. It generates payroll taxation differentiated by industrial sector or level of qualification. This differentiation may modify the structure of employement and unemployement as well as the structure of wages. The present paper aims at analysing these effects of differentiated payroll taxation in a model of search and matching with production function depending on differentiated worker type inputs and intra-firm wage bargaining by level of qualification. This allows to understands the distortions generated on the labor market as well as the distributive consequences.

It is well known that the tax burden does not fall only onto the individuals officially taxed by fiscal service. The burden is shared among the agents interacting with them on markets. This is also the case for payroll taxation; Gruber (1997) demonstrated that workers pay the main part of payroll taxes in Chile. For broader type of workers in western countries, their is a lack of empirical litterature but a recent empirical litterature has investigated the incidence of corporate tax on wages (Arulampalam et al. (2012); Dwenger et al. (2011); Liu and Altshuler (2011)) and shows how workers pay at least half of the corporate tax bill. Hence, one could anticipate that firms shift a large share of payroll taxation burden to their workers.

This issue is of main importance, not only for purely theoretical purpose but also for applied public economics matter. First of all, the motive of differentiated payroll taxes is often employement and incidence of payroll taxes is a key parameter of the success of such policies. There has been several empirical analysis of such policies in Europe, Kramarz and Philippon (2001) and Chéron et al. (2008) find significant impact for France when results of Bennmarker et al. (2009) for Suede and Huttunen et al. (2013) for Finland are more mitigated. The cause of the difference may lie in incidence and the fact that French payroll tax reduction was setted very close to the minimum wage and therefore induce that incidence is full burden for employers.

Furthermore, it is essential to know incidence for understanding equity of taxation. Equity of taxation is first of all understanded throught the actual distribution of the burden and not the official distribution. Different incidence of taxes - and particularly payroll taxes - may be of great influence on the way the redistributiveness of the whole fiscal system is measured. For exemple, Eidelman et al. (2012) estimated the global redistribution of the whole French system of tax and transferts and finds opposite results (strongly redistributive or quite flat) depending on incidence hypotheses of payroll taxes.

An other exemple of the main importance of payroll tax incidence may be found in the analysis by Farhi et al. (2011) of fiscal devaluation. They found that it is equivalent to monetary devaluation if incidence of VAT and payroll taxes are homogenous between sectors. This highlight the importance of understanding incidence not only globally but also at differentiated incidence at micro level depending on the characteristics of production and substitution between factors, what is one contribution of the present paper.

This also also crosses the issue of optimal labor taxation as payroll taxation and labor income taxation should have similar incidence even when labor taxation is not levied at source. However, optimal labor taxation literature has first focused on the labor supply side and the adverse selection problem. Mirrlees (1971) considered a descrete distribution of workers, Saez (2001) generalized the approach with continuous productivity of workers and Kleven et al. (2009) generalized to couples and labor supply in the extensive margins. However, this litterature does not consider any labor market as each unit of labor supplied finds an employer - there is no unemployement - and the wage is equal to the productivity of the worker.

The standart way of modeling labor markets has been developped by the search and matching literature (e.g. Pissarides (2000)). It provides a dynamic framework and reproduced the conditions of frictional unemployemment, the rent of employement being shared between firm and worker. Stole and Zwiebel (1996b,a) reniew the process of wages setting by the hypothesis that contract incompleteness does not enables neither firms nor workers to commit to future wages and employement decision, which leads to intra-firm bargaining engaged individually by workers within employement. It results in lower wages and more employement than in standart model. All these models does not take into account the structure of production and possible substitution between factors of production. Acemoglu (2001) built a matching model with two kinds of job (good job/bad job) and derive the incidence of minimum wage on the structure of production. However, it does not fit either the problematic of the present paper as there is only one type of worker and the two kinds of job are models as separate sectors of intermediate goods.

The present paper is therfore based on the matching model of with both a multi-factor production function and intra-firm bargaining of Cahuc et al. (2008), which models a large number of worker types being substitute or complement factor in the production function of each firms. This framework is used to understand the impact of taxation on the wage bargaining and the level of employement. The tax function may represent either payroll taxes or labor income taxes.

For the case of payroll taxation financing public social security systems, some countries separates employers and employees' parts of payroll taxation. This separation is not considered here because it is formall but has no economic reality except at the level of minimum wage. However, we consider such social security contributions that are not directly linked to the level of social security benefits. For exemple, in France, unemployement and retirement social security benefits are proportionnal to the contributions : it is mandatory consumption of assurance but not actual taxation. At the oppositte, family benefits and heatlh assurance benefits are deconnected from the contributions, wihch may therefore be considered as taxation.

The remainding of the paper is organized as follow. Section 2 present the general model, the global setup (2.1), the demand equation of firms (2.2), the wage bargaining process (2.3) and the general equilibrium (2.4). Section 3 investigate the case of a unique type of worker, annalytically for linear taxation (3.1) then numerically for quadratic tax functions (3.2). Section 4 investigate the case of segmented labor market with differentiated linear taxation first anallytically (4.1) then numerically (4.2). Section 5 concludes.

2 Theoretical framework

2.1 The general setup of the model

The model is based on Cahuc et al. (2008) to understand wage negociation in a matching framework with several categories of workers used as different factors in a unique production function.

We consider an economy with a numeraire good produced thanks to $n \geq 1$ labor types ($i = 1, ..., n$) supplied by a continuum of infinitely lived workers of size normalized to one (supplying each one unit. The production function is $F(N_1, ..., N_n)$ where $N_i \geq 0$ is the employment of workers of type i ($\vec{N} = (N_1, ..., N_n)$).

To recruit, firms post vacancies (group specific hiring cost γ_i per unit fo time and per vacancy) matched with pool of unemployed worker. Matching function $h_i(u_i, V_i)$ gives the mass of aggregate contacts depending on the mass of unemployed $u_i = 1 - N_i$ and the mass of vacancies V_i. With $\theta_i = V_i/u_i$ the group-specific tightness, the probability to fill a vacant job by unit of time is $q_i(\theta_i) = h_i(u_i, V_i)/V_i$ ($q_i'(\theta_i) < 0$ and $q_i(0) = +\infty$) and the probability to find a job by unit of time is $p_i = (u_i, V_i)/u_i = \theta_i q(\theta_i)$ (with $d[\theta_i q(\theta_i)]/d\theta_i > 0$). The group-specific exogenous probability of job destruction by unit of time is s_i. The wage $w_i(\vec{N})$ is continuously negociated after hiring (individually bargained but common byu symetry to all workers of type i).

Furthermore, a tax function T is considered such that the gross wage is $T[w_i(\vec{N})]$ when the net wage is $w_i(\vec{N})$. This tax function may represent most of tax schedule around the world, whatever social security contribution - mainly linear - or labor income taax scheddule - mainly piecewise linear. A quadratic version is also aanalysed numericallly to understand regressive abattement to social security contribution that target low weges in some countres, amid whose France.

The equilibrium on the market (2.4) is reach through the confrontation of a labor demand curve and a wage bargaining curve on each segment of the labor market - depending on the equilibria on the other labor markets. The demand for each level of labor is define ex ante by the quantity of vacancies posted on the labor market (2.2). It depends on the anticipation of the ex post wage bargaining, itself depending on the level of unemployement, the unemployement benefits and the marginal productivity of eache type of labor (2.3).

The overall model is dynamics and time is continuous. The equilibrium os calcullated through the use of Bellman equation for the values of profit flows for firms, and employement and unemployement for workers.

2.2 Labor demand

The demand for each type of labor is determine by the maximization by the firm of the value of its profit flows. The Bellman equation of the value of the firm for time between t and $t + dt$ is given by equation 1, subject to equation 2 giving the evolution of the number of each type of worker depending on the rate of destruction of jobs, the number of vacancies and the matchng function itself depending on the tightness of the segment of the labor market.

$$\Pi(\vec{N}) = \max_{\vec{V}} \frac{1}{1 + rdt} \left\{ \left[F(\vec{N}) - \sum_{j=1}^{n} \Big(T[w_j(\vec{N})]N_j + \gamma_j V_j \Big) \right] dt + \Pi(\vec{N}^{t+dt}) \right\} \tag{1}$$

$$N_i^{t+dt} = N_i(1 - s_i dt) + V_i q_i(\theta_i) dt \tag{2}$$

To resolve the maximization problem of firms, and consequently obtain the labor deman, the marginal profits (with respect to each type of workers) are noted $J_i(\vec{N}) = \partial\Pi(\vec{N})/\partial N_i$. There is two ways of calculating these marginal profits. The first one is the first order condition with respect to the number of vacancies V_i posted by firms: it gives equation 3 at steady state. The second one is derived from the envelop theorem: it gives equation 4.

$$J_i(\vec{N}) = \frac{\gamma_i}{q_i} \tag{3}$$

$$J_i(\vec{N}) = \frac{\frac{\partial F(\vec{N})}{\partial N_i} - T[w_i(\vec{N})] - \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i}}{r + s_i} \tag{4}$$

Indeed, first order condition with respect to V_i is $-\gamma_i dt + J_i(\vec{N}^{t+dt}) dN_i^{t+dt}/dV_i = 0$ where $dN_i^{t+dt}/dV_i = q_i dt$ from equation 2. At steady state, $\vec{N}^{t+dt} = \vec{N}$ which gives equation 3. In addition, the envelop theorem applied by differentiating equation 1 with respect to N_i gives:

$$\left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i} - T[w_i(\vec{N})]\right] dt + \frac{\partial N_i^{t+dt}}{\partial N_i} J_i(\vec{N}^{t+dt}) = J_i(\vec{N})(1 + rdt)$$

With $\frac{\partial N_i^{t+dt}}{\partial N_i} = (1 - s_i dt)$ from equation 2, which gives equation 4 at steady state. Combining equation 3 and 4 gives the decomposition of the marginal productivity with respect to the workers of type i in equation 5

$$\frac{\partial F(\vec{N})}{\partial N_i} = T[w_i(\vec{N})] + \frac{\gamma_i(r + s_i)}{q_i(\theta_i)} + \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i} \tag{5}$$

Where $\partial F(\vec{N})/\partial N_i$ is the marginal productivity of worker of type i; $T[w_i(\vec{N})]$ is its gross wage; $\gamma_i(r + s_i)/q_i(\theta_i)$ the hiring costs increasing with the vacancy posting cost γ_i and the rate of job destruction s_i and decreasing with the probability $q_i(\theta_i)$ that a vacancy meets an unemployed worker; $N_j \partial T[w_j(\vec{N})]/\partial N_i$ is the change in the wage bill for workers of type j due to the change in the level of employement of workers of type i through the intra-firm wage bargaining process.

This equation 5 gives a relation between wage bargaining function as anticipated by firms and the level of employement targeted by firm through their vacancies' posting. It correspond to the Labor demand curves. This demand is not such that overal marginal labor costs - gross wages plus the costs of hiring - equals the marginal productivity of workers. It depends also on the variations of the overall wage bill due to the change in the employement level because changing the level of employement (and therefore of unemployement) change the wages through changes in the outsidde options of workers and firms. As shown by Stole and Zwiebel (1996b,a) and confirmed by Cahuc et al. (2008), the labor demand may be such that the marginal productivity of a type of worker is lower than the overall margiinal cost of such type of labor.

2.3 Wage determination

This labor demand equation 5 gives a first relation between the number of employees of each type and their wages. The actual wages and employement levels for each type of worker need another relation to be fully determined, this second relation comes from the intra-firm bargaining determining function $w_j(\vec{N})$, which is determined in the present subsection.The Bellman equation of the value of being in employement E_i for worker of type i is equation

6, from which is directly derived equation 7.

$$rE_i = w_i(\vec{N}) + s_i(U_i - E_i) \tag{6}$$

$$E_i - U_i = \frac{w_i(\vec{N}) - rU_i}{r + s_i} \tag{7}$$

Given the type specific bargaining index β_i of workers of type i, the usual Nash bargaining equation is equation 8, according to the fact that the rent of employement for workers is the difference of values $E_i - U_i$ between employement and unemployement and the rent of employement for the firm is the marginal productivity $J_i(\vec{N})$ of workers of type i.

$$\beta_i J_i(\vec{N}) = (1 - \beta_i)(E_i - U_i) \tag{8}$$

According to equation 7 of the difference of value between employement and unemployement and equation 4 of the marginal productivity of workers of type i, equation 8 may be rewritten as the differential equation 9 of the wage as a function of the level of employemennt.

$$(1 - \beta_i)w_i(\vec{N}) + \beta_i T[w_i(\vec{N})] = (1 - \beta_i)rU_i + \beta_i \left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i} \right] \tag{9}$$

As the intra-firm bargaining take place individually for each worker allready employed by the firm, it didd not anticipate the possible change in employement resulting for the new wage, wich means that rU_i is constant in that ddifferential equation. This differential equation will be solve in the section of the actual resolution of the model. To solve this differential equation, a condition at the limit is needed. The condition considered is that the overall gross wage bill $N_i T[w_i(\vec{N_i})]$ for workers of type i tends towards zero when the employemennt N_i of such workers tends towards zero.

2.4 Labor market equilibrium

To determine the general equilibrium of this model, two relations are needed. The first one is the demand 5 of labor and the second the wage function given by differential equation 9. In addition, the bellman equation for the value of unemployement is:

$$rU_i = b_i + \theta_i q_i(\theta_i)(E_i - U_i)$$

Where b_i is the income flow at unemployment. As equation 8 gives $E_i - U_i = \beta_i/(1-\beta_i)J_i(\vec{N}) = \beta_i/(1-\beta_i)\gamma_i/q_i(\theta_i)$ because of equation 3, the value of unemployement is given by equation 10.

$$rU_i = b_i + \gamma_i \frac{\beta_i}{1 - \beta_i} \theta_i \tag{10}$$

Furthermore, as equation 2 gives $Ns = Vq(\theta)$, a first equation linking N to θ is equation 11. The second equation 12 is found by including equation 5 in equation 9.

$$\theta q(\theta) = \frac{sN}{1 - N} \tag{11}$$

$$w_i(\vec{N}) = b_i + \frac{\gamma_i \beta_i}{1 - \beta_i} \left(\theta_i + \frac{r_i + s_i}{q_i(\theta_i)} \right) \tag{12}$$

To calculate the structure of employement $\vec{N}$ and the structure wages $w(\vec{N})$, the solution of differentital equation 9 should be incorporated in this system, which gives two relations between the wage and employement and consequently the equilibrium wage and employelment (which are prooved to exist and be unique). This differential equation gives the result of wage determination inside firms through wage bargaining. It is done inside each firm considering the state of the labor market as given, and therefore it is a solution in partial equilibrium: the reservation wages rU_i and the labor market tightnesses θ_i are exogenous variables.

2.4.1 Full employer bargaining power

If the bargaining power is fully owned by the employer, that is if $\beta_i = 0$, differential equation 9 become equation 13 giving directly the bargained net wage.

$$w_i(\vec{N}) = rU_i = b_i \tag{13}$$

The employee without any bargaining power should accept its reservation wage and nothing more. In that case, the net wage is independant from the payroll tax which which is fully beared by the employer.

2.4.2 Full employee bargaining power

At the opposite if the full bargaining power is owned by the employee, differential equation 9 become:

$$T[w_i(\vec{N})] = \frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i}$$

Yet:

$$\frac{\partial \sum_{j=1}^{n} N_j T[w_j(\vec{N})]}{\partial N_i} = T[w_j(\vec{N})] + \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i}$$

And therefore differential equation 9 is equivalent to equation 14

$$\frac{\partial \left(\sum_{j=1}^{n} N_j T[w_j(\vec{N})] - F(\vec{N}) \right)}{\partial N_i} = 0 \tag{14}$$

And consequently $\sum_{j=1}^{n} N_j T[w_j(\vec{N})] - F(\vec{N})$ is constant with respect to $\vec{N}$. Yet it is zero when $\vec{N} = \vec{0}$. Hence, $\sum_{j=1}^{n} N_j T[w_j(\vec{N})] = F(\vec{N})$ and there is no equilibrium becauses the full output is paid in wage and nothing remain for the hiring costs.

3 Unique type of worker

In the simpler case with homogenous workers, differential equation 9 become equation 15.

$$(1-\beta)w(N) + \beta T[w(N)] = (1-\beta)rU + \beta \left[\frac{\partial F(N)}{\partial N} - N \frac{\partial T[w(N)]}{\partial N} \right] \tag{15}$$

However, even if it can be shown that equation 15 has solutions, they can not be exibited formally. Numerical solving for different tax functions T are presented in subsection 3.2. In a first subsection, differential equation 15 is solved formally in the special case of linear tax function.

3.1 The case of linear tax function

We consider here that the tax function is linear: $T(w) = (1+\tau)w$. In that case, the net wage is defined by differential equation 16.

$$(1-\beta\tau)w(N) = (1-\beta)rU + \beta\left[\frac{\partial F(N)}{\partial N} - (1+\tau)N\frac{\partial w(N)}{\partial N}\right] \tag{16}$$

With the condition at the limit being that the overall gross wage bill $NT[w(N)]$ tends towards zero when employement N tends towards zero, which is equivalent that that the net wage bill $Nw(N)$ tends towards zero when employement N tends towards zero because $T[w(N)] = (1+\tau)w(N)$.

Lemma 1. The solution of differential equation 16 subject to limit condition $\lim_{N\to 0} Nw(N) = 0$ is the wage function given by formula 17.

$$w(N) = \frac{1-\beta}{1+\beta\tau}rU + \int_0^1 v^{\frac{1-\beta}{\beta}+\tau}F'(v^{1+\tau}N)dv \tag{17}$$

Proof. The complete resolution is presented in appendix A.1.

And therefore, the wage decreases with respect to level of employement. This gives a decreasing relationship between wage and employement as equation 12 gives an increasing relationship between these variables, allowing to define a general equilibrium.

Proposition 1. *As soon as the production function has not increasing marginal productivity, the solution to equation 18 exists and is unique*

Proof. Given equation 17, equation 12 of the system linking N to *theta* become equation 18.

$$\int_0^1 v^{\frac{1-\beta}{\beta}+\tau}F'(v^{1+\tau}N)dv = \frac{(1+\tau)\beta}{1+\beta\tau}b + \frac{\gamma\beta}{1-\beta}\left(\frac{(1+\tau)\beta}{1+\beta\tau}\theta + \frac{r+s}{q(\theta)}\right) \tag{18}$$

The left hand side of this equation decreases in N as soon as the returns are decreasing. The right hand side increases in θ, which increases in N (because of equation 11 and $d[\theta q(\theta)]/d\theta > 0$). This right hand term tends towards $[(1+\tau)\beta/(1+\beta\tau)]b < b$ when N tends towards 0 (because of $\theta q(\theta)$ tends towards 0 due to equation 11). It increases towards infinity when N tends towards 1 (θ tends towards infinity and $q(\theta)$ is positive and decreasing). Hence, equation 18 has a unique solution. ***Q.E.D.***

It is also possible to derive from equation 18 the variations of the employement as a response to parameters changes, for the parameters deplacing left hand term or right hand term unambiguously. This results are summarized in table 1.

These results are globally straitforward: the level of employement increases with respect to the efficiency of production processes and the quality of matching between employers and employees; it decreases with respect to the employement benefits (the employees outside option), the cost of recruiting a new employee, the rate of job destruction. The impact of the interest rate may be understood as it give the employers' outside option, the reference of time discount.

However, the influence on wages is more ambiguous and, of more importance, the impact of taxes and bargaining powers needs more information on the production function to be derived. Next subsection assesses these influences through numerical analyses, and extend the results to the cases of non-linear payroll taxes.

Table 1: Impact of model parameters on the level of employement

	Parameter	Eq. 18 variation	Employement variation
Total factor produvitity	A	lht $\nearrow$	Increase
Matching function level	a	rht $\searrow$	Increase
Unemployement benefits	b	rht $\nearrow$	Decrease
Vacancy posting cost	γ	rht $\nearrow$	Decrease
Job destruction rate	s	rht $\nearrow$	Decrease
Interest rate	r	rht $\nearrow$	Decrease

3.2 Numerical analyses

3.2.1 Numerical analysis of the linear case

To go further, hypothesis should be made. We consider Cobb-Douglas production function $F(N) = AN^{\alpha}$. The matching function is assumed to be of the form $h(u, V) = au^{1-\eta}V^{\eta}$. Consequently, $q(\theta) = a\theta^{\eta-1}$ and $\theta q(\theta) = a\theta^{\eta} = sN/(1-N)$. Hence, equation 11 become 19

$$\theta = \left(\frac{s}{a}\right)^{\frac{1}{\eta}} \left(\frac{N}{1-N}\right)^{\frac{1}{\eta}} \tag{19}$$

Consequently, system of equations 19 and 18 imply equation 20 of the level of employment at equilibrium.

$$\frac{\alpha\beta AN^{\alpha-1}}{1-\beta+\alpha\beta(1+\tau)} = \frac{(1+\tau)\beta}{1+\beta\tau}b + \frac{\gamma\beta\left(\frac{s}{a}\right)^{\frac{1}{\eta}}}{1-\beta}\left[\frac{(1+\tau)\beta}{1+\beta}\left(\frac{N}{1-N}\right)^{\frac{1}{\eta}} + \frac{r+s}{s}\left(\frac{N}{1-N}\right)^{\frac{1}{\eta}-1}\right] \tag{20}$$

Left hand term decreases from infinity to a finite positive term when N goes from 0 to 1, while right hand term increases from 0 to infinity when N goes from 0 to 1. Hence, there exists a unique solution between 0 and 1. Now the question is: how this equilibrium N varies with respect to the parameters of the model (mainly β and τ) and how it impacts the equilibrium wage.

The program implemented to these numerical solving is presented in appendix D.1. The calibration is done with Cobb-Douglas production function with standart labor productivity parameter $\alpha = 2/3$, a marginal productivity setted to one when full employment, interest rate equal to three percent. The matching function parameters are calibrated according to Petrongolo and Pissarides (2000) survey of the empirical literature on the matching function and Borowczyk Martins et al. (2011) who corrects for a bias in the estimation due to endogenous search behavior from each side of the market.

For each parameter, variants are implemented to understand the effect of this parameters on the interest variables. Figures for each interest variable and each parameter are given in appendix B.1. It confirms the results of table 1 and are quite straitforward. Furthermore, the impact of to more parameters - the labor parameter α in the Cobb-Douglas production function and the bargaining power β - are presented. Concerning the level of wages, they are reported in figure 1.

The dependence on the bargaining power is straithforward: a larger bargaining power allows to get higher wages (as a share of the production). However, it is not the case for the parameter α of labor input in the Cobb-Douglas production function, an increase of those generate an equilibrium with lower wages. This come from a large impact of this parameter on the labor demand, which can be seen in figure 2 presenting the levels of unemployement for the different values of the parameters.

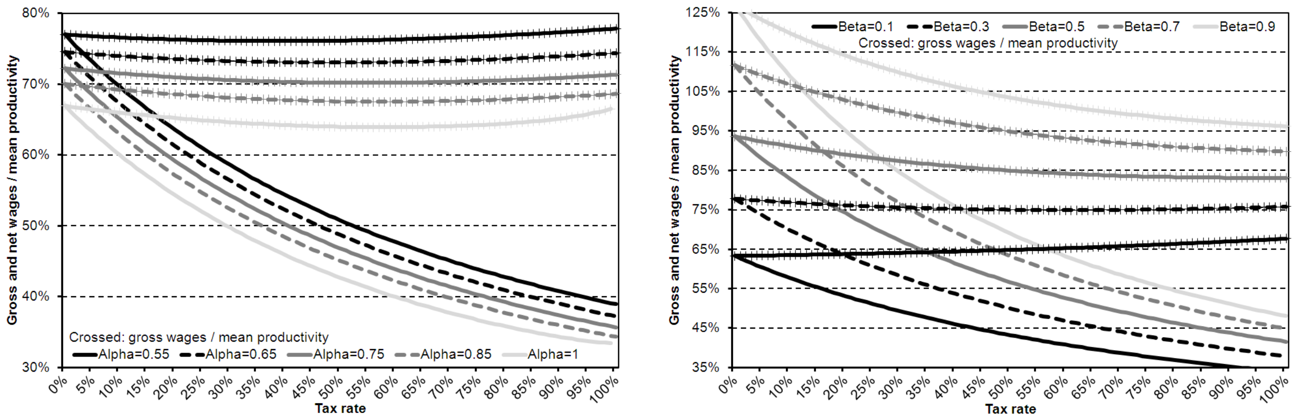

Figure 1: Impact of labor productivity and bargaining power on wages

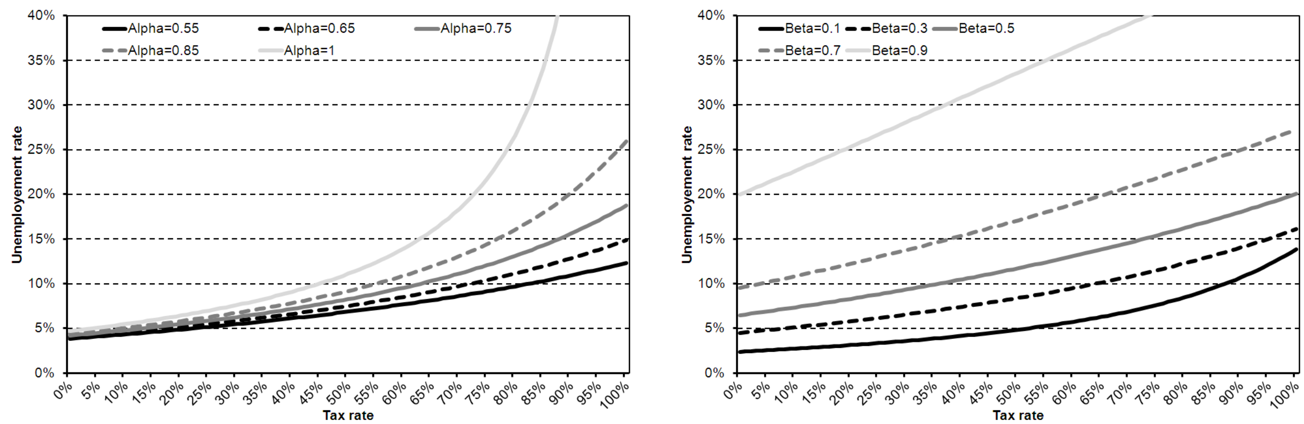

Figure 2: Impact of labor productivity and bargaining power on unemployement

The unemployement rate increases strongly with respect to that parameter α, and the negative impact of taxes on employement also increases more quickly when α is larger. Indeed, the impact of α is almost zero when taxation is low, it even has no impact at all without taxation. It highlights the phenomenon that some parameter other than taxation may be thought to have no impact on unemployment in models without taxation only because their effect is reveled by taxation. Furthermore, the bargaining power has also a negative impact on employement, due to its positive impact on wages. It is a standart result of search and matching literature. The crossed effect of bargaining power and taxes seems low even if each parameter reinforced the negtive impact on unemployment of the other. As a results of these dependencies, the incidence is given by figure 3.

The share of the tax burden falling onto employees increases with respect to both α and β parameters. It is not surprising in the case of the bargaining power parameter β: the employee get a larger share of the total surplus, and therefore leave a small share of that surplus to employers, that have therefore few surplus to actually pay the tax. However, it is more surprising in the case of the α parameter as with larger α employees have lower wages and larger share of the tax burden, even if this last effect is relatively small.

Moreover, figure 3 is very representative of the whole figure in appendix B.1 presneting the influence of taxation. For all reasonable combination of parameters, the incidence is both high - even larger than 100% - and strongly

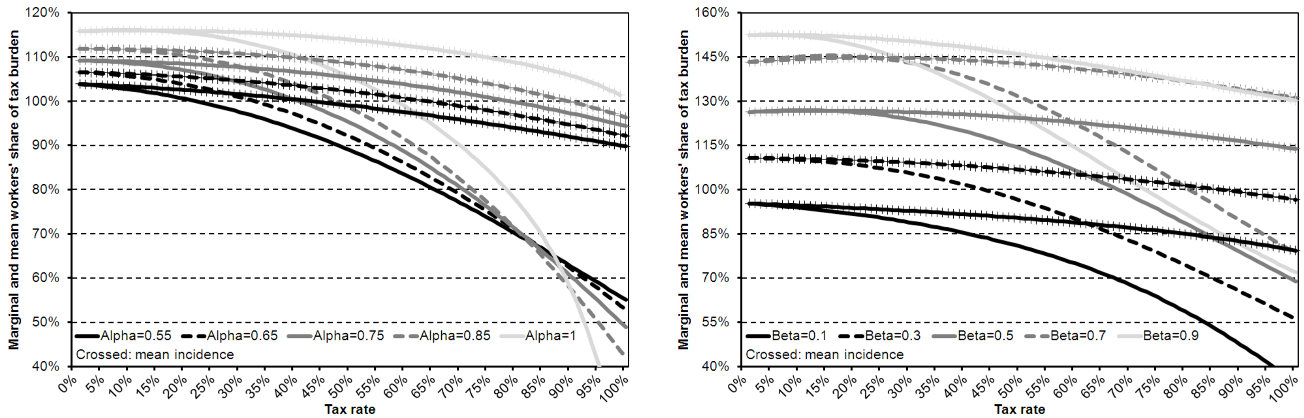

Figure 3: Impact of labor productivity and bargaining power on incidence

marginally decreasing. this last result has at least to important implications. First of all, concerning the interpretation of empirical results, they often under-estimates the overall incidence of taxation as they mainly measure marginal incidence (due to empirical strategy of identification) which is significantly lower than mean incidence.

Moreover, from a public policy point of view, it appears that as incidence is marginally decreasing, the impact of taxes on labor costs are marginaly increasing and therefore the impact of taxes on unemployement is marginally increasing. Consequently, the efficiency of employement policies consisting in payroll tax abatemment is marginally decreasing with the level of abatement. This results seems to be confirmed by the meta-analysis of Zemmour (2013) on the French payroll tax abatement policy.

3.2.2 Numerical analysis of a non linear case

If differential equation 15 can be solved formally only in the case of a linear tax function $T[.]$, numerical analyses can be done for different cases. In the present subsection, the focus is made on quadratic tax function, that are lineary increasing tax rate. Most tax function in the world are linear or piecewise linear, but there exists some quadratic form.

For exemple, France has setted degressive abatement of payroll tax, which correspond to a quadratic payroll tax. The first abattement was created in 1993, and has been several times completed since. In 2012, it consisted in 26% reduction in the payroll tax from one to 2.1 minimum wage, the reduction beeing lineary declining to zero from 2.1 to 2.4 minimum wage. Consequently, between 2.1 to 2.4 minimum wage, if the normal payrrol tax rate is X%[2], the tax function between 2.1 and 2.4 minimum wage is $T[w] = (X\% - 208\%)w + (260/3)w^2$.

The program to resolve numerically differential equation 15 is the case of quadratic tax function, and then to solve find the general equilibrium for different level of tax and tax progressivity is presented in appendix D.2, the results are compiled in figures presented in appendix B.2. The main results may be observed in figure 4.

It appears two surprising results, which are confirmed by looking at other variables or other scale of productivity.

[2]The actual normal payroll tax is not given here because calculating it requires differentiating what is actual taxation and what is mandatory insurrence in the French social contributions. However, this debate does not impact the present exemple concerns reduction of social contribution decollerated from any social benefit

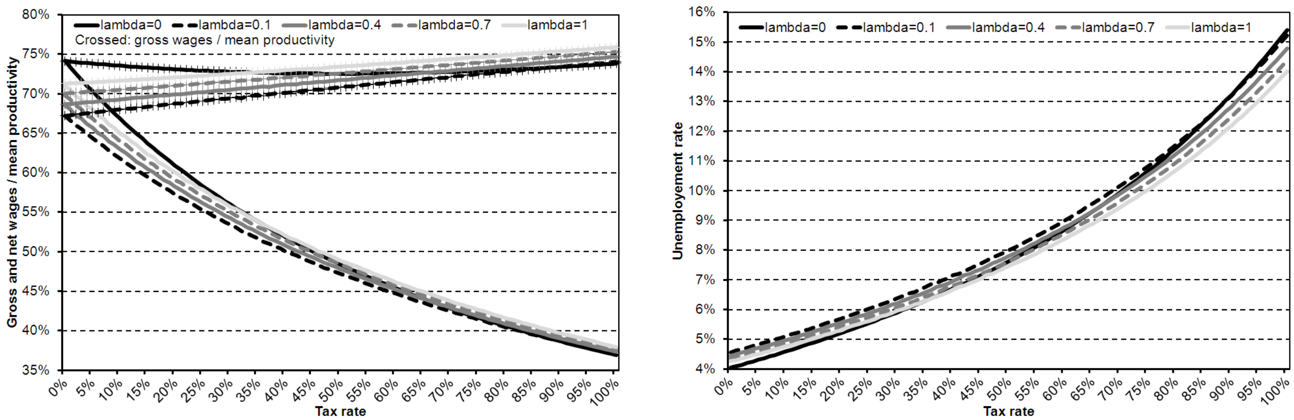

Figure 4: Impact of tax progressivity on wages and employement

First, the effect of progressivity is very strong when progressivity is very small, then it decreases and the curves go closer to the linear case when progressivity increases...

Second, the impact of progressivity is negative both on wages and employement when the overal tax rate is small, then less and less negative while the overal tax rate increases for ending positive when the overal tax rate is high...

4 General results

As in the previous, case, the differential equation may not be solved formally without additionnal assumption. A first subsection studies the linear case, a second interpret the results and a third exhibits numerical solutions for particular values of the tax function.

4.1 Linear tax function

We keep the linear tax function assumption, but allow for different rates for differents type of workers: $T_i[w_i(\vec{N})] = (1+\tau_i)w_i(\vec{N})$. The system of differential equations become as presented by equation 21.

$$(1+\beta_i\tau_i)w_i(\vec{N}) = (1-\beta_i)rU_i + \beta_i\left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{n}(1+\tau_j)N_j\frac{\partial w_j(\vec{N})}{\partial N_i}\right] \tag{21}$$

The problem with this system of differential equation is that each function $w_i(\vec{N})$ depends on the derivatives of the wage function of other type of workers. The first stage for solving this differential equation consists in desintengling partially this system. Appendix A.2 shows how it is possible and demonstrates that the system is equivalent to those of equation 22.

$$(1+\beta_i\tau_i)w_i(\vec{N}) = (1-\beta_i)rU_i + \beta_i\left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{n}(1+\tau_j)\chi_{ij}N_j\frac{\partial w_i(\vec{N})}{\partial N_j}\right] \tag{22}$$

Where the parameter $\chi_{ij} = \frac{\beta_j}{1-\beta_j}\frac{1-\beta_i}{\beta_i}$ give the comparison between the bargaining powers of workers of types i and j.

The second stage of the resolution is presented in appendix A.3. It consists in several changes of variables, the most important one being the change in polar coordinates allowing to actually resolve the differential equation.

Lemma 2. *The solution of the system of wage bargaining differential equations 21 with condition at limit being that the payroll bill for a given type of labor tends towards zero when the employement of workers of that type tends towards zero is given by equation 23.*

$$w_i(\vec{N}) = \frac{1-\beta_i}{1+\beta_i\tau_i} rU_i + \int_0^1 u^{\frac{1-\beta_i}{\beta_i}+\tau_i} \frac{\partial F(\vec{N}A_i(u))}{\partial N_i} du \tag{23}$$

Where matrix $A_i(u)$ is given by equation 24.

$$A_i(u) = \begin{pmatrix} u^{(1+\tau_1)\frac{\beta_1}{1-\beta_1}\frac{1-\beta_i}{\beta_i}} & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & u^{(1+\tau_j)\frac{\beta_j}{1-\beta_j}\frac{1-\beta_i}{\beta_i}} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & u^{(1+\tau_n)\frac{\beta_n}{1-\beta_n}\frac{1-\beta_i}{\beta_i}} \end{pmatrix} \tag{24}$$

Proof. Appendix A.3.

Equation 23 provides a decreasing relationship between the employement N_i and the net wage w_i as soon as factorial productivity is decreasing. As equation 12 provide an increasing relationship between these two variables, it allows to define a general equilibrium as in the following subsection. Furthermore, an increase of taxes or bargaining powers for one kind of workers generates a net wage increase for type of workers who are complement (the marginal productivity of one type of workers increases with respect to the numbers of other type of workers) and a net wage decrease for type of workers who are substitutes (the marginal productivity of one type of workers decreases with respect to the numbers of other type of workers).

Proposition 2. *As soon as the production function has not increasing marginal productivity, there exists a unique general equilibrium on the segmented labor market, solution of the system of equations 25.*

$$\int_0^1 u^{\frac{1-\beta_i}{\beta_i}+\tau_i} \frac{\partial F(\vec{N}A_i(u))}{\partial N_i} du = \frac{(1+\tau_i)\beta_i}{1+\beta_i\tau_i} b_i + \frac{\gamma_i\beta_i}{1-\beta_i}\left(\frac{1-\beta_i+2(1+\tau_i)\beta_i}{1+\beta_i\tau_i}\theta_i + \frac{r_i+s_i}{q_i(\theta_i)}\right) \tag{25}$$

Proof. Equalizing the net wages $w_i(\vec{N})$ in equations 12 and 23 gives equation 25. The marginal factorial returns of production function are not increasing, left hand term does not increase with respect to N_i. Right hand term of this equation increases from $(1+\tau_i)\beta_i b_i/(1+\beta_i\tau_i$ to infinity when N_i goes from zero to one because θ_i increases from zero to infinity when N_i goes from zero to one and $q(\theta_i)$ decreases with respect to θ_i and tends towards infinity when θ_i tends towards zero. ***Q.E.D.***

The impact of parameters on the employement level are the same as in table 1. For details on the impact of β_i and τ_i on the employement and wages of workers of type i, numerical analeses are presented in following subsection. Moreover, the impact of β_i, τ_i and N_i on other type of workers' employement and wages may be derived formally. An increase of any of these variables increases the left hand term of equation 25 for complement workers and decreases it for substitute workers; hence, it leads to an increase of complement workers' employement and a decrease of substitute workers'employement. It also leads to an increase of complement workers' wages and a decrease of substitute workers' wages (e.g.: equation 12).

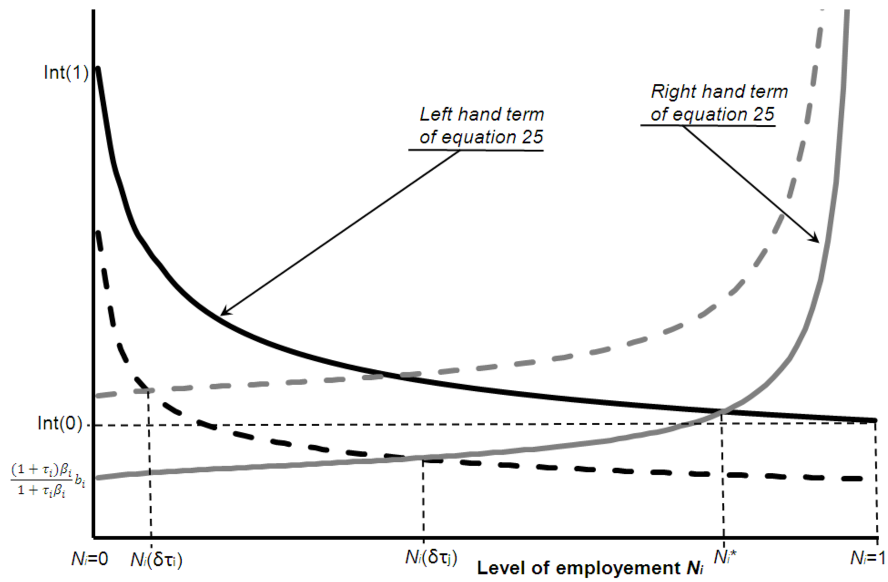

Figure 5: Equilibrium on a segemented labor market

Let be more accurate concerning the effects of the specific tax rates. Figure 5 presents the partial equilibrium on one segemented labor market given the other markets properties. First of all, if the tax rates on that market increases, the left hand of equation 25 term decreases (to the dashed black curve) and the right hand term of equation 25 increases (to the dashed gry curve); this leads to a new equilibrium $N_i(\delta\tau_i)$ with more unemployement and hence lower net wages.

When another tax rate increases, the right hand of equation 25 is not impacted and the equilibrium is changed only by the move of the left hand term. In the case of at least partially complement labor types, this left hand term decreases (the dashed black curve), leading to a new equilibrium with more unemployement and therefore lower wages; both net and gross wages decreases because the tax rate does not change for that type of work.

Proposition 3. *In the case of an increase of the tax rate on labor type j, the employement and wage decreases on market for labor type i (at least partially complement to labor type j) increases with respect to $[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]$, the relative bargaining power of workers of type j compared with the bargaining power of workers of type i.*

Proof. The left hand term of equation 25 decreases because the j inputin the production fonction under the intregral is $u^{(1+\tau_j)[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]}N_j$, which decreases both because N_j decreases (more strongly for workers j with larger bargaining power, e.g. subsection 3.2.1) and because $u^{(1+\tau_j)[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]}$ decreases because u is lower than 1; this last decreases is stronger when the parameter $[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]$ is larger. ***Q.E.D.***

Hence, increasing the taxes of high bargaining power workers affects more both their own employement and the employement of other workers than increasing taxes of low bargaining power workers. At the opposite, decreasing taxes on the low bargaining powers have less positive impact both on themselves and on the other workers. If bargaining power is positively correlated with qualification and income, it constitutes an efficiency point against progressive taxation in general, and payroll tax abattement targeted on low wages, as is heavyly stted in France.

4.2 Numerical solving

The aim of the numerical analysis is here to understand the effects of interactions between different workers subject to different taxes. To simplify the calculations, the number of worker types is limited at two. The idea is to simulate a production with two type of labor, one more qualified than the other. The difference of qualification is obtain by using different parameters of productivity in the production function. It is also assumed that the more qualified workers have more bargaining power than the less qualified one. The main idea is that qualification is rare and provides worker with some additional bargaining power due to the workers. To understand

To solve this problem, the system of two equations 25 is considered for the two worker types 1 and 2 with the two unknown variables N_1 and N_2. It should be given of functional form to the production function. The decreasing marginal productivity form is kept for the global workforce N, with $F(N) = AN^{\alpha}$. Moreover, the aim is to understand the impact of the level of complementarity/substituability between workers on their interaction in relation with payroll taxation. Hence, a constant elasticity of substitution form is assumed for the global workforce N depending on the number N_1 and N_2 of each type of worker, with various calculations for various elasticities of substitution, including 1 modeled by a Cobb-Douglas production function. The production function is therefore $F(N_1, N_2) = A\left(\alpha_1 N_1^{-\delta} + \alpha_2 N_2^{-\delta}\right)^{-\frac{\alpha}{\delta}}$ with $\alpha_2 = 1 - \alpha_1$.

As said in the proof of proposition 2, the left hand term of that equation 25 is monotonously increasing when the right hand term monotonously decreasing. The unique solution may be numirically approach as close as wanted for each N_i at N_j given by incremental variations of N_i depending on the sign of the difference between the left hand term and the right hand term. Each solving is done sequentially with the other N_j given as the solution of the previous solving since the sum of the square of the changes $((N_{i,t} - N_{i,t-1})^2 + (N_{j,t} - N_{j,t-1})^2)$ is inferior to the precision level. The program is presented in appendix D.3. The level of precision is here of 10^{-8}.

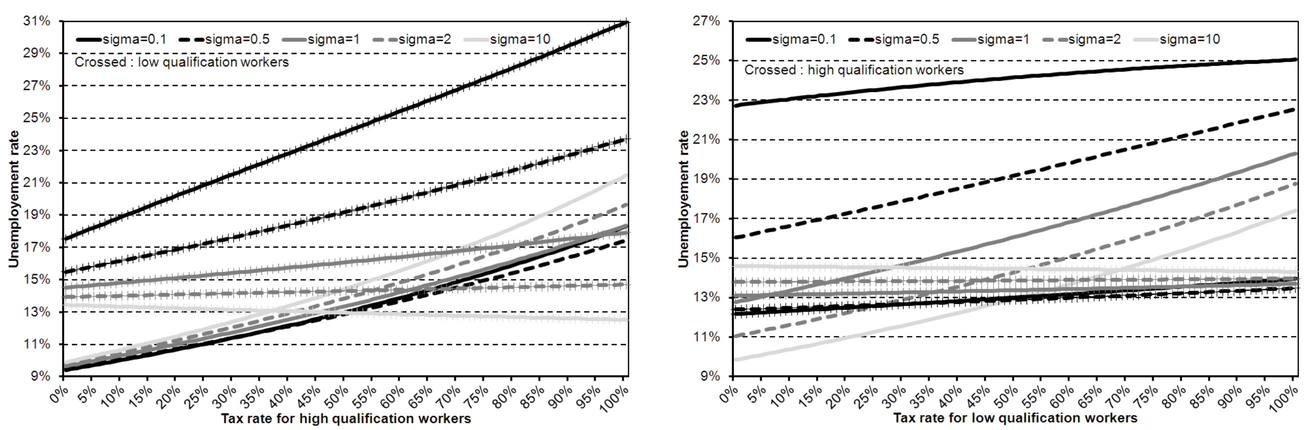

Figure 6: Impact of tax rate on high and low qualification workers' unemployment

The simulation is calculated for a high qualification worker with twice the productivity and the bargaining power as the low qualification worker ($\beta_h = 0.4$, $\beta_l = 0.2$, $\alpha_h = 2/3$, $\alpha_l = 1/3$). The simulations are done by changing the tax rate for one worker type from 0% to 100% keeping the other tax rate at 50%.

The full resluts are presented in appendix C. They show some similarities with the one worker case, particularly concerning the decreasing marginal incidence and therefore the marginal incidence being lower than the mean incidence. The theoretical results are confirmed, particularly concerning the crossed impact on employement. The level of unemployement resulting from the taxation of high or low qualification workers are presented in figure 6.

Taxing high qualification workers increase unemployement of low qualification workers except for very large elasticity of substitution between type of workers. The impact of high qualification workers'taxation on low qualification workers employment is quite large, even if it smaller than the direct impact on igh qualification workers' unemployment. The same result is appears qualitatively for high qualification workers when taxing low qualification workers but the effect is very small.

5 Conclusion and comments

The present article models a labor market with heterogenous workers in the search and matching framework. A global production function is considered to take into account the interactions between different level of qualifications of workers. Furthermore, the hypothesis of intra-firm bargaining is assumed as the main purpose is to understand incidence of taxation as a result of wage bargaining. The model is solve formally and some numerical analyses are run.

The results confirm that payroll taxation fall mainly on workers. Furthermore, it appears that marginal incidence decreases with respect to the level of taxation and therefore that marginal incidence is significantly lower than overall incidence. This has at least two applications. The first one is linked to interpretation of incidence estimations. Such empirical studies, if using an identification strategy in natural experiment, would estimate the marginal incidence and therefore underestimate the overall incidence. Those estimates would be accurate for marginal policy reforms but not to understand the overal distribution of the burden of taxation.

The second one is linked to the efficiency of employment policies consisting in lowering payroll taxation in order to deal with unemployment. Such policies' efficiency is marginally decreasing with the level of tax rebate. Indeed, an additional reduction of payroll tax take place with lower starting level of taxation, and indueces therefore more wages increase and less labor cost decrease than previous tax rebates.

In addition, Wages, unemployement and incidence increase with bargaining power and productivity of workers. This also matters for policies of tax rebates. By those policies, governments try to lower labor costs but this works only for low qualification and low bargaining power workers. For an exemple, French enlargement of tax rebate (CICE, up to 2.4 times the minimum wage) promise a low efficiency for lowering labor costs in France.

However, if the direct effect of taxation on unemployment is lower for high qualification workers than for low qualification workers, the crossed effect of high qualification workers taxation on low qualification workers unemployement is larger than the reciproqual.

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A Formal resolution of the differential equations

A.1 Differential equations for a unique worker type

The homogenous equation is $(1-\beta\tau)w(N)+(1+\tau)\beta N\partial w(N)/\partial N = 0$ whose solution is $W(N) = CN^{-[1+\beta\tau]/[(1+\tau)\beta]}$. The method of variation of the constant gives:

$$\frac{\partial w(N)}{\partial N} = \frac{\partial C}{\partial N}N^{-\frac{1+\beta\tau}{(1+\tau)\beta}} - \frac{1+\beta\tau}{(1+\tau)\beta}N^{-\frac{1+\beta\tau}{(1+\tau)\beta}-1}C$$

Taking the initial differential equation minus the constant term $(1-\beta)rU$:

$$(1+\tau)\frac{\partial C}{\partial N}N^{-\frac{1+\beta\tau}{(1+\tau)\beta}} - \frac{1+\beta\tau}{\beta}CN^{-\frac{1+\beta\tau}{(1+\tau)\beta}-1} + \frac{1+\beta\tau}{\beta N}CN^{-\frac{1+\beta\tau}{(1+\tau)\beta}} - \frac{1}{N}\frac{\partial F(N)}{\partial N} = 0$$

Which leads to the equation of the C function:

$$\frac{\partial C}{\partial N} = \frac{1}{1+\tau}N^{\frac{1-\beta}{(1+\tau)\beta}}\frac{\partial F(N)}{\partial N}$$

And consequantly:

$$C(N) = \int_0^N \frac{1}{1+\tau}z^{\frac{1-\beta}{(1+\tau)\beta}}\frac{\partial F(N)}{\partial N}dz + D$$

With the change of variable $z \to u = z/N$ ($dz = Ndu$, u goes from 0 to 1) the net wage is given by:

$$(1+\tau)w(N) = \frac{1}{N}\int_0^1 u^{\frac{1-\beta}{(1+\tau)\beta}-1}uNF'(uN)du + DN^{-\frac{1+\beta\tau}{(1+\tau)\beta}}$$

The intergration converge if $NF'(N)$ is continuous in 0 (what is assumed) because $(1-\beta)/[(1+\tau)\beta] - 1 > -1$. Furthermore $Nw(N) \to 0$ when $N \to 0$, hence $D = 0$ and the net wage is given by equation 17 (subject to variable change $u^{\frac{1}{1+\tau}} = v$).

A.2 Disentengling the system of differential equations

The partial derivative of equation 21 with respect to $l \neq i$ is:

$$(1+\beta_i\tau_i)\frac{\partial w_i(\vec{N})}{\partial N_l} = \beta_i\left[\frac{\partial^2 F(\vec{N})}{\partial N_i N_l} - \sum_{j=1}^{n}(1+\tau_j)N_j\frac{\partial^2 w_j(\vec{N})}{\partial N_i N_l} - (1+\tau_l)\frac{\partial w_l(\vec{N})}{\partial N_i}\right]$$

Yet, when $i \neq l$:

$$\frac{\partial^2}{\partial N_i \partial N_l}\sum_{j=1}^{n}(1+\tau_j)N_j w_j(\vec{N}) = \sum_{j=1}^{n}(1+\tau_j)N_j\frac{\partial^2 w_j(\vec{N})}{\partial n_l \partial N_i} + (1+\tau_i)\frac{\partial w_i(\vec{N})}{\partial N_l} + (1+\tau_l)\frac{\partial w_l(\vec{N})}{\partial N_i}$$

And therefore the derivative with respect to N_l of differential equation 21 for $i \neq l$ is:

$$(1-\beta_i)\frac{\partial w_i(\vec{N})}{\partial N_l} = \beta_i\frac{\partial^2}{\partial N_i \partial N_l}\left[F(\vec{N}) - \sum_{j=1}^{n}(1+\tau_j)N_j w_j(\vec{N})\right]$$

Comparing derivative with respect to N_l of differential equation 21 for $i \neq l$ and derivative with respect to N_i of differential equation 21 for $l \neq i$ gives equation 26.

$$\frac{\partial w_l(\vec{N})}{\partial N_i} = \frac{1-\beta_i}{\beta_i}\frac{\beta_l}{1-\beta_l}\frac{\partial w_i(\vec{N})}{\partial N_l} = \chi_{il}\frac{\partial w_i(\vec{N})}{\partial N_l} \tag{26}$$

Which implies that:

$$\sum_{j=1}^{n}(1+\tau_j)N_j\frac{\partial w_j(\vec{N})}{\partial N_i} = \sum_{j=1}^{n}(1+\tau_j)\chi_{ij}N_j\frac{\partial w_i(\vec{N})}{\partial N_j}$$

And differential equation 21 may be rewritten as differential equation 22.

A.3 Differential equations for multiple worker types

Now let consider a change of coordinate such that $\vec{M}_i = (M_{i1}, ..., M_{in})$, $v_i(\vec{M}_i) = w_i(\vec{N})$ and:

$$\sum_{j=1}^{n} M_{ij} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}} = \sum_{j=1}^{n} (1+\tau_j)\chi_{ij} N_j \frac{\partial w_i(\vec{N})}{\partial N_j}$$

It works in particular if for all j:

$$M_{ij} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}} = (1+\tau_j)\chi_{ij} N_j \frac{\partial w_i(\vec{N})}{\partial N_j}$$

Yet by definition:

$$\frac{\partial w_i(\vec{N})}{\partial N_j} = \frac{dM_{ij}}{dN_j} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}}$$

And therefore the differential equation in M_{ij} is $M_{ij} = (1+\tau_j)\chi_{ij} N_j dM_{ij}/dN_j$. One solution is $M_{ij} = N_j^{\chi_{ji}/(1+\tau_j)}$ as $1/\chi_{ij} = \chi ji$. Furthermore, we call $G(\vec{M}_i) = F(\vec{N})$, hence $\partial F(\vec{N})/\partial N_j = (\partial G(\vec{M}_i)/\partial M_{ij})(dM_{ij}/dN_j) = [\chi_{ji}/(1+\tau_j)] N_j^{\chi_{ji}/(1+\tau_j)-1} (\partial G(\vec{M}_i)/\partial M_{ij})$. And in particular $\partial F(\vec{N})/\partial N_i = [N_i^{-\tau_i/(1+\tau_i)}/(1+\tau_i)](\partial G(\vec{M}_i)/\partial M_{ii}) = [M_{ii}^{-\tau_i}/(1+\tau_i)](\partial G(\vec{M}_i)/\partial M_{ii})$.The differential equation become 27.

$$(1+\beta_i\tau_i) w_i(\vec{M}_i) = (1-\beta_i) r U_i + \beta_i \left[\frac{M_{ii}^{-\tau_i}}{1+\tau_i} \frac{\partial G(\vec{M}_i)}{\partial M_{ii}} - \sum_{j=1}^{n} M_{ij} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}} \right] \tag{27}$$

Another corrdinate change should now be made with spherical coordinate $(\rho_i, \phi_{i1}, ..., \phi_{i,n-1})$ where ρ_i is the canonical norm of the vector (eg: $\rho_i^2 = \sum_{j=1}^{n} M_{ij}^2$) and $\vec{\phi}_i = (\phi_{i1}, ..., \phi_{i,n-1})$ the angles. Let determine the angles as in equation 28.

$$\begin{cases} \phi_{i,1} & \textit{such that} \quad M_{i,n} = \rho_i \sin\phi_{i1} \\ \phi_{i,2} & \textit{such that} \quad M_{i,n-1} = \rho_i \cos\phi_{i1} \sin\phi_{i2} \\ & \qquad\qquad ... \\ \phi_{i,j} & \textit{such that} \quad M_{i,n+1-j} = \rho_i \cos\phi_{i1} ... \cos\phi_{i,j-1} \sin\phi_{ij} \\ & \qquad\qquad ... \\ \phi_{i,n-1} & \textit{such that} \quad M_{i,2} = \rho_i \cos\phi_{i1} ... \cos\phi_{i,n-2} \sin\phi_{i,n-1} \end{cases} \tag{28}$$

It follows that $M_{i,1}^2 = \rho^2 - \sum_{j=2}^{n} M_{i,j}^2$. Yet:

$$\begin{cases} M_{i,n}^2 + M_{i,n-1}^2 = \rho^2(1-\cos^2\phi_{i1} + \cos^2\phi_{i,1}\sin^2\phi_{i,2}) = \rho^2[1-\cos^2\phi_{i1}(1-\sin^2\phi_{i,2})] = \rho^2[1-\cos^2\phi_{i1}\cos^2\phi_{i,2}] \\ ... + M_{i,n-2}^2 = \rho^2[1-\cos^2\phi_{i1}\cos^2\phi_{i,2}(1-\sin^2\phi_{i,3})] = \rho^2[1-\cos^2\phi_{i1}\cos^2\phi_{i,2}\cos^2\phi_{i,3}] \\ ... \\ ... + M_{i,2}^2 = \rho^2[1-\cos^2\phi_{i1} ... \cos^2\phi_{i,n-2}(1-\sin^2\phi_{i,n-1})] = \rho^2[1-\cos^2\phi_{i1} ... \cos^2\phi_{i,n-1}] \end{cases}$$

And therefore $M_{i,1}$ is given by equation 29.

$$M_{i,1} = \rho \cos\phi_{i1} ... \cos\phi_{i,n-2} \cos\phi_{i,n-1} \tag{29}$$

Equation 28 and equation 29 implies that $\rho_i \partial M_{ij}/\partial\rho_i = M_{ij}$ and therefore:

$$\rho_i \partial v_i(\vec{M}_i)/\partial\rho_i = \rho_i \sum_{j=1}^{n} (\partial v_i(\vec{M}_i)/\partial M_{ij})(\partial M_{ij}/\partial\rho_i) = \sum_{j=1}^{n} M_{ij} \partial v_i(\vec{M}_i)/\partial M_{ij}$$

Hence, the differential equation become equation 30.

$$(1+\beta_i\tau_i)v_i(\rho_i,\vec{\phi}_i) = (1-\beta_i)rU_i + \beta_i\left[\frac{M_{ii}^{-\tau_i}}{1+\tau_i}\frac{\partial G(\vec{M}_i)}{\partial M_{ii}} - \rho_i\frac{\partial v_i(\rho_i,\vec{\phi}_i)}{\partial \rho_i}\right] \tag{30}$$

The solution of the homogenous equation is $v_i(\rho_i,\vec{\phi}_i) = C\rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}}$. The method of variation of the constant gives:

$$C(\rho_i,\vec{\phi}_i)\rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}} + \frac{\beta_i}{1+\beta_i\tau_i}\rho_i\left(\frac{\partial C(\rho_i,\vec{\phi}_i)}{\partial\rho_i}\rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}} - \frac{1+\beta_i\tau_i}{\beta_i}C(\rho_i,\vec{\phi}_i)\rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}-1}\right) = \frac{M_{ii}^{-\tau_i}}{1+\tau_i}\frac{\partial G(\vec{M}_i)}{\partial M_{ii}}$$

$$\frac{\beta_i}{1+\beta_i\tau_i}\rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}+1}\frac{\partial C(\rho_i,\vec{\phi}_i)}{\partial\rho_i} = \frac{M_{ii}^{-\tau_i}}{1+\tau_i}\frac{\partial G(\vec{M}_i)}{\partial M_{ii}}$$

And the result is:

$$v_i(\rho_i,\vec{\phi}_i) = \frac{1-\beta_i}{1+\beta_i\tau_i}rU_i + \rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}}\left(\kappa_i(\vec{\phi}_i) + \int_0^{\rho_i}\frac{M_{ii}^{-\tau_i}}{1+\tau_i}z^{\frac{1+\beta_i\tau_i}{\beta_i}-1}\frac{\partial G(\vec{M}_i)}{\partial M_{ii}}dz\right)$$

Where M_{ii} is indeed a function of z. It appears that $(u\rho_i,\vec{\phi}_i) = (uM_{i1},...,M_{in})$ so doing the change of variable $z = u\rho_i$ (u from 0 to 1, $dz = \rho_i du$, $\vec{M}_i(z) = u\vec{M}_i$). The integrale become:

$$\int_0^1 \frac{u^{-\tau_i}M_{ii}^{-\tau_i}}{1+\tau_i}u^{\frac{1+\beta_i\tau_i}{\beta_i}-1}\rho_i^{\frac{1+\beta_i\tau_i}{\beta_i}-1}\frac{\partial G(u\vec{M}_i)}{\partial M_{ii}}\rho_i du = \frac{M_{ii}^{-\tau_i}}{1+\tau_i}\rho_i^{\frac{1+\beta_i\tau_i}{\beta_i}}\int_0^1 u^{\frac{1-\beta_i}{\beta_i}}\frac{\partial G(u\vec{M}_i)}{\partial M_{ii}}du$$

In addition:

$$\frac{\partial G(u\vec{M}_i)}{\partial M_{ii}} = \frac{1+\tau_i}{u^{-\tau_i}M_{ii}^{-\tau_i}}\frac{\partial F(u\vec{M}_i)}{\partial N_i}$$

Let call $\mu_{ij} = uM_{ij}$, it is equal to $\nu_j^{\chi_{ji}/(1+\tau_j)}$ in the initial coordinates, yet $M_{ij} = N_j^{\chi_{ji}/(1+\tau_j)}$, so $\nu_j^{\chi_{ji}/(1+\tau_j)} = uN_j^{\chi_{ji}/(1+\tau_j)}$ and $\nu_j = u^{(1+\tau_j)\chi_{ij}}N_j$ and the net wage is given by equation 23.

B Numerical analyses in the case of unique worker type

B.1 One worker type and linear taxes

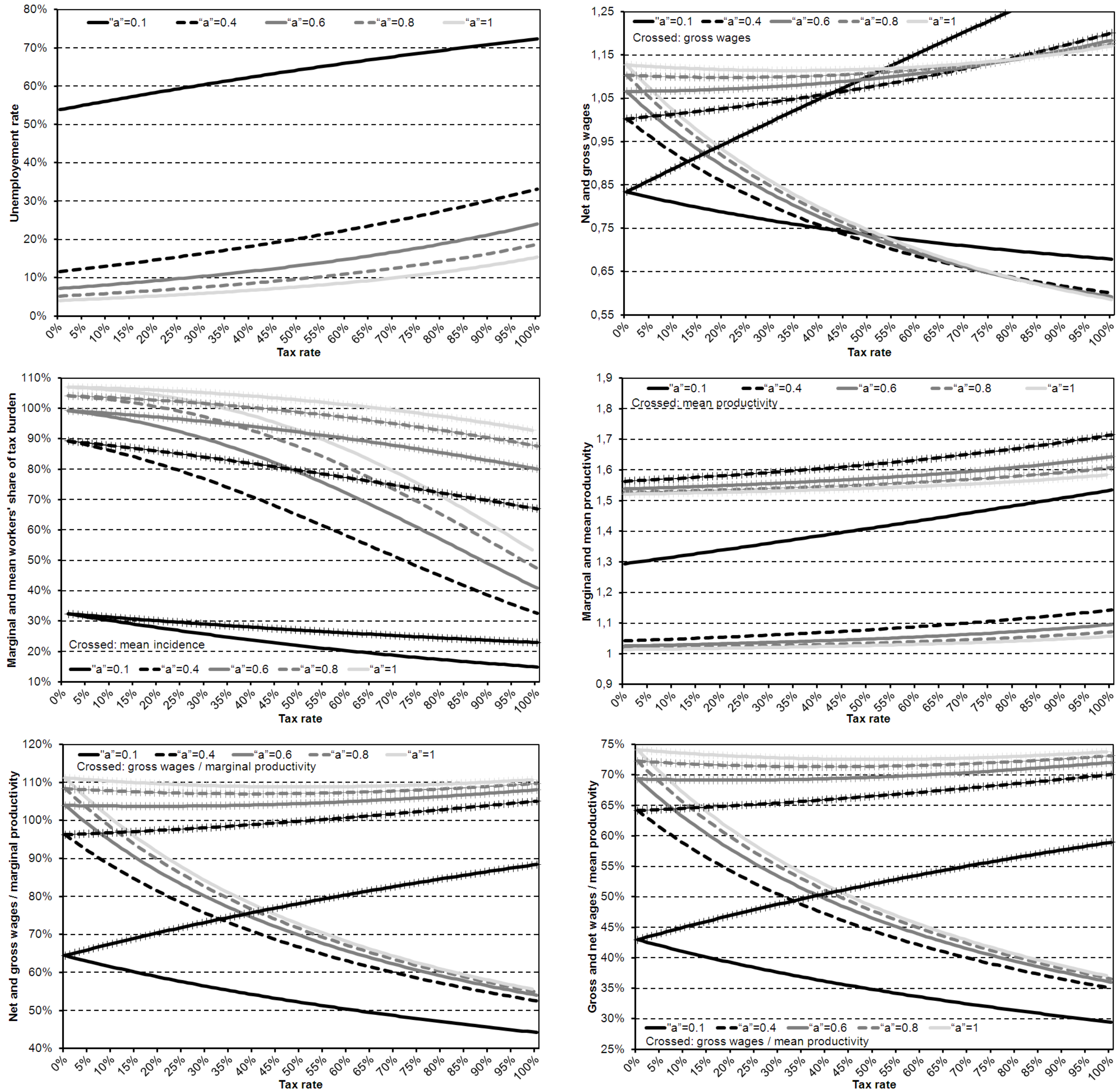

Figure 7: Impact of matching efficiency on unique labor market equilibrium

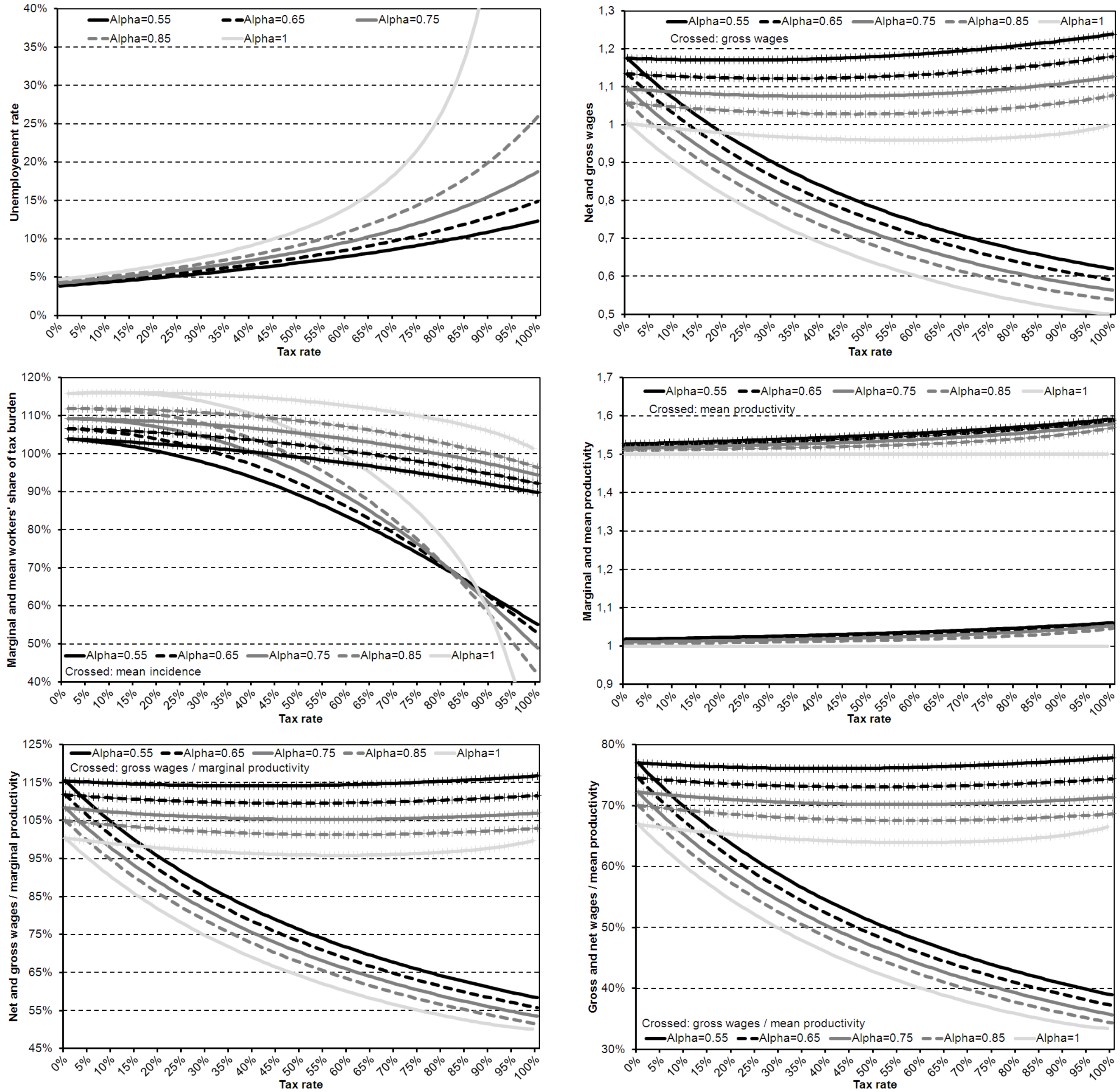

Figure 8: Impact of taxes and labor productivity on unique labor market equilibrium

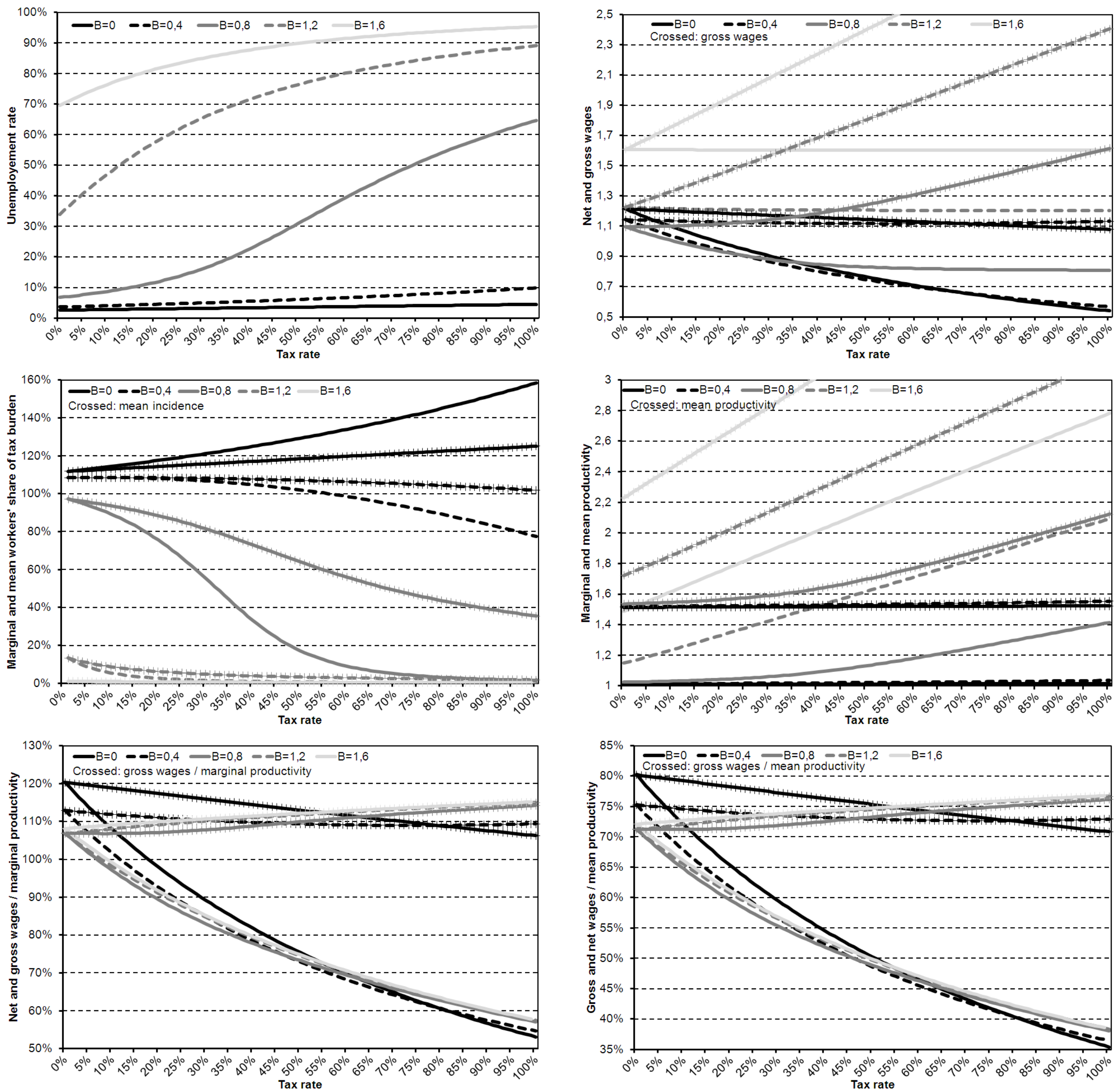

Figure 9: Impact of taxes and unemployement benefits on unique labor market equilibrium

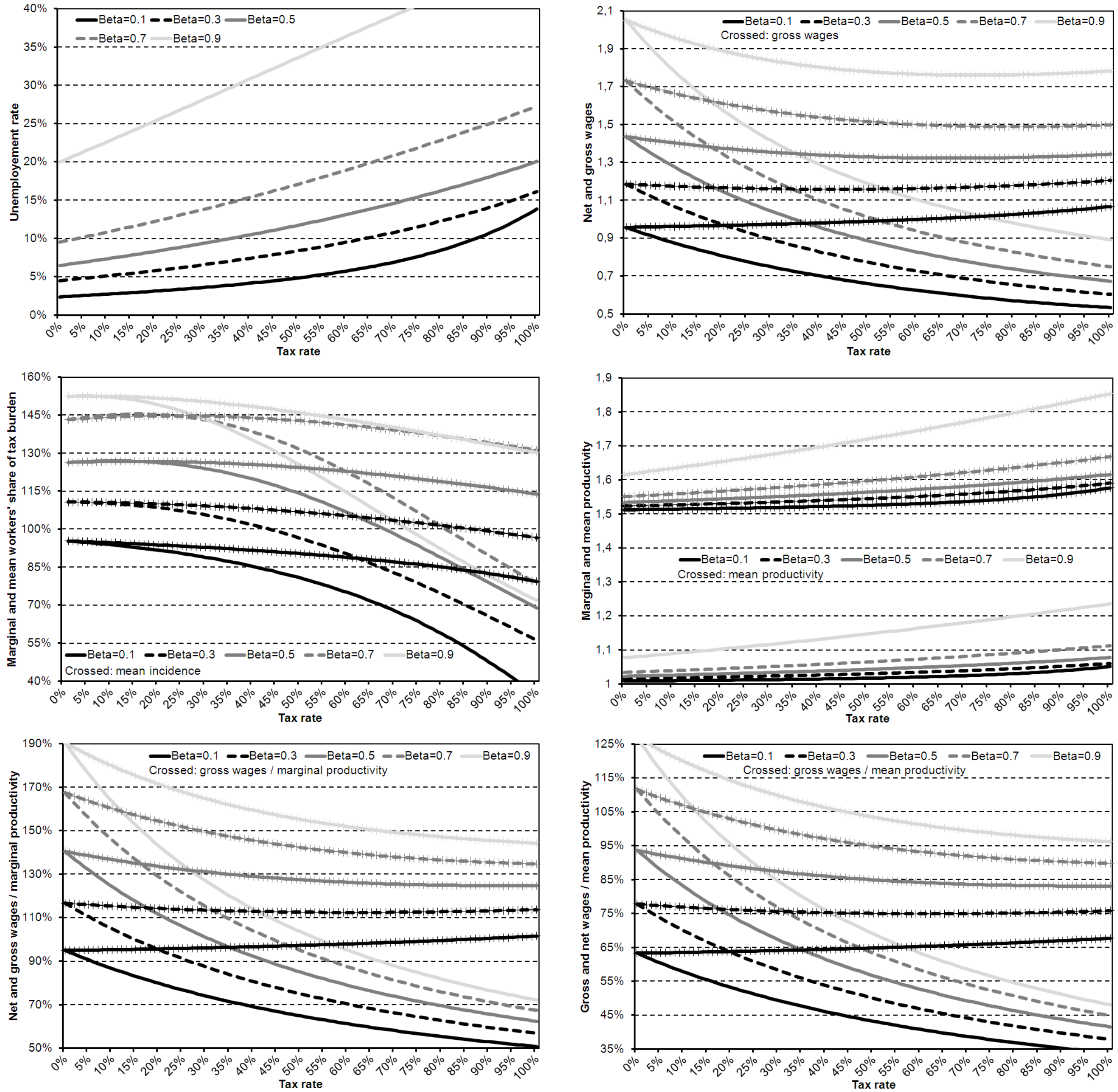

Figure 10: Impact of taxes and bargaining power on unique labor market equilibrium

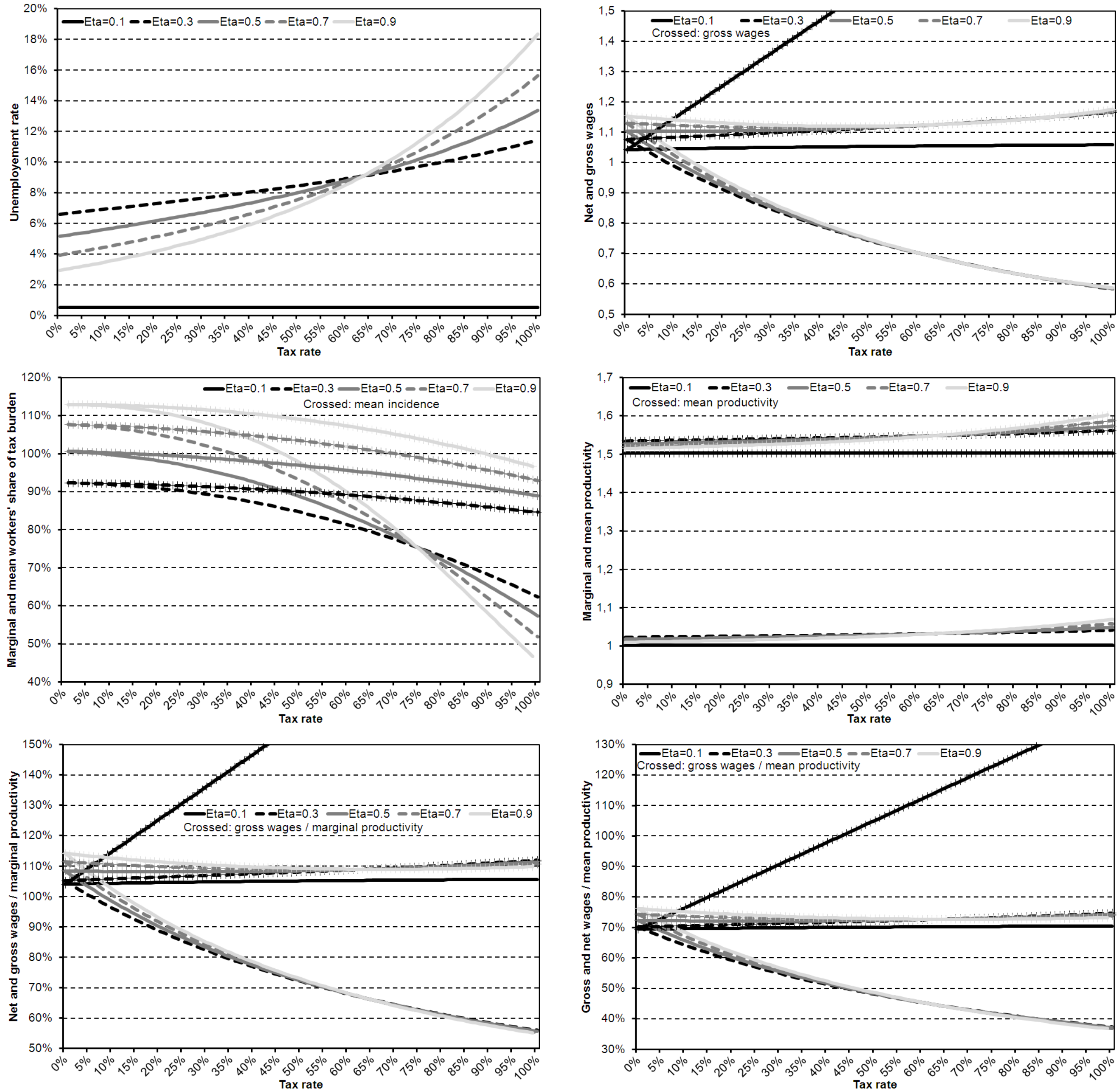

Figure 11: Impact of taxes and matching elasticity on unique labor market equilibrium

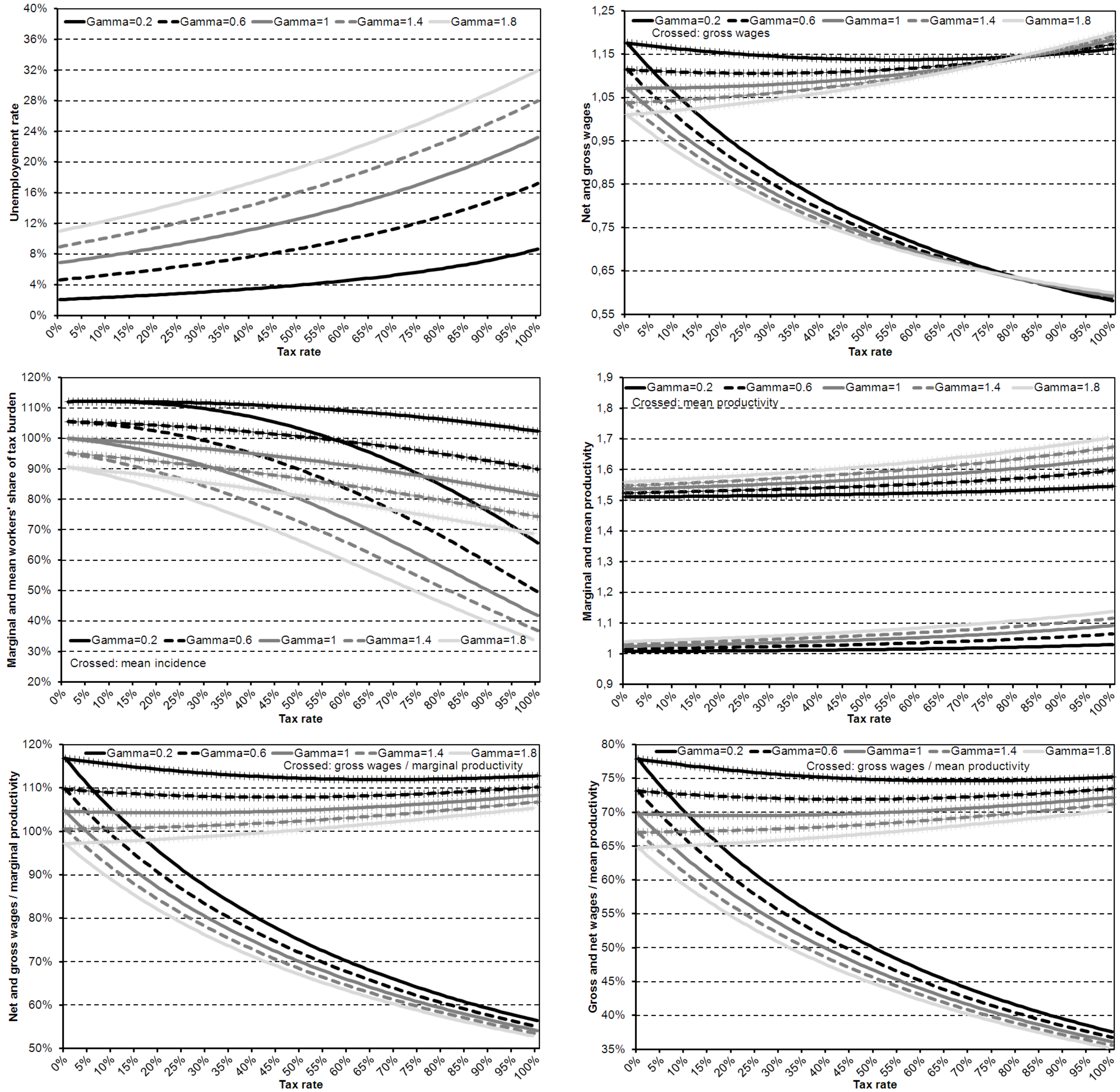

Figure 12: Impact of taxes and vacancy posting cost on unique labor market equilibrium

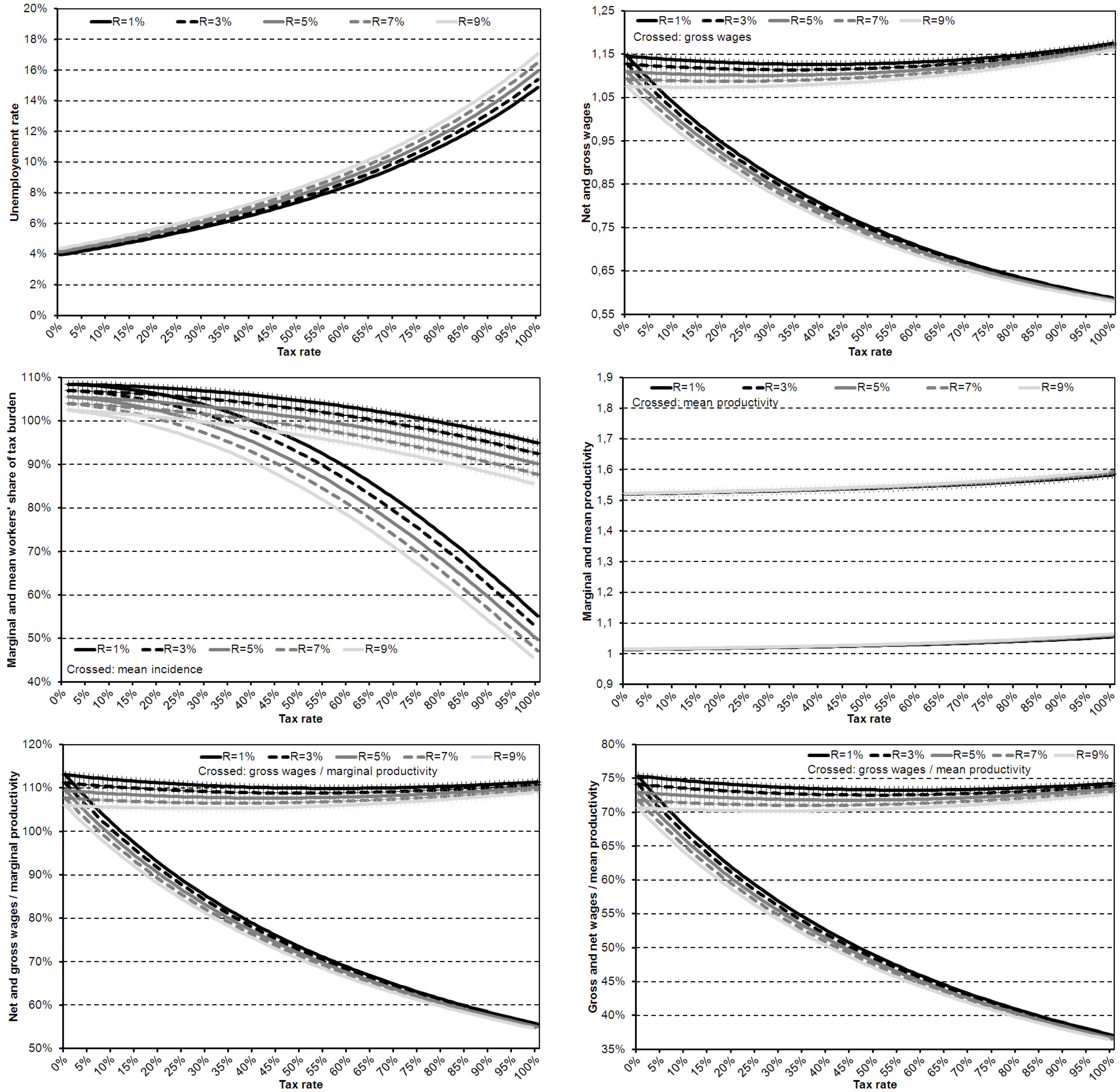

Figure 13: Impact of taxes and interest rate on unique labor market equilibrium

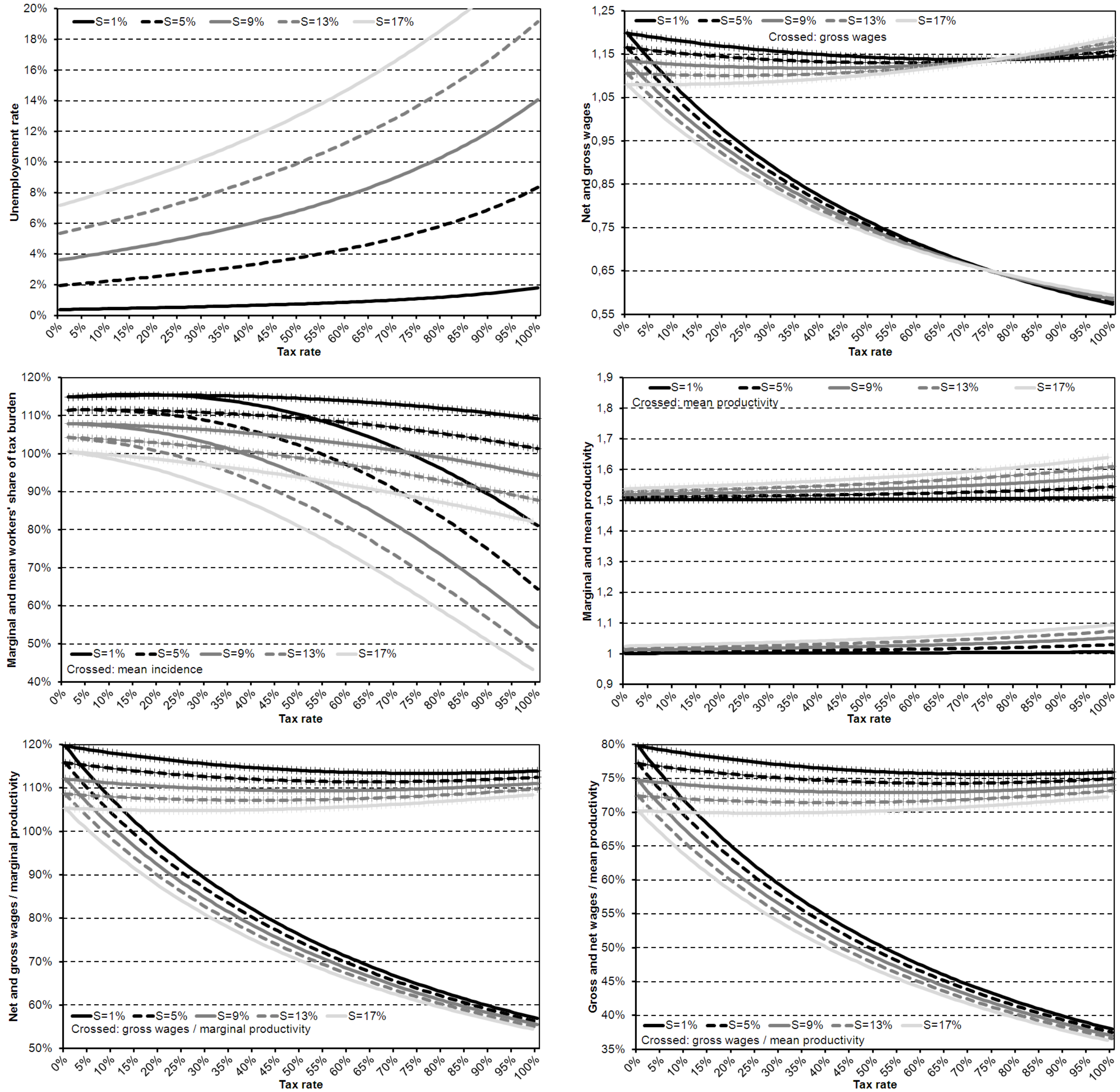

Figure 14: Impact of taxes and job destruction rate on unique labor market equilibrium

B.2 One worker type and quadratic taxes

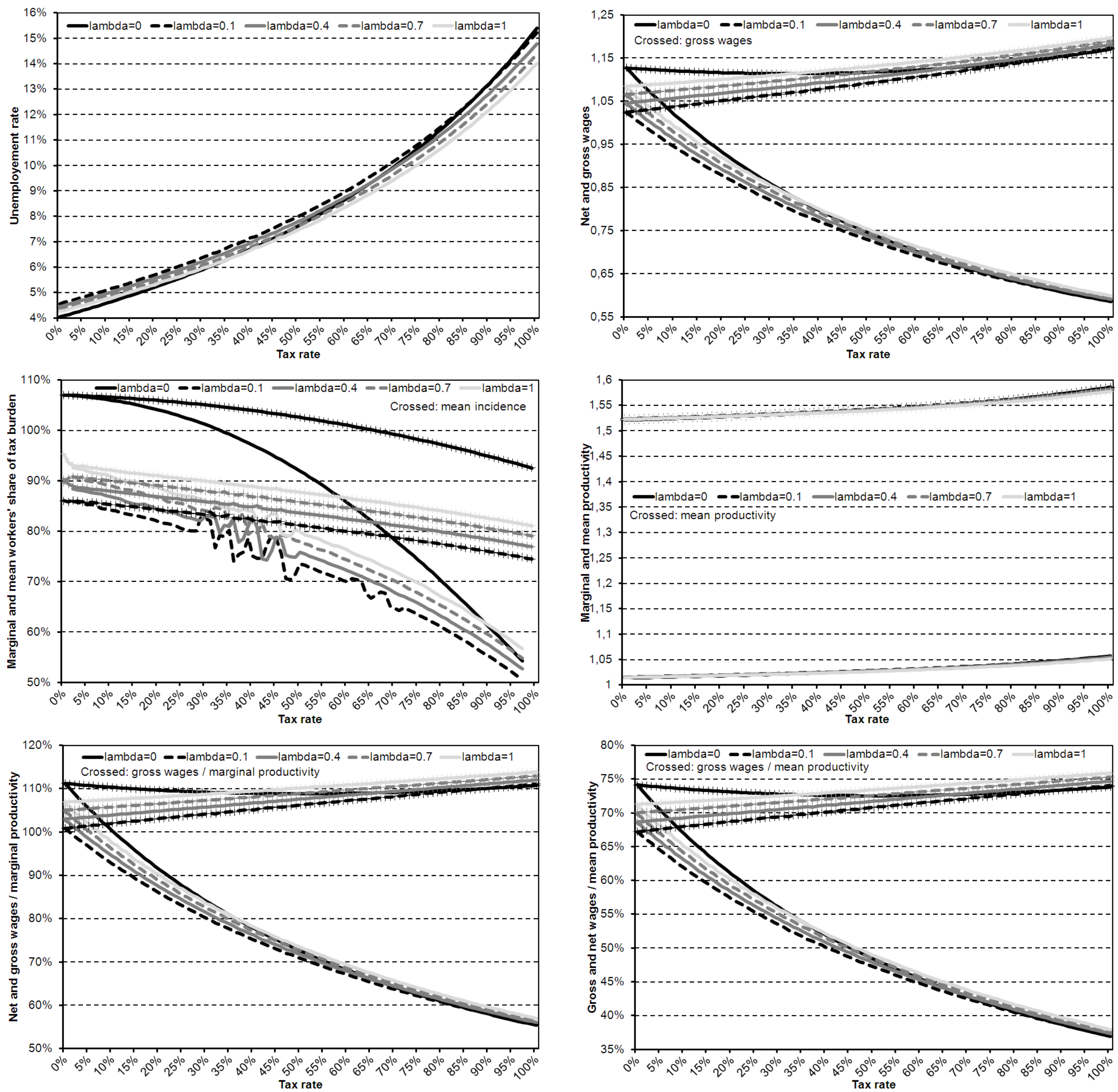

Figure 15: Impact of progressive taxes on unique labor market equilibrium

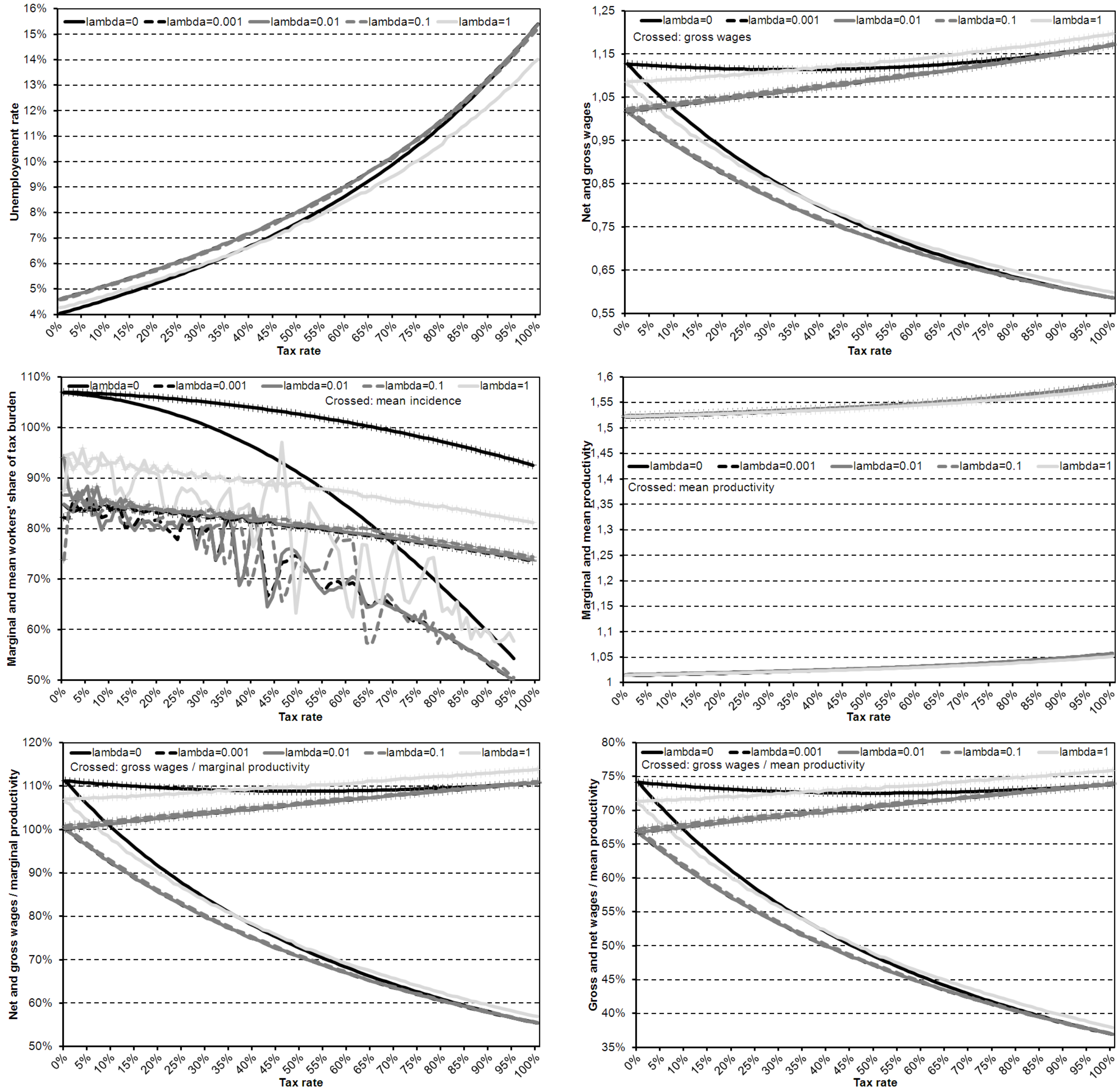

Figure 16: Impact of slightly progressive taxes on unique labor market equilibrium

C Numerical analyses in the case of two worker types

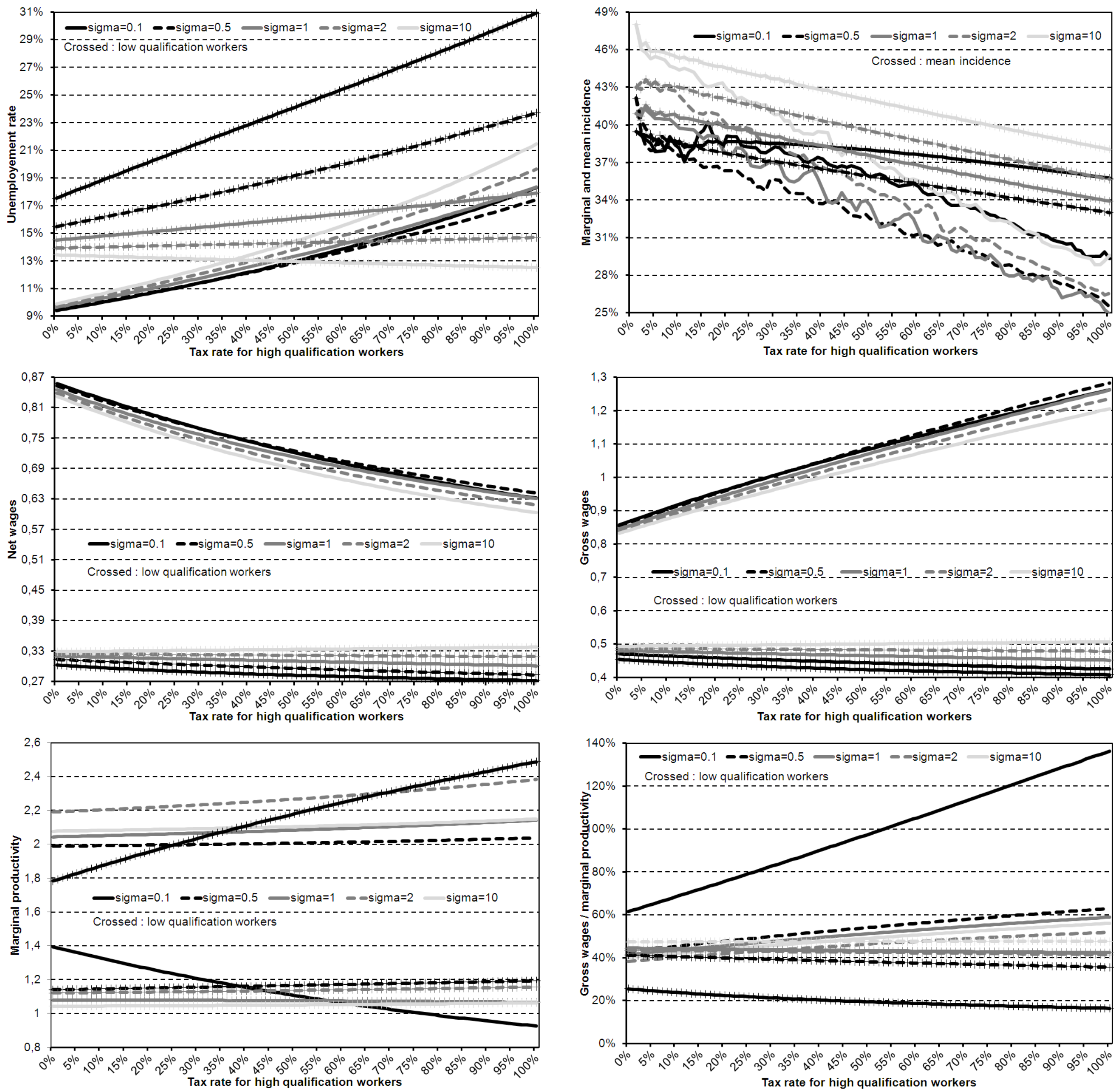

Figure 17: Impact of taxes on high qualification workers and substitution elasticity between workers

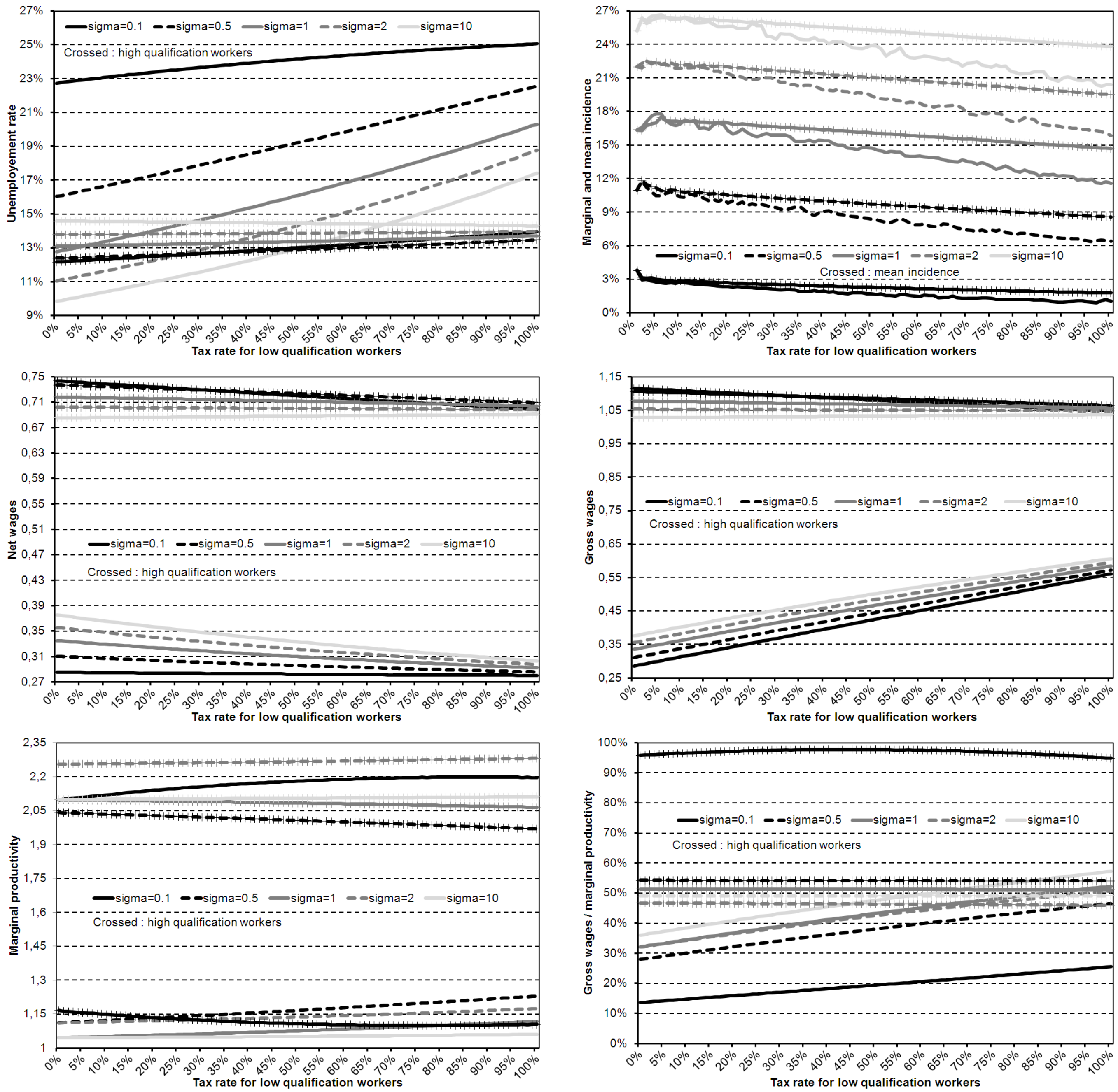

Figure 18: Impact of taxes on low qualification workers and substitution elasticity between workers

D Programs for numerical analyse

The numerical analyses has been implemented with *SciLab* software.

D.1 Programs for one worker type under linear tax

For the case of the analysis of the β, the program is as follows. It is the same with very little adaptation for the other parameter analyses.

```
N=zeros(10,101); Chom=zeros(10,101); Wnet=zeros(10,101); Wbrut=zeros(10,101);
Prodm=zeros(10,101);

bet=1/4; b=1/2; gam=1/2; s=1/10; r=3/100; eta=0.68; a=1;

deff('y=the(x)','y=(s/a)^(1/eta)*(x/(1-x))^(1/eta)');

deff('y=w(x)','y=b+gam*bet/(1-bet)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');

for be=1:1:10
      for ta=1:1:101
            alp=0.5+be/20;
            A=1/alp;
            tau=(ta-1)/100;
            deff('y=f(x)','y=((alp*bet*A*x^(alp-1))/(1-bet+alp*bet*(1+tau)))-
                  (((1+tau)*bet*b)/(1+bet*tau))-((gam*bet)/(1-
                  bet))*((1+tau)*bet/(1+bet)*(s/a)^(1/eta)*(x/(1-
                  x))^(1/eta)+(r+s)/s*(s/a)^(1/eta)*(x/(1-x))^(1/eta-1))');
            deff('y=fj(x)','y=((alp-1)*alp*bet*A*x^(alp-2))/(1-bet+alp*bet*(1+tau))-
                  (gam*bet)/(1-bet)*((1/eta)*(1+tau)*bet/(1+bet)*(s/a)^(1/eta)*(1/(1-
                  x)^2)*(x/(1-x))^(1/eta-1)+(1/eta-1)*(r+s)/s*(s/a)^(1/eta)*(1/(1-
                  x)^2)*(x/(1-x))^(1/eta-2))');
            if (be>9 & ta>99) then
                  N(be,ta)=fsolve(0.5,f,fj);
                  else
                  N(be,ta)=fsolve(0.99999999,f,fj);
            end
            Chom(be,ta)=1-N(be,ta);
            Wnet(be,ta)=w(N(be,ta));
            Wbrut(be,ta)=(1+tau)*Wnet(be,ta);
            Prodm(be,ta)=alp*A*N(be,ta)^(alp-1);
      end
end

[res]=file('open','resultalp.txt','new');
dispfiles()
write(1,"Fonction de alpha");
write(1," ");
write(1,"Chomage");
write(1,Chom);
write(1," ");
write(1,"Salaire net");
write(1,Wnet);
write(1," ");
write(1,"Salaire brut");
write(1,Wbrut);
write(1," ");
write(1,"Productivity");
write(1,Prodm);
file("close",1);
```

D.2 Programs for one worker type under quadratic tax

```
N=zeros(4,101); Chom=zeros(4,101); Wnet=zeros(4,101); Wbrut=zeros(4,101);

alp=2/3; bet=0.25; A=1/alp; b=1/2; gam=1/2; s=1/10; r=3/100; eta=0.68; a=1;

deff('y=the(x)','y=(s/a)^(1/eta)*(x/(1-x))^(1/eta)');

deff('y=w(x)','y=b+gam*bet/(1-bet)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');

for lambda=1:1:4
      if lambda==1 then lam=0.1; end
      if lambda==2 then lam=0.4; end
      if lambda==3 then lam=0.7; end
      if lambda==4 then lam=1; end

      for tax=1:1:101
            Wtest=0.75;
            test=1;
            tau=(tax-1)/100;

            while test>0.001,
                  kap-tau-Wtest*lam;
                  minecar=100000;
                  for enne=0.84:0.001:0.96
                        theta=the(enne);
                        deff('z=dif(x,y)','z=(1-bet)*b/bet+qam*theta+x^(alp-1)-(1-
                              bet)*(sqrt((1+kap)^2+4*lam*y/x)-(1+kap))/(2*bet*lam)');
                        nen=0.001:0.001:0.999;
                        sol=ode(0.001,0.001,nen,dif);
                        wage=(sqrt((1+kap)^2+4*lam*sol(1000*enne)/enne)-(1+kap))/(2*lam);
                        ecar=wage-b-gam*bet*(theta|(r|s)/(a*theta^(eta-1)))/(1-bet);
                        if (abs(ecar)<minecar) then
                              minecar=abs(ecar);
                              N(lambda,tax)=enne;
                              Chom(lambda,tax)=1-enne;
                              Wnet(lambda,tax)=wage;
                        end
                  end
                  Wtest=Wnet(lambda,tax);
                  tcst=abs(tau kap Wtcst*lam)
            end
            kap=tau-Wtest*lam;
            minecar=100000;
            Nmin=N(lambda,tax)-0.0005;
            Nmax=N(lambda,tax)+0.0005;
            for enne=0.84:0.00001:0.96
                  if enne>=Nmin then
                        if enne<=Nmax then
                              theta-the(enne);
                              deff('z=dif(x,y)','z=(1-bet)*b/bet+gam*theta+x^(alp-1)-(1-
                                    bet)*(sqrt((1+kap)^2+4*lam*y/x)-(1+kap))/(2*bet*lam)');
                              nen=0.00001:0.00001:0.99999;
                              sol=ode(0.00001,0.00001,nen,dif);
                              wage=(sqrt((1+kap)^2+4*lam*sol(100000*enne)/enne)-
                                    (1+kap))/(2*lam);
                              ecar=wage-b-gam*bet*(theta+(r+s)/(a*theta^(eta-1)))/(1-bet);
                              if (abs(ecar)<minecar) then
                                    minecar=abs(ecar);
                                    N(lambda,tax)=enne;
                                    Chom(lambda,tax)=1-enne;
                                    Wnet(lambda,tax)=wage;
                              end
                        end
                  end
            end
            Wbrut(lambda,tax)=(1+kap+lam*Wnet(lambda,tax))*Wnet(lambda,tax);
      end
end
```

D.3 Programs for one worker type under linear tax

```
Chom1=zeros(5,101); Chom2=zeros(5,101); Wnet1=zeros(5,101); Wnet2=zeros(5,101);
alp=2/3; alp1=1/3; alp2=2/3; A=9/2; bet1=0.2; bet2=0.4; b1=1/4; b2=1/2; gam=1/2; s=1/10; r=3/100; eta=0.68; a=1; tau2=0.5;

deff('y=the(x)','y=(s/a)^(1/eta)*(x/(1-x))^(1/eta)');
deff('y=weq1(x)','y=b1+gam*bet1/(1-bet1)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');
deff('y=weq2(x)','y=b2+gam*bet2/(1-bet2)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');

for sigma=1:1:5
   if sigma==1 then sig=0.1; end
   if sigma==2 then sig=0.5; end
   if sigma==3 then sig=1; end
   if sigma==4 then sig=2; end
   if sigma==5 then sig=10; end
   del=(1-sig)/sig;

   for ta=1:1:101
        sigma, ta,
        tau1=(ta-1)/100;
        if sigma==3 then
           deff('f=F1(x,y,z)','f=alp*alp1*A*x^(alp*alp1-1)*y^(alp*alp2)*z^(alp*(alp1*(1+tau1)
                +alp2*(1+tau2)*bet2*(1-bet1)/(bet1*(1-bet2))))');
           deff('f=F2(x,y,z)','f=alp*alp2*A*x^(alp*alp1)*y^(alp*alp2-1)*z^(alp*(alp1*(1+tau1)*bet1*(1-bet2)/(bet2*(1-bet1))
                +alp2*(1+tau2)))');
        else
           deff('f=F1(x,y,z)','f=alp*alp1*A*x^(-del-1)*z^(-del*(1+tau1))*(alp1*x^(-del)*z^(-del*(1+tau1))
                +alp2*y^(-del)*z^(-del*(1+tau2)*bet2*(1-bet1)/(bet1*(1-bet2))))^(-1-alp/del)');
           deff('f=F2(x,y,z)','f=alp*alp2*A*y^(-del-1)*z^(-del*(1+tau2))*(alp1*x^(-del)
                *z^(-del*(1+tau1)*bet1*(1-bet2)/(bet2*(1-bet1))) +alp2*y^(-del)*z^(-del*(1+tau2)))^(-1-alp/del)');
        end

        N1=0.85; N2=0.85; n1=0; n2=0; test1=(N1-n1)^2+(N2-n2)^2;
        if ta>1 then N1=1-Chom1(sigma,ta-1); N2=1-Chom2(sigma,ta-1); end

        while test1>0.00000002
           n1=N1; n2=N2;
           deff('u=sous1(t)','u=t^(tau1+(1-bet1)/bet1)*F1(N1,N2,t)');
           test2=intg(0,1,sous1)-(1+tau1)*bet1*b1/(1+bet1*tau1)-gam*bet1/(1-bet1)*(((1-bet1)
                +2*(1+tau1)*bet1)*the(N1)/(1+bet1*tau1)+(r+s)/(a*the(N1)^(eta-1)));

           if test2<0 then
                while (test2<0 & N1>0.0001);
                   N1=N1-0.00005;
                   deff('u=sous1(t)','u=t^(tau1+(1-bet1)/bet1)*F1(N1,N2,t)');
                   test2=intg(0,1,sous1)-(1+tau1)*bet1*b1/(1+bet1*tau1)-gam*bet1/(1-bet1)*(((1-bet1)
                        +2*(1+tau1)*bet1)*the(N1)/(1+bet1*tau1)+(r+s)/(a*the(N1)^(eta-1)));
                end
           else
                while (test2>0 & N1<0.9999);
                   N1=N1+0.00005;
                   deff('u=sous1(t)','u=t^(tau1+(1-bet1)/bet1)*F1(N1,N2,t)');
                   test2=intg(0,1,sous1)-(1+tau1)*bet1*b1/(1+bet1*tau1)-gam*bet1/(1-bet1)*(((1-bet1)
                        +2*(1+tau1)*bet1)*the(N1)/(1+bet1*tau1)+(r+s)/(a*the(N1)^(eta-1)));
                end
           end

           deff('u=sous2(t)','u=t^(tau2+(1-bet2)/bet2)*F2(N1,N2,t)');
           test2=intg(0,1,sous2)-(1+tau2)*bet2*b2/(1+bet2*tau2)-gam*bet2/(1-bet2)*(((1-bet2)
                +2*(1+tau2)*bet2)*the(N2)/(1+bet2*tau2)+(r+s)/(a*the(N2)^(eta-1)));

           if test2<0 then
                while (test2<0 & N2>0.0001);
                   N2=N2-0.00005;
                   deff('u=sous2(t)','u=t^(tau2+(1-bet2)/bet2)*F2(N1,N2,t)');
                   test2=intg(0,1,sous2)-(1+tau2)*bet2*b2/(1+bet2*tau2)-gam*bet2/(1-bet2)*(((1-bet2)
                        +2*(1+tau2)*bet2)*the(N2)/(1+bet2*tau2)+(r+s)/(a*the(N2)^(eta-1)));
                end
           else
                while (test2>0 & N2<0.9999);
                   N2=N2+0.00005;
                   deff('u=sous2(t)','u=t^(tau2+(1-bet2)/bet2)*F2(N1,N2,t)');
                   test2=intg(0,1,sous2)-(1+tau2)*bet2*b2/(1+bet2*tau2)-gam*bet2/(1-bet2)*(((1-bet2)
                        +2*(1+tau2)*bet2)*the(N2)/(1+bet2*tau2)+(r+s)/(a*the(N2)^(eta-1)));
                end
           end

           test1=(N1-n1)^2+(N2-n2)^2;
        end

        Chom1(sigma,ta)=1-N1;
        Chom2(sigma,ta)=1-N2;
        Wnet1(sigma,ta)=weq1(N1);
        Wnet2(sigma,ta)=weq2(N2);
   end
end
```