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Collective Decisions, Household Production, and the Labor Force Participation

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Abstract

In this paper, we generalize the collective model of household labor supply with domestic production to allow for the possibility of non-participation in the labor market. Firstly, we show that the main structural components of the decision process, in particular, the price of leisure, can be retrieved. Secondly, we estimate a system of market and domestic labor supply using the PSID and apply the theoretical results.

Kewwords: collective decisions, labor supply, household production, intra-household allocation, PSID

JEL Code: D13, J22

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1 Introduction

Over the recent years, the traditional 'unitary' representation of household behaviour has been challenged by models accounting for the presence of several decision-makers in the household, with possibly specific preferences. In particular, the collective model of labour supply, developed by Chiappori (1988, 1992), has turned out to be very popular. This model describes the household as a collectivity made up of two persons, each of whom is characterised by individual preferences and makes decisions about consumption and use of time. The household decision process, whatever its true nature, is assumed to lead to Pareto efficient outcomes. Then, it can be shown that, if consumption is purely private and household members are egoistic, the intra-household distribution of resources can be identified from the sole observation of spouses' labour supply functions. This property is important because it allows analysing welfare at the individual level, instead of exclusively concentrating on the distribution of well-being across households, as is generally the case with traditional models.¹

These features of the collective model have turned out to be very attractive, and the number of empirical studies based on Chiappori's initial framework is considerable. These include, among others, Bloemen (2010, Netherlands); Blundell et al. (2008, United Kingdom); Chau et al. (2007, China); Chiappori, Fortin, and Lacroix (2002, United States); Clark, Couprie and Sofer (2004, United-Kingdom); Crespo (2009, Spain); Donni (France, 2007); Donni and Moreau (2007, France); Fernandez-Val (2003, Spain); Fortin and Lacroix (1997, Canada); Haddad (2015, Iran); Lacroix and Radtchenko (2011, Russia); Moreau and Donni (2002, France); Sinha (2012, India); and Vermeulen (2005, Belgium). In these empirical applications, all non-market time is considered as leisure, implying that the time spent in home production activities is simply ignored. Such simplification may be misleading. In particular, Donni (2008) shows that it will generally lead to biased welfare analysis. Nevertheless, the extension of the initial identification results to models with domestic production is provided by Apps and Rees (1997) and Chiappori (1997). In the case of marketable domestic production, the intra-household distribution of resources can be identified from the observation of both domestic *and* market labour supply functions.² However, empirical applications of collective models accounting for home production

¹See Chiappori and Donni (2011) for a survey of collective models.

 $^{^{2}}$ The marketability assumption, though restrictive, turns out to be necessary to allow retrieving structural components of the decision process. It is justified by Chiappori (1997). The 'unitary' model of domestic pro-

remain quite rare. Examples are given by Apps and Rees (1996, Australia); Aronsson, Daunfeldt and Wikstrom (2001, Sweden); Cherchye, De Rock and Vermeulen (2012, Netherlands); Couprie (2007, United Kingdom); Donni and Matteazzi (2012, United-States); and Rapoport, Sofer and Solaz (2009, France). More importantly, none of them takes into account the possibility of corner solutions for labour market participation. Samples are simply restricted to two-working couples (without always taking into account the problem of endogenous sample selection). Identification theory with corner solutions has still to be developed.³

The objective of the present paper is to fill this gap. We provide a first collective model of labour supply with domestic production that can be estimated on a sample including household members who do not work in the market. We prove that the sharing of household resources can be identified over the whole domain of exogenous variables from the observation of spouses' market and domestic labour supply functions. This theoretical extension is important because it allows the econometrician to recover some important structural components, including the price of leisure, for individuals who do not work in the market. In addition, it also allows using larger samples in empirical estimations and thus obtaining more precise results. For the sake of illustration, we finally use the 2009 wave of PSID data and consider the case where women have to decide whether to participate or not in the labour market. A system of domestic and market labour supply equations is estimated by the simulated maximum likelihood method taking into account unobservable heterogeneity. The estimations are discussed and compared in the light of the existing empirical literature.

The paper is structured as follows. Section 2 presents the model and the main theoretical results. Section 3 discusses the empirical specification. Section 4 is dedicated to data and the estimation method. Section 5 presents the empirical results. Section 6 concludes.

duction developed by Gronau (1977), and its numerous extensions (Gronau, 1980; Graham and Green, 1984; Solberg and Wong, 1992), is based on this very assumption as well.

³See Donni (2003) and Blundell et alii (2008) for a study of corner solutions without domestic production.

2 The Collective Model

2.1 Preferences, Technology, and the Decision Process

In what follows, we shall focus on a labour supply model, within a two-member household, accounting for domestic production and labour market participation decisions. Let i = 1, 2 denote, respectively, the husband and the wife. The total labour supply of spouse i, defined as the sum of market and domestic labour time, is denoted by L_i and her private consumption by c_i . Following Chiappori (1988, 1992), we assume that individuals' preferences are of the egoistic type, i.e., each spouse is characterised by a smooth, monotonic, and strictly quasi-concave utility function of the form:⁴

$$u_i = u_i \left(L_i, c_i \right), \tag{SU}$$

with i = 1, 2. This function also satisfies the following condition, $\lim_{c_i \to 0} \partial u_i / \partial c_i = \infty$, so that household members always consume a positive quantity of goods. The total labour supply can be broken down into market labour supply h_i and domestic labour supply t_i , i.e.,

$$L_i = t_i + h_i.$$

The household technology is represented by a smooth, monotonic, and strictly concave function of spouses' time input, i.e.,

$$z \le f(t_1, t_2),\tag{1}$$

where z denotes the quantity of the household produced good. The production function also satisfies the following conditions, $\lim_{t_i\to 0} \partial f/\partial t_i = \infty$ with i = 1, 2, which implies that spouses' domestic labour supply is always positive. It is worth noting that the quantity produced by the household may be smaller or greater than the quantity consumed. Indeed, following Gronau (1977, 1980), we suppose that the domestic produced good can be exchanged on a competitive market at a constant price. This is the marketability assumption mentioned in the introduction. The price of market and domestic goods is set to one. The following non-negativity restrictions must also be satisfied:

$$h_1 \ge 0, \quad h_2 \ge 0, \tag{2}$$

⁴As usual, all the results are still valid if preferences are of the 'caring' type; see Chiappori (1992).

so that we allow non-participation in the labour market. The household budget constraint is defined as:

$$c_1 + c_2 \le w_1 h_1 + w_2 h_2 + y + z, \tag{3}$$

where y denotes household non-labour income or, alternatively, total expenditure net of labour income, and w_i is spouse *i*'s market wage rate. Note that market wage rates are assumed to be always observed by the economist, even when the wife or the husband does not work.

Following the basic idea of the collective approach, we assume that the decision process, whatever its true nature, always generates Pareto-efficient outcomes. This approach can be justified by referring to the theory of repeated games under perfect information. Since the household is a typical example of such repeated games, it is plausible that mechanisms leading to efficient outcomes could be developed by its members. The household equilibrium is thus located on the Pareto frontier; a change in the exogenous variables may, in general, lead to a change in the form of this frontier as well as to a move of the equilibrium along it. If we suppose for convenience that utility functions are not only quasi-concave but also concave, the efficient outcomes can be simply obtained as follows:

$$\max_{\{c_1, c_2, h_1, h_2, t_1, t_2\}} \mu(w_1, w_2, y) u_1(t_1 + h_1, c_1) + (1 - \mu(w_1, w_2, y)) u_2(t_2 + h_2, c_2)$$
(PO)

subject to (1)–(3), where $\mu(w_1, w_2, y)$ is a Pareto weight that represents the bargaining power of the husband relatively to the wife. If $\mu(w_1, w_2, y)$ increases, the balance of power shifts in favour of the husband. Generally, the Pareto weight depends on wage rates and non-labour income; it may also depend on preference factors and distribution factors (i.e., variables that influence the decision process without affecting the budget constraint or preferences; see Bourguignon, Browning, and Chiappori (2009)). This will be the case in the empirical application.

2.2 The Structure of labour Supply Functions

For the simple model where corner solutions are ignored, the household decision process can be represented as a two-stage budgeting one. First, spouses choose their respective contribution to domestic chores in order to maximise household profits. At this stage, they also agree on an allocation of total non-labour income (i.e., the sum of non-labour income and profit), according to a given sharing rule depending on the bargaining power of the household members. Second, spouses maximise, each one independently from the other, their own utility subject to the production and the sharing decisions made at the first stage. This interpretation is well-known since Apps and Rees (1997) and Chiappori (1997). However, the decentralization process is a little more complicated if the spouses have the possibility of not participating in the labour market. For working spouses, the price of time is simply equal the corresponding market wage rate. For non-working spouses, the price of time – which corresponds to the number of units of consumption that spouses are willing to give up to get one additional unit of time – is endogenously determined by the marginal productivity in the household production. The decision process can then be decentralised as described in the following proposition.

Proposition 1. Suppose that spouses' utility functions are of the SU form. Then, under Pareto efficiency, the optimal allocation $\{h_1^*, h_2^*, t_1^*, t_2^*, c_1^*, c_2^*\}$ is the solution of a decentralised decision process. More precisely, there exist a pair of prices for spouses' time $\{w_1^*, w_2^*\}$ and a pair of shares for total non-labour income $\{\varphi_1(w_1^*, w_2^*), \varphi_2(w_1^*, w_2^*)\}$ such that:

A. At the first stage, the domestic labour supply functions $\{t_1^*, t_2^*\}$ are solutions of

$$\max_{\{t_1,t_2\}} \left\{ f(t_1,t_2) - t_1 w_1^* - t_2 w_2^* \right\} = \pi(w_1^*,w_2^*);$$

B. At the second stage, the market labour supply and consumption demand functions $\{h_i^*, c_i^*\}$ with i = 1, 2 are solutions of

$$\max_{\{c_i,h_i\}} u_i(c_i,h_i+t_i^*)$$

subject to $c_i \leq w_i^* (h_i + t_i^*) + \varphi_i(w_1^*, w_2^*)$, and $h_i \geq 0$, where $\varphi_1(w_1^*, w_2^*) + \varphi_2(w_1^*, w_2^*) = y + \pi(w_1^*, w_2^*)$.

C. In the participation case, i.e., $h_i^* > 0$, the price of time of spouse *i*, with i = 1, 2, is equal to her market wage, i.e., $w_i^* = w_i$.

If Conditions A–C are fulfilled for some $\{w_1^*, w_2^*\}$ and $\{\varphi_1(w_1^*, w_2^*), \varphi_2(w_1^*, w_2^*)\}$, then the optimal allocation is Pareto-efficient.

Proof. From the First and the Second Theorems of Welfare Economics, any efficient allocation can be obtained as a decentralised competitive equilibrium, and conversely. \Box

The decentralization process is similar to that in Chiappori (1997) or Donni and Matteazzi (2012) except that the wage rates are replaced by the prices of time. The specific structure of market and domestic labour supply functions can be derived from the optimization problems of the first and second stage. Condition A states that the household maximises profits taking into account the possible endogeneity of the price of spouses' time. Solving the optimization problem gives domestic labour supply functions: $t_i^* = g_i(w_1^*, w_2^*)$, for some function $g_i(\cdot)$ which has the traditional properties of input demand functions. One of these properties is that, if the price of spouses' time is constant, the level of non-labour income should not affect the number of hours devoted to domestic chores. Condition B can be interpreted as follows. The spouses agree on a particular distribution of resources (non-labour income and household profits) between them, and receive a share of it. Then, the spouses choose the consumption and market labour supply that maximise their utility, under the constraints that their domestic labour supply is (optimally) fixed and they cannot spend more than the amount provided by their respective shares. Solving the individual utility maximization problem above gives the traditional Marshallian total labour supply functions: $L_i^* = F_i(w_i^*, \varphi_i)$, for some function $F_i(\cdot)$. The price of spouses' time is equal to the wage rate if they work in the market. This is established by Condition C. If the husband (wife) does not work in the market, then the price of his (her) time is equal to the marginal rate of substitution between his (her) time devoted to labour and his (her) consumption or, alternatively, to the marginal productivity of his (her) time devoted to the domestic production. As suggested by Neary and Roberts (1980), the price of time can be computed by inverting the total labour supply function with the hours of market labour supply set to zero. Let us suppose, for instance, that the wife does not work in the market. Hence, the price of wife's time, w_2^* , solves the following equation:

$$g_2(w_1, w_2^*) = F_2(w_2^*, \varphi_2(w_1, w_2^*)).$$
(4)

To summarise, the structure of market and domestic labour supply functions derived from

Proposition 1 is described by the following system of equations:

$$h_1^* = F_1(w_1^*, \varphi_1(w_1^*, w_2^*)) - g_1(w_1^*, w_2^*),$$
(5)

$$t_1^* = g_1(w_1^*, w_2^*), (6)$$

$$h_2^* = F_2(w_2^*, \varphi_2(w_1^*, w_2^*)) - g_2(w_1^*, w_2^*), \tag{7}$$

$$t_2^* = g_2(w_1^*, w_2^*), (8)$$

where $\varphi_1(w_1^*, w_2^*) + \varphi_2(w_1^*, w_2^*) = y + \pi(w_1^*, w_2^*)$, with the constraints:

$$[F_1(w_1^*,\varphi_1(w_1^*,w_2^*)) - g_1(w_1^*,w_2^*)] \cdot (w_1^* - w_1) = 0, \qquad (9)$$

$$[F_2(w_2^*,\varphi_2(w_1^*,w_2^*)) - g_2(w_1^*,w_2^*)] \cdot (w_2^* - w_2) = 0.$$
(10)

that define the price of leisure. The conditions in (9) and (10) imply that if $w_i^* > w_i$ then individuals do not participate. Otherwise, if $w_i^* = w_i$ then individuals may supply some positive hours in the labour market. This results is provided in the last statement of Proposition 1.

To go further we first have to note that the market wage rates may influence the intra-household distribution of resources even in the case of non-participation of one or both spouses.⁵ Hence, the sharing functions can be written as $\varphi_i(w_1^*, w_2^*, w_1, w_2, y)$ or, more compactly, with a slight abuse of notation, as $\varphi_i(w_1, w_2, y)$ as the prices of time are themselves functions of the current wage rates and non-labour income. The idea is that when spouses do not work they may exert their bargaining power within the family on the grounds of their potential wage, i.e., the current wage they may earn entering the labour force, as justified by more formal models of household bargaining in which divorce is used as threat point (Manser and Brown, 1980; McElroy and Horney, 1981). Other variables (e.g., distribution factors) may also enter the sharing functions but they will be discussed in the empirical section.

Finally, let us note that the structure (5)-(10) is restrictive – and could be empirically tested – in the sense that any system of labour supply functions will generally *not* be characterised by such a specific structure.

⁵In that case, market wage rates can be assimilated to distribution factors because they do not influence the budget constraint (nor preferences).

2.3 Identification

The present model is made of four "observable" "reduced-form" equations, that is, $(t_1^*, t_2^*, h_1^*, h_2^*)$ as functions of (w_1, w_2, y) . If both spouses participate in the labour market, the time they devote to household chores is completely determined by the production technology, i.e., the form of the "reduced-form" domestic labour supply functions is independent of the characteristics of the utility functions and the intra-household distribution of resources.⁶ Then identification of the sharing functions can be obtained from the observation of both market and domestic labour supply functions using a result of Chiappori (1997) and, more precisely, Donni and Matteazzi (2012) for the case without distribution factors. However, this result cannot be applied here because household members do not necessarily participate in the labour market.

A generalization of this identification result is thus proposed below. Let us start by demonstrating that, in general, the price of leisure can be recovered from observed behaviour. To do this, we need the following assumption.

Assumption I.1. Wage rates and non-labour income (w_1, w_2, y) continuously vary in $\mathbb{R}^2_+ \times \mathbb{R}$. For any $(w_1, w_2) \in \mathbb{R}^2_+$, there exists some y such that $h_1^* > 0$ and $h_2^* > 0$.

In other words, whatever the level of wage rates, it is always possible to find a level of household non-labour income such that both the wife and the husband work. This assumption allows us to prove the next important result.

Proposition 2. Under Pareto efficiency and I.1, the price of spouses' time can be identified as functions of (w_1, w_2, y) from the observation of two market and domestic labour supply functions. The profit function can be identified up to a constant as well.

The proof is given in the Appendix. Intuitively, for any value of the wage rates, one can find households where both spouses work in the market. Hence the spouses' domestic labour supply functions can be recovered over their domain as a function of wage rates only. Then these functions indirectly define the price of spouses' time because the latter coincide with individual's productivity. It is worth noting that the result is not specific to collective models: under I.1, the price of spouses' time can be identified in unitary models as well. To the best of our knowledge, and as simple as it may be, the result had not been proved until now.

⁶This is the traditional separability principle used in models of agricultural households.

To complete the identification of individuals' preferences and the household decision process, more structure on the decision process is useful, if not necessary. In the unitary model of labour supply, the labour market participation decision can be modelled by a reservation wage equation, where the reservation wage rate is justly defined as the price of time when the worker is indifferent between being in the labour force or not. Yet Blundell et al. (2007) and Donni (2003, 2007) point out that, in a collective model of labour supply, the participation frontier is not necessarily well-behaved. In particular, the uniqueness of the reservation wage rate does not result from the theoretical framework that we have adopted, but must be explicitly postulated. This is made with the following assumption.

Assumption I.2. There exists a pair of functions $w_1^R(w_2, y)$ and $w_2^R(w_1, y)$ such that $h_1^* > 0$ if $w_1 > w_1^R(w_2, y)$ and $h_2^* > 0$ if $w_2 > w_2^R(w_1, y)$.

The reservation wage functions $w_1^R(w_2, y)$ and $w_2^R(w_1, y)$ partition $\mathbb{R}^2_+ \times \mathbb{R}$ into four connected subsets. This assumption could be relaxed at the cost of some complications but it is rather natural and should not be the subject of controversy. It is discussed in detail by Donni (2003).

We also use the following additional assumptions.

Assumption I.3. For any $w_1 \in \mathbb{R}_+$, there exists some y such that $w_2 = w_2^R(w_1, y)$. For any $w_2 \in \mathbb{R}_+$, there exists some y such that $w_1 = w_1^R(w_2, y)$.

This assumption is sufficient to obtain global identification, and not only local identification in the neighbourhood of the participation frontiers.⁷

Identification cannot be achieved over all the values of (w_1, w_2, y) . We thus define $\mathcal{P} \subset \mathbb{R}^3_+$ as the subset of (w_1, w_2, y) such that at least one spouse participates in the labour market. The following proposition can now be put forward.

Proposition 3. Suppose that spouses' utility functions are of the SU form. Under Pareto efficiency and I.1–I.3, and some technical conditions listed in the proof, the shares of non-labour income as functions of (w_1, w_2, y) can be recovered on \mathcal{P} up to a pair of additive constants from the observation of the two market and domestic labour supply functions. More precisely, if φ_1^* and φ_2^* is a particular pair of sharing functions which is consistent with the household behaviour,

⁷This assumption is a little stronger than the preceding one, but it could relaxed to obtain identification on a more restricted subset of $\mathbb{R}^2_+ \times \mathbb{R}$.

then any other pair defined as $\varphi_1 = \varphi_1^* + k_1$ and $\varphi_2 = \varphi_2^* + k_2$ for some constants k_1 and k_2 , is also consistent with the same household behaviour.

The proof is given in the Appendix. Interestingly, two constants are unidentified instead of one as in the other collective models. The reason is that total non-labour income, i.e., the sum of non-labour income and profit, is not directly observable by the economist, but must be recovered from the data. Once the constants are chosen, individual utility functions can be identified as well (up to an increasing transformation).

In a model without domestic production, Donni (2003) demonstrates that the sharing functions can be identified (up to a unique additive constant) when one spouse at most does not participate in the labour market. The present result is thus a generalization of this result.⁸

2.4 Robustness

The canonical model described in the previous subsections could be amended in various directions. The critical issue is the exogeneity or constancy of the domestic good price.

For the price of the domestic good to be exogenous, one of the key assumptions is that home time produces a good that has a perfect (or close at least) substitute sold on the market at a constant price. This (strong) assumption is discussed in details by Chiappori (1997), so we will just underline an important implication of relaxing it. If the domestic good cannot be sold or purchased on the market, then its price will be endogenously determined within the household. Consequently, the separation principle according to which domestic labour supplies are determined by the sole technology of production is no longer valid even if both partners are working in the market. In particular, a change in the intra-household distribution of bargaining power will affect market and domestic labour supply through a change in the price of the domestic good, and not only through a change in the sharing rule, which should incite us to be careful when interpreting the empirical results. Nonetheless, the present model can still be seen as a good approximation if produced goods and purchased goods are strongly,

⁸It is worth saying that the identification result by Donni (2003) is local in the sense that it is valid only in the neighborhood of the participation frontier while the identification result of Proposition 3 is global. This technical distinction is explained in the proof.

if not perfectly substitutable. We will come back on this interpretation in the empirical part of this paper.⁹

Another necessary assumption for the exogeneity of the domestic good is that household consumption is purely private. If consumption had some public components, the price of the consumption good would be endogenously determined (as in the non-market case) and specific to each individual living in the household.¹⁰ Our claim here is that the collective model of labour supply with private consumption may still be a good approximation to this alternative, more general model if the sensitivity of individual prices to changes in wage rates and non labour income remains limited. This seems to be an acceptable assumption.

3 The Empirical Specification

In this section, we present the functional form for market and domestic labour supply functions and discuss the stochastic specification. Since the participation rate of men in the data is close to one, we consider as a simplification of the theoretical model that the husbands' labour supply is always positive. The wife's labour supply may vary between zero and any positive number.

3.1 labour Supplies, Sharing Rule, and Preferences

Let us first consider the functional forms for domestic labour supply functions, then for market labour supply functions, and lastly for sharing functions.

We suppose that the household profit function can be approximated by a Generalised Leontief specification (Diewert, 1973), i.e.,

$$\pi(w_1^*, w_2^*) = \Delta(\boldsymbol{x}) - 2a_1 (w_1^*)^{1/2} - 2a_2 (w_2^*)^{1/2} - b_1(\boldsymbol{x})w_1^* - b_2(\boldsymbol{x})w_2^* - 2c (w_1^*)^{1/2} (w_2^*)^{1/2},$$

where $\Delta(\mathbf{x})$, $b_1(\mathbf{x})$ and $b_2(\mathbf{x})$ are functions of socio-demographic factors \mathbf{x} including stochastic terms that represent unobservable heterogeneity in technology (but $\Delta(\mathbf{x})$ is not identified),

⁹The collective model of household behavior with domestic production relaxing the marketability assumption is studied by Apps and Rees (1997) and Chiappori (1997). It raises some identification issues.

¹⁰The sum of individual prices will be equal to one if the domestic good is purely private or larger than one if it is not a purely public good.

 a_1 , a_2 and c are parameters, and $w_1^* = w_1$ since the husband always works in the market.¹¹ This function is globally regular (i.e., decreasing and convex in its arguments) if a_1 , a_2 , $b_1(\boldsymbol{x})$, $b_2(\boldsymbol{x})$ and c are positive. To guarantee positivity, $b_1(\boldsymbol{x})$ and $b_2(\boldsymbol{x})$ are supposed to be quadratic functions of \boldsymbol{x} . Then, applying the Hotelling's Lemma, the spouses' domestic labour supply functions are derived, i.e.,

$$t_1 = a_1 (w_1^*)^{-1/2} + b_1(\boldsymbol{x}) + c (w_1^*)^{-1/2} (w_2^*)^{1/2}, \qquad (11)$$

$$t_2 = a_2 (w_2^*)^{-1/2} + b_2(\boldsymbol{x}) + c (w_1^*)^{1/2} (w_2^*)^{-1/2}.$$
(12)

For the sake of homogeneity, we also suppose that spouses' market labour supply can be approximated by the following linear form, i.e.,

$$h_1 = \alpha_1(\boldsymbol{x}) + \beta_1 \left(w_1^* \right)^{1/2} + \gamma_1 \varphi_1 - a_1 \left(w_1^* \right)^{-1/2} - b_1(\boldsymbol{x}) - c \left(w_1^* \right)^{-1/2} \left(w_2^* \right)^{1/2}, \quad (13)$$

$$h_2 = \alpha_2(\boldsymbol{x}) + \beta_2 (w_2^*)^{1/2} + \gamma_2 \varphi_2 - a_2 (w_2^*)^{-1/2} - b_2(\boldsymbol{x}) - c (w_1^*)^{1/2} (w_2^*)^{-1/2}, \qquad (14)$$

where $\alpha_1(\boldsymbol{x})$ and $\alpha_2(\boldsymbol{x})$ are functions of socio-demographic factors \boldsymbol{x} including stochastic terms, β_1 , β_2 , γ_1 and γ_2 are parameters. The Slutsky conditions are globally satisfied if β_1 , β_2 are positive and γ_1 , γ_2 are negative.

The next step consists in specifying the functional form for the sharing functions. The husband's share is expressed as a deviation with respect to equal sharing. It has the following form:

$$\varphi_1 = \frac{y + \pi(w_1^*, w_2^*)}{2} + \Gamma$$

where

$$\Gamma = \kappa_0(\boldsymbol{x}) + \kappa_1 w_1^{1/2} + \kappa_2 w_2^{1/2} + \kappa_3 y + \kappa_4 s,$$

where $\kappa_0(\boldsymbol{x})$ is an unidentified function of the socio-demographic factors, $\kappa_1, \ldots, \kappa_4$ are parameters, and s is a distribution factor. Note that, from the previous theoretical considerations, the effect of the profit on individual shares cannot be isolated or identified. The wife's share has then the following form:

$$\varphi_2 = \frac{y + \pi(w_1^*, w_2^*)}{2} - \Gamma,$$

¹¹The variables \boldsymbol{x} entering in $b_1(\boldsymbol{x})$, $b_2(\boldsymbol{x})$, $\Delta(\boldsymbol{x})$ or $\alpha_1(\boldsymbol{x})$, $\kappa_0(\boldsymbol{x})$ and $\alpha_2(\boldsymbol{x})$ (see below) are not necessarily the same.

If the wife does not participate in the labour market, the price of her leisure enters the profit function. It is equal to her marginal productivity in domestic activities or, alternatively, to her marginal rate of substitution between leisure and consumption.

3.2 Coherency Conditions and the Price of Wife's Time

In a unitary model, it is well known that the coherency conditions – which guarantee that the endogenous variables are determined unambiguously by the structural equations – are closely related to the Slutsky conditions (Ransom, 1987; van Soest and Kooreman, 1990).¹² The same conclusion can be derived here. In the estimated model, the Slutsky conditions (as well as the regularity conditions of the profit function) are thus imposed and not empirically tested. Under these coherency conditions, the equation that defines the price of wife's leisure, i.e.,

$$\alpha_{2}(\boldsymbol{x}) + \beta_{2} (w_{2}^{*})^{1/2} + \gamma_{2} \left(\frac{y + \pi(w_{1}^{*}, w_{2}^{*})}{2} - \Gamma \right) = a_{2} (w_{2}^{*})^{-1/2} + b_{2}(\boldsymbol{x}) + c (w_{1}^{*})^{1/2} (w_{2}^{*})^{-1/2},$$

has one and only one solution. This equation can be solved by numerical algorithms such as Newton-Raphson. The husband is assumed to be always working so that the price of his time is observed and given by his market wage rate.

4 Data and Estimation Method

4.1 Data

The empirical analysis is based on the 2009 wave of the Panel Study of Income Dynamics (PSID). The PSID makes available a large set of information on housing, employment, income, wealth, savings, housework, household expenditures, besides detailed information on individual and family characteristics.¹³ The original sample counts 8,690 families. We select a subsample

 $^{^{12}}$ Bloemen (2006) derives coherency conditions in collective models of labor supply without domestic production and suggests that the wage parameters of the sharing functions must satisfy non-negativity conditions. It can be shown that these conditions are automatically satisfied here because the sharing functions are expressed as a deviation with respect to equal sharing.

¹³Housework reported in the PSID is less detailed than in time use diaries like the American Time Use Survey (ATUS). It also tends to have an upward bias (Juster, Ono and Stafford, 2003) but it certainly has a smaller variance. More importantly, the PSID reports housework for both spouses and includes comprehensive

of heterosexual couples, either legally married or cohabiting, aged between 25 and 60. Given that the male participation rate is close to one, we include only working men but we include both working and non-working women. After the exclusion of observations with relevant missing information, the sample size reduces to 2,528 households. Following the suggestion of Fortin and Lacroix (1997), we also consider the sub-sample of households with childless couples.¹⁴ It includes 942 households.

As underlined by Blundell and McCurdy (1999), market working hours must be expressed as a function of net total expenditure, and not non-labour income, for the empirical results to be more easily interpretable.¹⁵ The household net total expenditure is defined as household total expenditure minus household labour earnings. Although the PSID does not provide information about household total expenditure, it gathers information on some broad categories of household expenses, such as food, health, transport, and utilities, that are crucial to assign a measure of total consumption to families in the PSID. As suggested by Skinner (1987) and Guo (2010), linear predictions of total household expenditure can be imputed in the PSID using the estimated parameters of a regression of total expenditure on a series of consumption items and other variables available in both the PSID and the Consumer Expenditure Survey (CEX). From the CEX of 2008, we thus select a sample of 503 heterosexual couples, either legally married or cohabiting, with age between 25 and 60 and a working husband. We compute a measure of total household expenditure on non-durable goods¹⁶ and then regress it on a set of variables that are available both in the CEX and the PSID to obtain the coefficients for the imputation procedure.¹⁷

information about individual earnings and working conditions, information that is necessary for our purpose.

¹⁴The theoretical model presented in the previous subsections supposes a household with only two members and thus excludes children. Children may generally be seen as a public good.

¹⁵In the present context with household production, however, the advantages of the construction suggested by Blundell and McCurdy (1999) are not so evident as net-expenditure does not coincide with net-consumption.

¹⁶The durable goods are implicitly supposed separable from the non-durable goods. In the definition of household total expenditure we include family expenses for food, alcoholic beverages, tobacco, utilities, domestic services, clothing and footwear, transportation, health, education and entertainment.

 $^{^{17}}$ For this regression, the coefficient of determination amounts to about 0.80. The variables include a second order polynomial in age, the number of years spent in education, the ethnic origin (a dummy variable for Hispanic or not) and race (non-white or not) of both spouses, the number of children aged between 0 and 6 and between 7 and 18 years of age, the region of residence (Midwest, South, West or other), the 2008 unemployment

Variables	Full sample	Childless sample
Men's annual market working hours	2 155	2 137
Women's annual market working hours	1 434	1 631
Men's annual domestic working hours	385	354
Women's annual domestic working hours	868	706
Men's annual total working hours	2 540	2 491
Women's annual total working hours	2,297	2,337
Men's participation rate	1.00	1.00
Women's participation rate	0.84	0.89
Men's market hourly wage	28.31	28.29
Women's market hourly wage (working women)	21.23	21.47
Annual household net expenditure	-55 749	-62 419
Men's age in years	41.90	46.11
Women's age in years	40.18	44.64
Men's education in years	13.80	13.92
Women's education in years	14.17	14.18
Number of children (0-6)	0.40	0.00
Number of children (7-18)	0.84	0.00
Region: Midwest	0.26	0.25
Region: South	0.39	0.39
Region: West	0.20	0.19
Sex ratio	0.49	0.49
Number of observations	2,528	942

Table 1 - Mean Values of the Main Variables

The main variables used in the empirical application are described in Table 1. As previously explained, the full sample includes households with children – the average number of children under 18 is about 1.2 per household – while the restricted sample includes only households without children. The female participation rate is equal to 84% in the full sample and to 89% rate by state, the amount of household non-labor income (including income from rent, dividends, interests, and inheritances in 2008), the dwelling type (a one-family house or not), the actual number of rooms in the referred dwelling, the existence of a mortgage on the referred dwelling (yes or not), the husband's hourly wage, the wife's hourly wage (replaced by 0 if she is not participating), and annual expenses for food, transport, utilities and health. With the exception of the number of children, the region of residence and individual characteristics, all the other variables are excluded from labor supply equations and are only used for identification purpose.

in the restricted sample. These differences between both samples are also reflected in annual hours worked. On average, women's working time in the labour market amounts to about 1,400 hours per year, including overtime, in the full sample, and about 1,600 hours in the restricted sample. Women's working time at home in activities like cooking, cleaning and doing other works around the house amounts to about 870 hours per year in the full sample, and 700 hours in the restricted sample. Differently, men's working times in the labour market and at home are of the same order of magnitude in both samples and amount to about 2,500 hours and 350 hours, respectively. Fitted annual household net total expenditure amounts to about -\$56,000 per year for the full sample and -\$62,000 for the restricted sample. These figures are negative as family labour earnings are larger than what is spent on the sole durable goods. On average, for both samples, men's hourly wage is about \$28, against \$21 for women. Finally, regarding the socio-demographics, the mean values for the two samples are of the same order of magnitude, with the exception that, on average, spouses are slightly older in the restricted sample.

4.2 Estimation Method

In what follows, we consider the estimation of the complete system, (11)-(14), of structural market and domestic labour supply equations. Before discussing the estimation methodology, the issue of endogenous covariates must be accounted for.

4.2.1 Endogeneity and Selection

For working individuals, hourly wages are computed as a ratio between labour earnings and hours of work, which may create a division bias (Borjas, 1980). For non-working women, hourly wages are thus missing and have to be imputed from a wage equation. To reduce the division bias, however, we predict hourly wages for all individuals, and not only for non-working women.

To do so, we use the full sample and apply the Heckman's (1979) two-step procedure. In the first step, we estimate a Probit model for wives' labour force participation. The regressors include the number of years of education, a second order polynomial of age, ethnicity (a dummy variable for Hispanic or not), race (non-white or not), health (bad health or not), religion (protestant, Jew, catholic or other) and labour market experience (defined as the number of years worked for money since the age of 18) for both the husband and the wife. We also control for the region of residence (Midwest, South, West or other), the number of children between 0 and 6 and between 7 and 18 years of age, the educational level of both the wife's mother and father (the parent has a post-secondary degree or not), the employment status of the wife's mother (employed or not), household non-labour income and the 2008 unemployment rate by state. In the second step, we estimate a selectivity-corrected log hourly wage equation for women. In addition to the inverse of the Mill's ratio computed from the first stage estimates, we include her personal characteristics, the same as those included also in the participation equation, her seniority with the current employer (defined as the number of years of experience with the present employer), the region of residence of the household and the number of children aged between 0 and 6 and between 7 and 18. As for men, we estimate a log hourly wage equation including the same variables as in female log hourly wage equation (except for the selection-correction term) but referring to him. From these estimations, we compute the fitted log hourly wages for men and women, denoted by log \hat{w}_1 and log \hat{w}_2 , the standard error of the regression terms and their correlation coefficient.

The household net total expenditure is computed as the difference between fitted household total expenditure on non-durables and services and household labour earnings. This variable is likely to be endogenous in the labour supply model because of unobservables in the labour supply equations that enter household net total expenditure. We address the endogeneity problem by adopting a two-stage approach and replace household net total expenditure by its fitted value. The household net total expenditure equation includes the fitted hourly wages and all the individual characteristics of the husband and the wife (excluding labour market experience and seniority), the region of residence of the household non-labour income, the dwelling type, the actual number of rooms and the existence of a mortgage on the referred dwelling, and the annual household expenditures on food, health, transportation and utilities items.

4.2.2 Simulated Log Likelihood

We suppose that "desired" hours of domestic and market labour supply are not directly observed because of an additive perturbation due to measurement or optimization errors. Thus, we write the system of equations (11)–(14) as:

$$\begin{aligned} t_1^* &= t_1(\psi, w, \nu) + \epsilon_1, \\ t_2^* &= t_2(\psi, w, \nu) + \epsilon_2, \\ h_1^* &= h_1(\psi, w, \nu) + \epsilon_3, \\ h_2^* &= h_2(\psi, w, \nu) + \epsilon_4, \end{aligned}$$

where $w = (w_1, w_2)$ is the vector of market hourly wages, $\psi = (x, y)$ is a vector of other observable variables, ν is a vector of error terms representing unobservable heterogeneity in preferences and technology and $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$ the vector of error terms representing measurement or optimization error, supposed to be distributed independently from each other, with a normal probability density function ϕ_{ϵ} . The unobservable heterogeneity terms ν may be correlated, with a normal joint probability density function ϕ_{ν} . Other variables include years of education, a second order polynomial of age of the corresponding person, the number of children between 0 and 6 and between 7 and 18 years of age, ethnicity and race, the region of residence and household net total expenditure.

To compute the log-likelihood, we consider two regimes according to the wife's participation in paid employment.

Regime 1. In this regime, the wife does work in the market and her market labour supply is unconstrained. For the moment, we suppose that heterogeneity error terms are observed. Then, the conditional likelihood function is simply given by:

$$L_1(\psi, w, \nu) = \phi_{\epsilon} \left(t_1 - t_1^*(\psi, w, \nu), t_2 - t_2^*(\psi, w, \nu), h_1 - h_1^*(\psi, w, \nu), h_2 - h_2^*(\psi, w, \nu) \right)$$

where ϕ_{ϵ} corresponds to the product of marginal density functions of the normal distribution. As explained above, wives' and husbands' hourly wages are replaced by fitted values to take account of the division bias. Thus, $w = (\hat{w}_1\zeta_1, \hat{w}_2\zeta_2)$ where $\zeta = (\zeta_1, \zeta_2)$ are multiplicative error terms, with a log-normal joint probability density distribution ϕ_{ζ} , the standard error and the correlation coefficient of which are computed from the estimation of the log hourly wage equations. The wage error terms and the unobserved heterogeneity terms have to be integrated out. Thus, the simulated log-likelihood function for this regime can be obtained by drawing R error terms ν_r and ζ_r in the aforementioned distributions ϕ_{ν} and ϕ_{ζ} and then computing:

$$\log \mathcal{L}_1 = \log \left(\sum_{r=1}^R L_1(\psi, \widehat{w}\zeta_r, \nu_r) \right).$$

Regime 2. In this regime, the wife does not work in the market. Hence her hourly wage is replaced in the profit function and the labour supply functions by a measure of the price of her time, computed with numerical methods. The functional form of labour supply functions switches, as explained above. The conditional likelihood function is then given by:

$$L_2(\psi, w, \nu) = \int_{-\infty}^{-h_2^*(\psi, w, \nu)} \phi_{\epsilon}(t_1 - t_1^*(\psi, w, \nu), t_2 - t_2^*(\psi, w, \nu), h_1 - h_1^*(\psi, w, \nu), h_2 - h_2^*(\psi, w, \nu)) \cdot d\epsilon_4.$$

The computation of this integral is straightforward because measurement and optimization errors are assumed independent from each other. As previously, the simulated log-likelihood function for this regime can be obtained by drawing R error terms ν_r and ζ_r and then computing:

$$\log \mathcal{L}_2 = \log \left(\sum_{r=1}^R L_2(\psi, \widehat{w}\zeta_r, \nu_r) \right).$$

The maximization of the simulated log likelihood function, i.e., the sum of $\log \mathcal{L}_1$ and $\log \mathcal{L}_2$, yields the estimation of the parameters of interest.

5 Estimation Results and Discussion

Before discussing the estimation results of the complete system, (11)-(14), of structural market and domestic labour supply equations, we present estimates of household expenditure and spouses' log hourly wage equations.

5.1 Auxiliary Estimations

As previously explained, market hourly wages are predicted for all individuals. The estimates of male and female log hourly wage equations are shown in Table 2. We note that hourly wages increase sharply with worker's age (but at a decreasing pace) as well as with education and seniority. Experience has no significant effect, though. On average, blacks and Hispanics get lower wages (similar results are found by O'Neill, 1990; Trejo, 1997; Antecol and Bedar, 2002)

Variables	Husbands' Log Hourly Wage	Wives' Log Hourly
		Wage
Intercept	0.296	0.035
	(0.216)	(0.295)
Age	0.068	0.057
	(0.010)	(0.012)
Squared Age	-0.001	-0.001
	(0.000)	(0.000)
Years of Education	0.092	0.112
	(0.004)	(0.008)
Years of Seniority	0.011	0.020
	(0.001)	(0.002)
Years of Experience	0.000	0.000
	(0.000)	(0.001)
Black	-0.134	0.027
	(0.024)	(0.028)
Hispanic	-0.061	-0.157
	(0.038)	(0.045)
Protestant	0.001	-0.008
	(0.026)	(0.033)
Catholic	0.068	0.083
	(0.032)	(0.042)
Jew	0.334	0.332
	(0.082)	(0.094)
Bad Health	-0.367	-0.110
	(0.123)	(0.123)
Number of Children (0-6)	0.032	-0.063
	(0.016)	(0.034)
Number of Children (7-18)	0.015	-0.070
	(0.010)	(0.017)
Region: Midwest	-0.085	-0.029
	(0.032)	(0.039)
Region: South	-0.076	-0.115
	(0.031)	(0.036)
Region: West	0.044	0.041
	(0.035)	(0.040)
Inverse of Mill's Ratio	_	0.562
	_	(0.199)

 Table 2 - Estimated Parameters of Log Hourly Wage Equations

Notes: standard errors are in parentheses.

while Jews and Catholics obtain higher wages. In line with other studies (e.g., Hersch and Stratton, 2000; Lundberg and Rose, 2000, 2002), the presence of children is associated with a wage premium for fathers but not for mothers. Having health problems negatively affects hourly wages. Finally, the coefficient associated with the inverse of the Mill's ratio is, as expected, positive and significant, suggesting that working women are positively selected in the labour market.

Lastly, as explained above, we also predict the household net total expenditure. The results are not presented here but are available upon request.

5.2 Structural market and domestic labour supply equations

The estimation results for structural domestic and total labour supply equations are reported in Tables 3 and 4, respectively. We consider two specifications. The first one corresponds to the previous functional form estimated on the sample of all the households and the second one to the same functional form estimated on the sample of childless households.

5.2.1 Domestic labour Supply Equations

For each specification, the domestic labour supply equations have thirteen parameters and most of them are significantly different from zero at usual levels of significance. We first focus on the full sample case. The coefficients a_i associated with the inverse of the square root of hourly wages in the equations are positive and, at least in the wives' equation, statistically significant. It suggests that both husbands and wives reduce their contribution to domestic chores when their wage increases. On the other hand, the coefficient c is positive, implying that spouses' time inputs are substitute in the household production process. The hypothesis of a (very moderate) substitutability between time inputs is supported by several empirical studies based on PSID data (Gronau, 1977, 1980; Graham and Green, 1984). Overall, these conclusions are confirmed by the estimations obtained with the childless sample (even if the estimated parameters a_i for husbands is constrained to be non-negative for consistency reasons).

Given the difficulty to provide any direct interpretation of the parameters associated with wages, since they enter labour supply equations nonlinearly, we compute the marginal effects of hourly wages on domestic labour supplies for each observation and each realization of the

 b Intercept b Age b Squared Age ÷ 100 b Number of Children 0-6 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage b Intercept 	A. Husbands' Dor 1.952 (0.091) -0.085 (0.005) 0.097 (0.005) -0.128 (0.007) 0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135 (0.013)	mestic Labor Supply 1.677 (0.205) -0.062 (0.009) 0.071 (0.011) 0.000 () 0.000 () -0.009 (0.004) -0.348 (0.029)
 b Age b Squared Age ÷ 100 b Number of Children 0-6 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.091) -0.085 (0.005) 0.097 (0.005) -0.128 (0.007) 0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	(0.205) -0.062 (0.009) 0.071 (0.011) 0.000 () -0.009 (0.004) -0.348
 b Squared Age ÷ 100 b Number of Children 0-6 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	-0.085 (0.005) 0.097 (0.005) -0.128 (0.007) 0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	-0.062 (0.009) 0.071 (0.011) 0.000 () -0.009 (0.004) -0.348
 b Squared Age ÷ 100 b Number of Children 0-6 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.005) 0.097 (0.005) -0.128 (0.007) 0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	(0.009) 0.071 (0.011) 0.000 () 0.000 () -0.009 (0.004) -0.348
 b Number of Children 0-6 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	$\begin{array}{c} 0.097\\ (0.005)\\ -0.128\\ (0.007)\\ 0.053\\ (0.005)\\ -0.001\\ (0.002)\\ -0.454\\ (0.015)\\ -0.034\\ (0.013)\\ 0.135 \end{array}$	0.071 (0.011) 0.000 () -0.009 (0.004) -0.348
 b Number of Children 0-6 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.005) -0.128 (0.007) 0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	(0.011) 0.000 () 0.000 () -0.009 (0.004) -0.348
 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	-0.128 (0.007) 0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	0.000 () 0.000 () -0.009 (0.004) -0.348
 b Number of Children 7-18 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.007) 0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	() 0.000 () -0.009 (0.004) -0.348
 b Years of Education b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	0.053 (0.005) -0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	0.000 () -0.009 (0.004) -0.348
 b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	-0.001 (0.002) -0.454 (0.015) -0.034 (0.013) 0.135	-0.009 (0.004) -0.348
 b Region: Midwest b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.002) -0.454 (0.015) -0.034 (0.013) 0.135	(0.004) -0.348
 b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	-0.454 (0.015) -0.034 (0.013) 0.135	-0.348
 b Region: South b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.015) -0.034 (0.013) 0.135	
 b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	-0.034 (0.013) 0.135	10.0491
 b Region: West b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.013) 0.135	-0.002
 b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	0.135	(0.020)
 b Black b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0, 013)	-0.113
 b Hispanic a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.010)	(0.022)
 a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	0.080	-0.040
 a Inv. Sq. Root of Hourly Wage c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage 	(0.011)	(0.023)
c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage	0.123	0.036
c Sq. Root of Partner's Hourly Wage x Inv. Sq. Root of Hourly Wage	(0.014)	(0.031)
Root of Hourly Wage	0.034	0.000
Root of Hourly Wage	(0.063)	()
	0.125	0.122
b Intercept	(0.014)	(0.008)
b Intercept	B. Wives' Dome	estic Labor Supply
	0.626	-0.473
	(0.118)	(0.220)
b Age	0.011	0.042
b Squared Age ÷ 100	(0.006) 0.008	(0.010) -0.021
D Squared Age + 100	(0.007)	(0.011)
b Number of Children 0-6	0.308	0.000
~	(0.009)	()
b Number of Children 7-18	0.040	0.000
	(0.005)	()
b Years of Education	-0.049	-0.038
	(0.002)	(0.004)
b Region: Midwest	-0.165	0.032
t Degiant South	(0.015)	(0.029) 0.025
b Region: South	-0.301 (0.016)	(0.023)
b Region: West	-0.125	0.058
	(0.018)	(0.022)
b Black		0.169
	-0.005	(0.021)
b Hispanic	(0.013)	0.105
	(0.013) 0.046	(0.046)
a Inv. Sq. Root of Hourly Wage	(0.013) 0.046 (0.018)	0 =0 1
c Sq. Root of Partner's Hourly Wage x Inv. Sq.	(0.013) 0.046 (0.018) 0.812	0.594
Root of Hourly Wage	(0.013) 0.046 (0.018)	0.594 (0.068) 0.122

Table 3 - Estimated Parameters of Domestic Labor Supply Equations

Notes: Standard errors are in parentheses. Working times are in thousands of hours. Sociodemographics are arguments of a quadratic function. error terms. The main percentiles of the distribution of these marginal effects are presented in Table 5.¹⁸ For the median household, for example, the marginal effects for men and women amount to -0.002 and -0.009, respectively. That is, a one-dollar increase in hourly wages implies that wives' and husbands' domestic labour supplies decrease by about two hours and to nine hours, respectively. This is in line with the theory. In addition, wives' domestic labour supply is a little more sensitive to changes in hourly wage than the husbands' one. This can be explained by referring to the traditional gender division of work and to the evidence of higher own-wage market labour supply responsiveness of women with respect to men. While men primarily spend their time in market work and leisure, women share their time among paid work, leisure and domestic work. Thus, as compared to men, women have closer substitutes for time spent in market activities and larger substitution effects on both women's domestic and market labour supply are expected.

To be complete, we can also examine the role of socio-demographics. For mothers, the time allocated to domestic chores increases with the number of children and the effect is larger the younger the child.¹⁹ This does not seem to be the case for fathers: the effect of children on men's homework is small, and even negative. For the other socio-demographics, the two specifications give similar results. For wives, education has a negative effect on the time devoted to domestic activities while, for husbands, it is not significant. The black and Hispanic dummies have also a positive (or non significant) effect on working hours. Finally, age has not a clear effect on domestic labour supplies.

5.2.2 Total and Market labour Supply Equations

The estimated parameters of the total labour supply equations are presented in Table 4. We first note that an increase in hourly wage has only a very moderate and not significant effect on total hours of work for wives with the full sample. For husbands, the effect is even negative and, for consistency reasons, constrained to zero. Total labour time may, however, be indirectly

 $^{^{18}}$ We focus on marginal effects here because we have judged that they are less ambiguous than elasticities as working time may be close to zero.

¹⁹Socio-demographics enter labor supply equations nonlinearly, i.e., via a quadratic function. The marginal effect of a change in the variable has, however, the same sign as the corresponding estimated parameter (at least at the average point of the sample).

		Model with full sample	Model with childless sample
		A. Husbands' 7	fotal Labor Supply
α	Intercept	2.644	1.809
		(0.274)	(0.569)
α	Age	-0.023	0.010
	Squared Age ÷ 100	(0.012)	(0.019) -0.015
α	Squared Age + 100	0.024	
~	Number of Children <6	(0.014) 0.044	(0.022) 0.000
α	Number of Children <0	(0.019)	()
α	Number of Children 7-18	0.023	0.000
u	Number of children 7 10	(0.013)	()
α	Years of Education	-0.016	-0.044
u	Tears of Badeation	(0.005)	(0.009)
α	Region: Midwest	0.064	0.083
u		(0.040)	(0.059)
α	Region: South	0.123	0.103
u		(0.038)	(0.053)
α	Region: West	0.070	-0.039
u		(0.042)	(0.063)
α	Black	-0.004	0.022
u	Ditter	(0.028)	(0.053)
α	Hispanic	0.025	0.066
u	mopanio	(0.039)	(0.078)
β	Square Root of Hourly Wage	0.000	0.000
Ρ	Square noor of moury mage	()	()
γ	Share of Total Net Income	-0.004	-0.006
Y		(0.001)	(0.001)
			tal Labor Supply
α	Intercept	4.283	4.889
u	meercept	(0.816)	(1.492)
α	Age	-0.008	0.021
u		(0.017)	(0.025)
α	Squared Age ÷ 100	0.005	-0.034
ũ	1	(0.021)	(0.029)
α	Number of Children <6	-0.175	0.000
a		(0.026)	()
α	Number of Children 7-18	-0.001	0.000
		(0.015)	()
α	Years of Education	-0.026	-0.027
		(0.009)	(0.014)
α	Region: Midwest	0.030	0.039
	0	(0.051)	(0.074)
α	Region: South	-0.009	0.026
	-	(0.049)	(0.069)
α	Region: West	-0.120	-0.084
-	-	(0.058)	(0.090)
α	Black	-0.051	-0.179
		(0.056)	(0.101)
α	Hispanic	0.089	-0.119
	-	(0.053)	(0.089)
β	Square Root of Hourly Wage	0.010	0.000
	_	(0.087)	()
γ	Share of Total Net Income	-0.029	-0.024
		(0.002)	(0.003)

Table 4 - Estimated Parameters of Total Labor Supply Equations

Notes: Standard errors are in parentheses. Working times and shares of total expenditure are in thousands of hours.

Table 5 - Estimated Percentiles of the Distribution of the SimulatedMarginal Effects of Hourly Wages on Domestic, Market and Total LaborSupply

		A	A. Full sampl	e	
Percentiles	P10	P25	P50	P75	P90
Husbands' Market Labor Supply Own Marginal Effect	0.003	0.004	0.006	0.008	0.011
Wives' Market Labor Supply Own Marginal Effect	0.009	0.015	0.024	0.037	0.053
Husbands' Domestic Labor Supply Own Marginal Effect	-0.006	-0.004	-0.002	-0.001	-0.001
Wives' Domestic Labor Supply Own Marginal Effect	-0.031	-0.017	-0.009	-0.004	-0.001
Husbands' Total Labor Supply Own Marginal Effect	0.002	0.003	0.003	0.004	0.005
Wives' Total Labor Supply Own Marginal Effect	0.006	0.008	0.011	0.019	0.029
	B. Childless sample				
Percentiles	P10	P25	P50	P75	P90
Husbands' Market Labor Supply Own Marginal Effect	0.004	0.005	0.007	0.010	0.013
Wives' Market Labor Supply Own Marginal Effect	0.007	0.011	0.018	0.028	0.040
Husbands' Domestic Labor Supply Own Marginal Effect	-0.006	-0.004	-0.002	-0.001	-0.001
Wives' Domestic Labor Supply Own Marginal Effect	-0.024	-0.014	-0.007	-0.004	-0.002
Husbands' Total Labor Supply Own Marginal Effect	0.003	0.004	0.005	0.006	0.007
Wives' Total Labor Supply Own Marginal Effect	0.004	0.005	0.008	0.014	0.021

Note: Simulations are obtained by drawing the error terms in log hourly wage equations, in domestic labor supply equations (which enters the profit function) and in market labor supply equations (which enters the price of wives' leisure).

affected by wages through a change in the individual shares of total expenditure. The effect of an increase in the share of total expenditure is indeed significant and negative. It is also larger for women than for men, as in many studies (Blau and Khan, 2007; Heim, 2009). A one-thousand-dollar increase in the annual share thus implies a decline in women's total labour supply by about thirty hours and in men's total labour supply by about four hours.

To have a better comprehension, the main percentiles of the distribution of the marginal effects of hourly wages on total labour supplies, incorporating the indirect effect through individual shares, are provided in Table 5. For the model estimated on the full sample, the medians for men and women amount, respectively, to 0.003 and 0.011. In other words, a one-dollar increase in his or her hourly wage implies an increase in total labour supply for men by three hours and for women by eleven hours. Marginal effects of the same order of magnitude are obtained for the restricted sample. Total labour supply (or leisure) is thus rather rigid with respect to wages.

This rigidity of total labour supply may be of great consequence in terms of welfare analysis as it implies that leisure time is not affected by wages. On the other hand, the marginal effects of hourly wages on market labour supplies are larger than the marginal effects on total labour supplies. For the full sample case, the medians for men and women amount to 0.006 and 0.024, respectively. Therefore, an increase in wage has essentially a substitution effect between domestic and market labour supply. In addition, women have larger labour supply responses than men as in many studies (e.g, Blau and Khan, 2007; Heim, 2007, 2009).

Finally, socio-demographic variables are generally not significant. Only the years of education and the number of young dependent children matter for total labour supply. One additional child between 0 and 6 implies a reduction in wives' total labour supply by about 175 hours per year and an increase in husbands' total labour supply by 44 hours. Years of education have a negative effect on total labour supply.

5.2.3 Sharing Equations

The estimated parameters of the sharing equations are reported in Table 6 (recall that the parameters corresponds to the husbands' share). Standard errors are relatively small, especially if compared to those estimated by Fortin and Lacroix (1997) and Donni and Moreau (2007), for instance. To begin with, we note that the effect of net total expenditure on husbands' share is positive and significant, i.e., men receive the larger part of any increase in total expenditure or non-labour income. From a one-thousand-dollar increase in household net total expenditure, the husbands receive about \$884 and the wives about \$116. This result is in line with what is obtained by Blundell et al. (2007), Bloemen (2010), Chiappori et al. (2002), Fernandez-Val (2003), and Donni (2007), as well as Donni and Matteazzi (2012) who also suppose that net total expenditure or non-labour income has a linear effect on market labour supply.²⁰

 $^{^{20}}$ Most other studies do not provide estimates of the sharing functions or, if they do, give standard errors that are excessively large. One of the rare exceptions is given by Haddad (2015) who finds that women, in Iran, receives the largest part of any increase in non-labor income.

	Model with full sample	Model with childless sample
Husband's Sq. Root Hourly Wage	-4.507	-4.028
	(0.502)	(0.934)
Wife's Sq. Root Hourly Wage	0.001	0.271
	(2.095)	(1.095)
Household Net Total Expenditure	0.384	0.303
	(0.026)	(0.048)
Sex ratio in percentage	-0.651	-1.908
	(0.504)	(1.192)

 Table 6 - Estimated Parameters of Individual Share Equations

Notes: Standard errors are in parentheses. Share equations and household net total expenditure are in thousands of dollars. Wife's and husband's hourly wage are multiplied by 2.

The other parameters are less precisely estimated. Interestingly, however, the husband's share of total expenditure tends to decrease with his hourly wage. To interpret the sign, let us note that an increase in one spouse's hourly wage (when the household profit is maintained constant) may have two opposite effects. First, it may reduce the need of a transfer from his (or her) partner in her (or his) favour. Second, it may improve her (or his) bargaining power within the household decision process. Our results thus suggest that, in the case of an increase in the husbands' hourly wage, the first effect dominates the second one. To complete our discussion, let us note that the husbands' share is not significantly affected by an increase in the wives' hourly wage. Since the wives' participation rate and their working hours are lower, the first effect is likely to be smaller, so that the two aforementioned effects simply offset each other. Interestingly, the same conclusions are obtained by Clark, Couprie and Sofer (2004) for the United Kingdom. Still Fernandez-Val (2003) and Blundell et al. (2007) for Spain and the United Kingdom, respectively, show that the husband's share increases with the husband's wage, while Chiappori, Fortin and Lacroix (2002), for the United States, do not discern any significant effect.

In the household production context, however, the total income of the household also includes the profit generated by production activities. Thus, an increase in one spouse's hourly wage has a third effect which is negative due to the reduction of the household profit. To take into account this third effect which is highly nonlinear, the main percentiles of the marginal effect of spouses' wage on individual shares are computed and presented in Table 7. For both specifications, the effects on husbands' share are large in absolute value, and negative. For the median household

		А	. Full samp	e	
Percentiles	P10	P25	P50	P75	P90
Marginal Effect of Husbands' Hourly Wage on Husbands' Share	-1.369	-1.074	-0.837	-0.666	-0.548
Marginal Effect of Wives' Hourly Wage on Husbands' Share	-1.024	-0.683	-0.430	-0.297	-0.224
Marginal Effect of Husbands' Hourly Wage on Wives' Share	0.059	0.292	0.457	0.630	0.881
Marginal Effect of Wives' Hourly Wage on Wives' Share	-0.804	-0.464	-0.207	-0.102	-0.056
		В. С	hildless sar	nple	
Percentiles	P10	B. C P25	hildless sar P50	nple P75	P90
Percentiles Marginal Effect of Husbands' Hourly Wage on Husbands' Share	P10 -1.204			-	P90 -0.514
Marginal Effect of Husbands' Hourly Wage on	-	P25	P50	P75	
Marginal Effect of Husbands' Hourly Wage on Husbands' Share Marginal Effect of Wives' Hourly Wage on	-1.204	P25 -0.963	P50 -0.765	P75 -0.618	-0.514

Table 7 - Estimated Percentiles of the Distribution of the SimulatedMarginal Effects of Hourly Wages on Individual Shares

Notes: Standard errors are in parentheses. Share equations are in thousands of dollars. Simulations are obained by drawing the error terms in log hourly wage equations and in domestic labor supply equations (which enters the profit function).

in the full sample, for instance, a one-dollar increase in the husband's and wife's hourly wage implies a reduction in his share by \$837 and \$430, respectively, while a one-dollar increase in the husband's hourly wage implies an augmentation in the wife's share by \$457.

One last comment is in order. Even if the effect of the sex ratio is not statistically significant, its sign fits with the intuition. For the full sample, a one-percentage-point increase in the sex ratio (i.e., an increase in the number of men compared to the number of women) will induce husbands to transfer an additional \$650 of income to their spouse and, for the restricted sample, an additional \$1,910, the same order of magnitude as what is obtained by Chiappori, Fortin and Lacroix (2002) in their estimations with the PSID.

5.2.4 Testable restrictions

It must be clear that the collective model developed in the previous pages also generates testable restrictions (as pointed out in the discussion of Proposition 1). Basically, the observed labour supply equations have to be such that they can be written under the separable form (5)-(10).

Implementing empirical tests using our specification may, however, be problematic. If $w_1^{1/2}$ and $w_2^{1/2}$ are directly incorporated into the wife's and the husband's market labour supply equations, respectively, to relax their functional structure, the model is no longer identifiable. On the other hand, negativity restrictions have to be imposed, instead of being tested, to guarantee the uniqueness of the price of wife's time. Nevertheless, a simple restriction to test is that the distribution factor (i.e., the sex ratio) must not have a direct effect on labour supply (in addition to the intra-household distribution effect). This test has been carried out and the corresponding Likelihood Ratio statistics amount to 18.09 for the complete sample (and 15.85 for the restricted sample) with three degrees of freedom. The null hypothesis is thus clearly rejected. Interestingly, however, only the parameters in domestic labour supply equations are significantly different from zero: for both samples, in addition to the intra-household effect, an increase in the sex ratio has a positive effect on domestic labour supply but no effect on market labour supply. A simple explanation is that the sex ratio is not a traditional distribution

Hourly market wages	P10	P25	P50	P75	P90
Simulated women's hourly market wage (working women)	8.079	11.830	18.251	28.075	41.536
Simulated men's hourly market wage (working women)	11.604	16.437	24.171	35.738	50.546
Simulated women's hourly market wage (non-working women)	5.145	7.556	11.161	15.880	21.902
Simulated men's hourly market wage (non-working women)	10.471	15.288	22.860	34.070	49.922
Wife's Hourly Prices of Leisure					
Model with full sample (non-working women)	11.459	16.863	24.781	33.413	40.535
Model with childless sample (non- working women)	11.537	16.774	24.649	33.883	41.861

Table 8 - Estimated Percentiles of the Distribution of the Simulated Pricesof Leisure and Simulated Hourly Wages

Note: Simulations are obtained by drawing the error terms in log hourly wage equations, in domestic labor supply equations (which enters the profit function) and in market labor supply equations (which enters the price of wives' leisure).

factor in the sense that it affects the household technology. Yet the mechanism through which it might happen seems rather obscure. Another, more realistic possibility is that the price of the domestic good is endogenous, suggesting that either the marketability assumption or the privateness assumption (see subsection 2.4) are violated.²¹ To address this problem, however, it is necessary to develop a more general model, which is beyond the scope of the present paper. We will just invite the reader to be cautious when interpreting the empirical results.

5.2.5 Price of Leisure

Finally, the present framework can be used to evaluate the price of leisure. For each realization of the error terms, the price of leisure for the wives who do not work in the market is computed. The main percentiles of this distribution as well as, for the sake of comparison, simulated hourly market wages are presented in Table 8. We observe that the price of wives' leisure has a large dispersion, ranging from \$11 to \$42. For both samples, the median of the price of wives' leisure amounts to about \$25, which is a reasonable value. By comparison, the median of the simulated hourly wage of non-working women is about \$11.

6 Conclusion

In this paper, we generalise the collective model of household labour supply with domestic production to allow for the possibility of non-participation in the labour market. We show that the sharing of the full non-labour income (non-labour income plus the profit generated by production activities) can be identified up to a pair of constants. The household technology and the price of leisure, when individuals do not work in the market, can also be identified. This generalization is important for several reasons. It is admitted that omitting household production may lead to biased welfare comparisons (Apps and Rees, 1997; Donni, 2008). To obtain precise parameter estimates, however, it is necessary to consider large samples including spouses who do not participate in the labour market. More fundamentally, the structural components such as the price of spouses' leisure or their individual shares are likely related to spouses' employment status.

Our theoretical results are applied to the 2009 PSID data. We estimate by simulated maximum likelihood a system of structural market and domestic labour supply equations that accounts

²¹The fact that the sign of the effect of the sex ratio is the same in both domestic labor supply equations suggests that the price of the domestic good changes in the same direction for both spouses. We are thus inclined to believe that the critical point concerns the marketability of the domestic good.

for the wives' possible non-participation in the labour market – the husbands being supposed to always participate in the labour market. Some empirical results are worth mentioning in this conclusion. Firstly, we show that total labour supplies (and thus leisure) are both very rigid, compared with domestic and market labour supplies which are slightly more flexible. As previously explained, this empirical observation may have implications in terms of welfare analysis. Secondly, we show that husbands receive the larger part of any increase in net total expenditure or non-labour income. Quite interestingly, such conclusion is confirmed by the large majority of empirical studies and can be seen as one of the most robust results in this literature. Thirdly, we show that the price of wives' leisure is comprised between \$11 and \$42 (if we ignore the tails of the distribution). On average, the price of non-working wives' leisure is rather large. It stochastically dominates the hourly wages of non-working wives (but also of working wives).

Finally, our empirical results also suggest that the price of the domestic good is likely not exogenous. The prime suspect here is the marketability assumption which is probably too restrictive. Estimating a collective model of domestic production without the marketability assumption is left for future research.

Appendix

Proof of Proposition 2

Thanks to Assumption I.1, the domestic labour supply functions (g_1, g_2) can be observed on \mathbb{R}^2_+ as a function of (w_1, w_2) . (a) Suppose that the wife does not work in the market; her domestic labour supply is $t_1^* = g_1(w_1^*, w_2)$, where w_1^* is endogenously determined. Since $\partial g_1/\partial w_1 < 0$, g_1 can be inverted, giving: $w_1^* = g_1^{-1}(t_1^*(w_1, w_2, y), w_2) = w_1^*(w_1, w_2, y)$. Hence, the price of the wife's time is identified. (b) Suppose the husband does not work. The procedure to identify the price of his time is exactly the same. (c) Suppose that both spouses do not work in the market so that domestic labour supply functions are given by $t_1^* = g_1(w_1^*, w_2^*)$ and $t_2^* = g_2(w_1^*, w_2^*)$. Because of the properties of factor demand functions, the Jacobian matrix $\nabla(g_1, g_2)$ is nonsingular. Hence, inverting domestic labour supply functions satisfy the symmetry restriction, $\partial g_1/\partial w_2 = \partial g_2/\partial w_1$, the profit function can be retrieved by integrating the system of two equations, $\partial \Pi / \partial w_1 = -t_1^*$ and $\partial \Pi / \partial w_2 = -t_2^*$ derived from Hotelling Lemma. Identification is obtained up to a constant.

Proof of Proposition 3

Thanks to assumption I.2, the set $\mathbb{R}^2_+ \times \mathbb{R}$ on which the (market and domestic) labour supply functions are defined can be partitioned into four connected subsets $\{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}\}$ such that the husband and the wife work in the market if $(w_1, w_2, y) \in \mathcal{A}$, the husband, but not the wife, works in the market if $(w_1, w_2, y) \in \mathcal{B}$, the wife, but not the husband, works in the market if $(w_1, w_2, y) \in \mathcal{C}$, neither the husband nor the wife works in the market if $(w_1, w_2, y) \in \mathcal{D}$. The proof then follows in four steps.

Step 1. We consider the identification of the sharing functions in set \mathcal{A} . In his original paper, Chiappori (1997) has shown that the sharing functions can be recovered and Donni and Matteazzi (2012) have generalised this result to the case where there is no distribution factors. This is the result we present here. For any (w_1, w_2, y) such that $(\partial h_1^*/\partial y) \cdot (\partial h_2^*/\partial y) \neq 0$, we introduce the following definitions:

$$A(w_1, w_2, y) = \left(\frac{\partial h_1^*}{\partial w_2} + \frac{\partial t_1^*}{\partial w_2}\right) \left(\frac{\partial h_1^*}{\partial y}\right)^{-1} \text{ and}$$
$$B(w_1, w_2, y) = \left(\frac{\partial h_2^*}{\partial w_1} + \frac{\partial t_2^*}{\partial w_1}\right) \left(\frac{\partial h_2^*}{\partial y}\right)^{-1}.$$

The first result is that the sharing functions can be identified, if regularity conditions are satisfied, when both spouses are in the labour force.

Lemma A-1. Let us assume that $(\partial h_1^*/\partial y) \cdot (\partial h_2^*/\partial y) \neq 0$ and $AB_y - B_{w_2} \neq BA_y - A_{w_1}$ almost everywhere. Then individual shares of total non-labour income are identified (up to two specific constants) on \mathcal{A} .

Proof. See Donni and Matteazzi (2012). \Box

Step 2. The following lemma then states that, under regularity conditions, the sharing functions are uniquely defined on \mathcal{B} .

Lemma A-2. Let us assume I.1–I.3; let us also assume that $(\partial L_1^*/\partial y) \cdot (\partial w_2^*/\partial y) \neq 0$ almost everywhere and

$$\frac{\partial L_1^*/\partial w_2}{\partial L_1^*/\partial y} \neq \frac{\partial w_2^*/\partial w_2}{\partial w_2^*/\partial y}.$$

Then the individual shares are identified (up to two specific constants) on $\mathcal B$.

Proof. (a) For any $(w_1, w_2, y) \in \mathcal{B}$, the total labour supply function of the husband can be written as: $L_1^*(w_1, w_2, y) = F_1(w_1, \varphi_1(w_1, w_2, y))$. From its implicit definition, i.e., $g_2(w_1, w_2^*) = F_2(w_2^*, \varphi_2)$, the price of leisure can be written as: $w_2^*(w_1, w_2, y) = m_2(w_1, \varphi_2(w_1, w_2, y))$, for some function $m_2(\cdot)$, where the left-hand side function is known from Proposition 2. Differentiating these expressions with respect to w_2 and y gives:

$$A(w_1, w_2, y) = \frac{\partial L_1^* / \partial w_2}{\partial L_1^* / \partial y} = \frac{\partial \varphi_1 / \partial w_2}{\partial \varphi_1 / \partial y}, \qquad B(w_1, w_2, y) = \frac{\partial w_2^* / \partial w_2}{\partial w_2^* / \partial y} = \frac{\partial \varphi_2 / \partial w_2}{\partial \varphi_2 / \partial y}$$
(15)

where the left-hand side functions are known. Together with $\phi_1(w_1, w_2, y) + \phi_2(w_1, w_2, y) = y + \pi(w_1, w_2, y)$, the system of two equations (15) has two unknowns. Solving it gives:

$$\frac{\partial \varphi_1}{\partial w_2} = \alpha(w_1, w_2, y), \qquad \frac{\partial \varphi_1}{\partial y} = \beta(w_1, w_2, y)$$
(16)

where

$$\alpha(w_1, w_2, y) = -A \frac{B + t_2^* (\partial w_2^* / \partial w_2)}{(A - B)}, \qquad \beta(w_1, w_2, y) = -\frac{B + t_2^* (\partial w_2^* / \partial w_2)}{(A - B)}$$

Provided that a cross-derivative restriction (i.e., $\partial \alpha / \partial y = \partial \beta / \partial w_2$) is satisfied, these two partial differential equations define the husband's sharing function up to a function k_1 of w_1 , i.e., for any sharing function $\varphi_1^*(w_1, w_2, y)$ consistent with the system (16), then the sharing function $\varphi_1(w_1, w_2, y)$ defined as

$$\varphi_1(w_1, w_2, y) = \varphi_1^*(w_1, w_2, y) + k_1(w_1)$$

is also consistent with this system. Similarly, the sharing function of the wife can be identified up to a function k_2 of w_1 (because the sum of sharing functions is equal to total non-labour income, the latter being identified up to a constant, $k_1(w_1) + k_2(w_1) = \Delta \equiv \text{a constant}$). In addition, the condition $\partial \alpha / \partial y = \partial \beta / \partial w_2$ is a testable restriction.

(b) To recover the function $k_1(w_1)$, let us recall that the sharing functions are continuous along the participation frontier. From Lemma 1, the sharing function $\varphi_1(w_1, w_2, y)$ is identified up to a constant δ_1 on \mathcal{A} and, by continuity, is also identified along the participation frontier defined by $w_2 = \gamma_2(w_1, y)$. Thus,

$$\varphi_1(w_1, \gamma_2(w_1, y), y) = a_1^*(w_1, y) + \delta_1,$$

where $a_1^*(w_1, y)$ is a known function. Similarly, the sharing function along the frontier obtained from (16) can be written as:

$$\varphi_1(w_1, \gamma_2(w_1, y), y) = b_1^*(w_1, y) + k_1(w_1).$$

where $b_1^*(w_1, y)$ is a known function. Because of continuity, we thus have

$$k_1(w_1) = a_1^*(w_1, y) - b_1^*(w_1, y) + \delta_1.$$

The functions $a_1^*(w_1, y)$ and $b_1^*(w_1, y)$ are defined for any (w_1, y) such that there exists some $w_2 = w_2^R(w_1, y)$ and, therefore as a consequence of Assumption I.2, are defined for any $w_1 \in \mathbb{R}_+$ and some y. Hence the function $k_1(w_1)$ is identified up to a constant δ_1 for any $w_1 \in \mathbb{R}_+$. In addition, the fact that $a_1^*(w_1, y) - b_1^*(w_1, y)$ is independent of y is a testable restriction. \Box

Step 3. The next lemma is analogous to the preceding one. It states that, under regularity conditions, the sharing functions are defined on C.

Lemma A-3. Let us assume B-3; let us also assume that $(\partial L_2^*/\partial y) \cdot (\partial w_1^*/\partial y) \neq 0$ almost everywhere and

$$\frac{\partial L_2^* / \partial w_1}{\partial L_2^* / \partial y} \neq \frac{\partial w_1^* / \partial w_1}{\partial w_1^* / \partial y}$$

Then the individual shares are identified (up to two specific constants) on \mathcal{C} .

Proof. The proof is symmetric to the preceding one. \Box

The preceding lemmas involve that the sharing functions are globally identified on $\mathcal{A} \cup \mathcal{B} \cup \mathcal{C}$ but not on \mathcal{D} . Our conjecture is that sharing functions can still be identified on \mathcal{D} at least in the neighbourhood of the participation frontiers. Indeed, the identification of the price of spouses' leisure provides a new partial differential equation in φ_1 over \mathcal{D} . The argument is then similar to that used in Donni (2003) but necessitates a more general result of partial differential equation theory, namely, the Theorem of Cauchy-Kovalevsky (Zachmanoglou and Thoe, 1976), because the partial differential equation is not of the first order.

References

- Antecol, H. and K. Bedard (2002), The relative earnings of young Mexican, black, and white women, *Industrial and labour Relations Review*, 56, 122-135.
- [2] Apps, P., and R. Rees (1996), Labour supply, household production and intra-family welfare distribution, *Journal of Public Economics*, 60, 199-219.
- [3] Apps, P., and R. Rees (1997), Collective labor supply and household production, Journal of Political Economy, 105, 178-190.
- [4] Aronsson, T., S.O. Daunfeldt and M. Wikström (2001), Estimating intra-household allocation in a collective model with household production, *Journal of Population Economics*, 14, 569-584.
- Blau, F.D. and L.M. Kahn (2007), Changes in the labor supply behavior of married women: 1980–2000, Journal of labour Economics, 25, 393-438.
- [6] Blundell, R., P.A. Chiappori, T. Magnac, and C. Meghir (2007), Collective labour supply: Heterogeneity and non-participation, *Review of Economic Studies*, 74, 417-445.
- Blundell R. and T. McCurdy (1999), labor supply: a review of alternative approaches, in:
 O. Ashenfelter and D. Card (ed.), Handbook of labour Economics 3, Elsevier.
- [8] Bloemen, H. (2010), An empirical model of collective household labour supply with nonparticipation, *Economic Journal*, 120 (543), 183–214.
- [9] Borjas, G.J. (1980), The Relationship between Wages and Weekly Hours of Work: The Role of Division Bias, *Journal of Human Resources*, 15, 409-423.
- [10] Bourguignon, F., M. Browning, and P.A. Chiappori (2009), Efficient intra-household allocations and distribution factors: Implications and identification, *Review of Economic* Studies, 76(2), 503-528.
- [11] Browning M., P.-A. Chiappori, and A. Lewbel (2013), Estimating Consumption Economies of Scale, Adult Equivalence Scales, and Household Bargaining power, *Review of Economic Studies*, 80, 1267-1303.
- [12] Chau, T.W., H. Li, P.W. Liu, and J. Zhang (2007), Testing the collective model of household labor supply: Evidence from China, *China Economic Review*, 18 (4), 389-402.

- [13] Cherchye, L., B. De Rock, and F. Vermeulen (2012), Married with children: a collective labor supply model with detailed time use and intrahousehold expenditure information, *American Economic Review*, 102, 3377-3405.
- [14] Chiappori, P.A. (1988), Rational household labor supply, *Econometrica*, 56, 63-90.
- [15] Chiappori, P.A. (1992), Collective labor supply and welfare, Journal of Political Economy, 100, 437-467.
- [16] Chiappori, P.A. (1997), Introducing household production in collective models of labor supply, Journal of Political Economy, 105, 191-209.
- [17] Chiappori, P.A. and O. Donni (2011), Non-unitary models of household behavior: A survey, in: A. Molina (ed.), *Household Economic Behaviors*, Berlin: Springer.
- [18] Chiappori, P.A., B. Fortin, and G. Lacroix (2002), Household labor supply, sharing rule and market marriage, *Journal of Political Economy*, 110, 37-72.
- [19] Clark, A., H. Couprie, and C. Sofer (2004), La modelisation collective de l'offre de travail: mise en perspective et application aux données britanniques, *Revue économique*, 55 (4), 767-790.
- [20] Couprie, H. (2007), Time allocation within the family: Welfare implications of life in a couple, *Economic Journal*, 117, 287-305.
- [21] Crespo, L. (2009), Estimation and testing of household labour supply models: Evidence from Spain", Investigaciones Económicas, 33(2), 303-335.
- [22] Diewert, E. (1973), Functional forms for profit and transformation functions, Journal of Economic Theory, 6, 284-316.
- [23] Donni, O. (2003), Collective household labor supply: Nonparticipation and income taxation, Journal of Public Economics, 87, 1179-1198.
- [24] Donni, O. (2007), Collective female labour supply: Theory and application, Economic Journal, 117 (516), 94-119.
- [25] Donni, O. (2008), labor supply, home production and welfare comparisons, Journal of Public Economics, 92, 1720-1737.

- [26] Donni, O. and E. Matteazzi (2012), On the importance of household production in collective models: Evidence from U.S. data, Annals of Economics and Statistics, 105-106, 99-126.
- [27] Donni, O. and N. Moreau (2007), Collective labor supply: A single-equation model and some evidence from French data, *Journal of Human Resources*, 42, 214-246.
- [28] Fernandez-Vál, I. (2003), Household labor supply: Evidence for Spain, Investigaciones Económicas, 27(2), 239-275.
- [29] Fortin, B. and G. Lacroix (1997), A test of the unitary and collective models of household labour supply, *Economic Journal*, 107, 933-955.
- [30] Graham, J.W. and C.A. Green (1984), Estimating the parameters of a household production function with joint production, *Review of Economics and Statistics*, 66, 277–282.
- [31] Gronau, R. (1977), Leisure, home production and work The theory of time allocation revisited, Journal of Political Economy, 85, 1099-1123.
- [32] Gronau, R. (1980), Home production A forgotten industry. Review of Economics and Statistics, 62, 408–416.
- [33] Guo, S. (2010), The superior measure of PSID consumption: An update, Economics Letters, 108, 253-256.
- [34] Haddad, G.K. (2015), Gender ratio, divorce rate, and intra-household collective decision process: Evidence from Iranian urban households labour supply with non-participation, *Empirical Economics*, 48(4), 1365-1394.
- [35] Heim, B.T. (2007), The incredible shrinking elasticities: married female labor supply, 1978–2002, Journal of Human Resources, 42 (4), 881-918.
- [36] Heim B.T. (2009), Structural estimation of family labor supply with taxes: Estimating a continuous hours model using a direct utility specification, Journal of Human Resources, 44 (2), 350-385.
- [37] Hersch J. and L.S. Stratton (2000), Household specialization and the male marriage wage premium, *Industrial and labour Relations Review*, 54, 78-94.
- [38] Heckman J.J. (1979), Sample Selection Bias as a Specification Error, Econometrica, 47, 153-161.

- [39] Juster, F.T., H. Ono, F. Stafford (2003), An Assessment of Alternative Measures of Time Use, Sociological Methodology, 33, 19-54.
- [40] Lacroix, G. and N. Radtchenko (2011), The changing intra-household resource allocation in Russia, Journal of Population Economics, 24(1), 85-106.
- [41] Lundberg, S. and E. Rose (2000), Parenthood and the earnings of married men and women, Labour Economics, 7, 689-710.
- [42] Lundberg,S. and E. Rose (2002), The effects of sons and daughters on men's labor supply and wages, The Review of Economics and Statistics, 84, 251-268.
- [43] Manser, M. and M. Brown (1980), Marriage and household decision making: a bargaining analysis, International Economic Review, 21, 333-349.
- [44] McElroy, M. and M.J. Horney, 1981), Nash-bargained decisions: Toward a generalisation of the theory of demand, *International Economic Review*, 22, 333–349.
- [45] Moreau N. and O. Donni (2002), Estimation of a collective model of labor supply with taxation, Annales d'économie et de statistique, 65, 55-83.
- [46] Neary, P. and K.W.S. Roberts (1980), The theory of household behavior under rationing, European Economic Review, 13, 25–42.
- [47] Ransom M. (1987), A comment on consumer demand systems with binding non-negativity constraints, *Journal of Econometrics*, 34, 355-359.
- [48] Rapoport, B., C. Sofer, and A. Solaz (2011), Household production in a collective model: some new results, *Journal of Population Economics*, 24(1), 23-45.
- [49] Sinha, K. (2012), Endogenous, intra-household decision-making and its impact on the labour supply of couples in rural India, Asian Economic Review, 26, 137–157.
- [50] Skinner, J. (1987), A superior measure of consumption from the Panel Study of Income Dynamics, *Economic Letters*, 23, 213-16.
- [51] O'Neil, J. (1990), The role of human capital in earnings differences between black and white men, *Journal of Economic Perspectives*, 4, 25-45.
- [52] Solberg, E.J. and D.C. Wong, 1992. Family time use: leisure, home production, market work, and work related travel, *Journal of Human Resources*, 27, 485–510.

- [53] Trejo, S.J. (1997), Why Do Mexican Americans Earn Low Wages?, Journal of Political Economy, 105, 1235-68.
- [54] Van Soest A. and P. Kooreman, 1990, Coherency of the indirect translog demand systemwith binding nonnegativity constraints, *Journal of Econometrics*, 44, 391-400.
- [55] Vermeulen, F. (2005), And the winner is... An empirical evaluation of two competing approaches to household labour supply. *Empirical Economics*, 30, 711–734.
- [56] Zachmanoglou, E.C., Thoe, D.W. (1976), Introduction to Partial Differential Equations with Applications. Dover Publications, New-York.