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Regulating unverifiable quality by fixed-price contracts

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Abstract

We apply the idea of relation contracting to a very simple problem of regulating a single-product monopolistic firm when the regulatory instrument is a fixed-price contract, and quality is endogenous and observable, but not verifiable. We model the interaction between the regulator and the firm as a dynamic game, and we show that, provided both players are sufficiently patient, there exist self-enforcing regulatory contracts in which the firm prefers to produce the quality mandated by the regulator, while the regulator chooses to leave the firm a positive rent as a reward to its quality choice. We also show that the socially optimal self-enforcing contract implies a distortion from the second best, which is greater the more impatient is the firm and the larger is the (marginal) effect of the contractual price on the profits the firm would make by deviating from the offered contract. Whenever the punishment profits are strictly positive, even if the firm were infinitely patient, the optimal contract would ensure a Ramsey condition but with positive profits to the firm. Our result also illustrates that, whenever the firm's output has some unverifiable component, optimal regulatory lag in fixed-price contract should be reduced to limit the reward of the firm's opportunistic behaviour.

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1 Introduction

The quality of goods and services provided by regulated firms is an extremely sensitive issue: indeed, how to regulate quality has been a subject widely explored since the early days of the economics of regulation (Spence, 1975, Sheshinsky, 1976). Quality has many distinctive features: for instance, it may be difficult to observe, it has a non-deterministic component, consumers' preferences towards it may be difficult to observe. Our focus here is on unverifiability: unverifiability occurs whenever a variable cannot be proven in front of a court and, as a consequence, cannot be contracted upon. In regulated industries it is often the case that a quality dimension of the regulated firm's output is not verifiable: possible examples are courtesy to the customers, voltage of electricity provided in a particular moment, noise of a call, and so forth.

When quality is not verifiable, the regulated firm cannot simply be *directly* rewarded or penalized for the levels of service quality provided. This implies that the regulatory tools developed by the existing literature and commonly applied in practice may turn out not to be very effective. Theoretically, optimal contracts under asymmetric information may provide the firm incentives to supply quality which are intrinsically in conflict with those to reduce cost (Laffont and Tirole, 1993). On more applied grounds, the regulatory instruments commonly in use, such as quality standards and links between the quality provided and his allowed revenues or prices, are typically able to influence only those quality dimension which are readily verifiable (Waddams Price et al., 2008; De Fraja and Iozzi, 2008).

In this paper we suggest an alternative way of regulating quality, based on the idea of *relational contracts*.[1] These are informal agreements and unwritten codes of conduct that are sustained by the value of future relationships, and are applicable in the cases where the outcome of a repeated

[1] See, among the others, Bull (1987), MacLeod and Malcomson (1989), Baker et al. (2002), MacLeod (2003), Levin (2003), Rayo (2007), Fuchs (2007), MacLeod (2007) and Iossa and Spagnolo (2009)

relationship is based on some unverifiable variables. This type of contract fits in naturally with the nature of the interaction between the regulator and the regulated firm. This is a relationship which is typically repeated over time, and in which both parties have quite large discretionary space of manoeuvre, are well informed on many variables affecting the outcome of the relationship, even if part of this knowledge cannot be proven in court or be written in a contract, and in which both parties may have mutual gains from concerted behaviour.

We apply the idea of relation contracting to a very simple problem of regulating a single-product monopolistic firm when the regulatory instrument is a fixed-price contract, and quality is endogenous and observable, but not verifiable. Since quality is not verifiable, the regulator cannot include the quality dimension in the regulatory contract she offers to the firm; however, we consider the possibility that the regulator can use its discretionary powers in choosing the regulatory contract to impose informal punishments to restrain the firm's quality choice. More specifically, the regulator discretionary sets the regulated price to underpin an informal agreement in which the regulated firm is allowed some positive profits only if some target quality measure is met.

We model the interaction between the regulator and the firm as a dynamic game, and we show that, provided both players are sufficiently patient, there exist self-enforcing regulatory contracts in which the firm prefers to produce the quality mandated by the regulator, while the regulator chooses to leave the firm a positive rent as a reward to its quality choice. Despite this result being an application of the Folk theorem in a repeated sequential game, it is nevertheless of interest in that it illustrates, in a way previously unexplored in the literature, a way in which the regulator can use its only instrument, the constraint on the (verifiable) price chosen by the firm, to elicit from the firm the (unverifiable) desired level of quality.

We also characterize the optimal contract, that is the self-enforcing contract which induces the highest social welfare. We show that, under normal circumstances, this contract implies distortion from the second best. We find that this distortion is greater the more impatient is the firm and the larger is the (marginal) effect of the contractual price on the profits the firm would make by deviating from the offered contract. Moreover, the profits the firm obtains under the optimal contract are clearly positively correlated to the profits it would obtain when punished for a deviation from such a contract: if the punishment profits were strictly positive, even if the firm were infinitely patient, the optimal contract would entail a Ramsey condition of tangency between the isowelfare and the isoprofit, but would still grant positive profits to the firm.

Since the distortion with respect to the second best of our optimal contract is larger the smaller is the discount factor, our paper also makes a contribution to the issue of the optimal regulatory lag in fixed-price contract. According to the received literature, when the firm's cost is exogenous, there is a simple trade-off in setting the timing between price reviews: the longer is the regulatory lag, the higher is the incentive the firm has to undertake cost-reducing efforts but also the higher is the probability of allocative inefficiency arising from the excessive profits (Armstrong et al., 1994; and Armstrong et al., 1995). Instead, our result illustrates that, in case the firm's output has some unverifiable component, an increase in the frequency of the price revision reduces the inefficiency of the optimal contract since diminishes the reward of an opportunistic behaviour by the firm.

This paper is clearly related to the literature on quality regulation, recently presented in the excellent survey by Sappington (2005). Price cap regulation plans give the firm insufficient incentive to deliver the socially optimal level of service quality. Therefore, these schemes typically incorporate explicit rewards and penalties to ensure the delivery of desired (and observable) levels of service quality (Waddams et al., 2008). De Fraja and Iozzi (2009) propose an extension of the traditional Laspeyres price cap mechanism (Vogelsang and Finsinger, 1979) where the dynamic nature of price cap regulation allows the regulator to write a price constraint which gives the firm the correct trade-off between price and (observable) quality and induces it to set, in the long run, the optimal price and quality pair. Unverifiability of quality is explicitly taken into account by the optimal regulation literature: in a static context, Laffont and Tirole (1991) shows that the power of the optimal incentive scheme has to reduced (relatively to the case of exogenous quality) to limit the firm's perceived cost of supplying quality. Auray at al. (2008) extend the analysis of incentives in quality regulation to a dynamic framework, albeit restricting the analysis to the case of observable quality. More closely related to our paper is Dalen (1997), who analyses a two-period model in which the regulator must decide between low and high-powered incentive scheme according to the revealed information provided by the firm's past performance. He shows that offering a low-powered incentive contract reduces the value of keeping private information on own efficiency to secure future information rent. Our paper is also complementary in some sense to Lewis and Sappington (1991) who, by dealing with unverifiable quality in a procurement problem, identify the conditions under which verifiability would increases the welfare of both the buyer and supplier. Therefore, they conclude that, when these conditions arise, both parties would be likely to agree on institutional structures that facilitate third-party verification. We extend the scope of their conclusion since we show that, when an institutional structures facilitating third-party verification does not exist, or it is too costly, the buyer (regulator) and the supplier (firm) might still find convenient to find an informal agreement that would make both parties better off. Indeed, the relational contract analyzed in our paper can allow this kind of agreements.

This paper is also related to the recently growing and cross-field literature on relational contracts. Since most of our results are readily applicable to the case of repeated procurement, our paper is linked to Klein and Leffler (1981), Kim (1998), Doni (2006), Calzolari and Spagnolo (2009) (see also Che (2008) for a survey). A first attempt to introduce relational agreements in repeated public procurement is due to Klein and Leffler (1981). More recently, Kim (1998), Doni (2006) and Spagnolo and Calzolari (2009) study the incentive of relational contracts to deliver non-contractible quality in procurement repeated auctions with more than one supplier. As in our paper, they show that an optimal strategy for the buyer to enforce unverifiable quality is leaving future rents to the contractor. However, given their application of relational agreements to repeated auctions, the punishment for an opportunistic behaviour is the termination the relationship, what certainly is not applicable in our regulator-single firm relationship.

The rest of the paper is organised as follows. Section 2 presents the model. The equilibria of the static and the dynamic game are characterized in section 3, which also contains two examples that shed further light on the nature of the optimal contract arising in the dynamic game equilibrium. Section 4 concludes.

2 The Model

We analyse an infinite horizon game in which two parties, a regulator and a monopolistic firm, interact at dates $t = 0, 1, \ldots, \infty$. Let δ be the discount factor common to the firm and the regulator.

The monopolist produces one good, whose demand is $x(p, q)$, with p denoting the price of the good and q its quality; we assume that $p \in \mathbb{R}_+$ and $q \in Q \equiv [\underline{q}, \overline{q}] \subseteq \mathbb{R}_{++}$. The firm's technology is described by the cost function $c(x, q)$. The firm's profits are therefore given by $\pi(p, q) = x(p, q)p - c(x(p, q), q)$.

The demand function is assumed to have standard properties: for all quality levels q, it is continuous and twice differentiable, with $\frac{\partial x}{\partial p} < 0$ and $\frac{\partial x}{\partial q} > 0$ whenever $x > 0$. The cost function satisfies, plausibly, $\frac{\partial c}{\partial x} > 0$ and $\frac{\partial c}{\partial q} > 0$. To avoid corner solutions, we assume that $\lim_{q \to \underline{q}} \frac{\partial c}{\partial q} = 0$ and that $\lim_{q \to \overline{q}} c(x, q) = +\infty$: a marginal increase of quality is costless when quality is at its minimum and maximal quality is infinitely costly.

The regulator's objective function is given by the social welfare function $V(p, q)$ which is assumed to be quasi-convex, continuously differentiable and satisfy the following plausible conditions: $\frac{\partial V(.)}{\partial p} \leq 0$, and $\frac{\partial V(\cdot)}{\partial q} > 0$.[2] The social value of not having the good produced by the firm is equal to V_0; we assume that having the good produced is always beneficial for the society, so that $V(p,q) > V_0$ for any value of p and q.

The dynamic game we consider is an infinite repetition of the following sequential stage game:

Stage 1: the regulator makes an offer $\mathcal{F} = \{p', q'\}$ in which it asks the firm to produce a good of quality q' and sets the market price p' at which the good has to be sold in the market;

Stage 2: the firm chooses whether or not to accept the contract; if the firm does not accept the contract, the game ends, otherwise the game proceeds to the following stage;

Stage 3: the firm chooses the effective quality level q''; at the end of this stage the regulator observes q'', and the payoffs $V(p', q'')$ and $\pi(p', q'')$ are realized.

Observe that, because of the assumption on the regulator's reservation value, the regulator will always make offers such that $\pi(p', q') \geq \pi_0$, where we denote with π_0 the firm's reservation profits, which we normalise to zero. This implies that the second stage of the game can be ignored in the rest of the analysis, since the firm will never find it profitable to reject the offer and quit the game.

We analyse a game of complete but imperfect information. However, despite the realisation of price and quality being fully observable by both players, quality is not enforceable in a court of law, in that the regulator cannot impose any directly enforceable penalty on the firm when it observes $q' \neq q''$.

[2] We purposely do not impose any further restriction on the consumers and regulator's preferences. A less general but equally natural setting would be with many consumers with quasi-linear preferences and a benevolent regulator. In such a setting, many standard properties, such as the equivalence between consumers' surplus and their welfare and Roy's identity would hold; none of these properties would however be necessary for our results.

Before proceeding into the analysis of the game, we state the following:

Definition 1. *Let the following definitions hold:*

a) for any price p, let $\widehat{q}(p) = \arg\max_q \ \pi(p,q)$;

b) for any price p, let $\widehat{\pi}(p) = \pi(p, \widehat{q}(p))$;

c) let p^0 be the price such that $\widehat{\pi}(p^0) = 0$.

In words, $\widehat{q}(p)$ is the quality level that delivers the highest profit to the firm for any possible price. We assume it to exist and be unique. Similarly, $\widehat{\pi}(p)$ is the profit the firm can make, for any given price, when it optimally chooses its quality. Also, p^0 is the price level which ensures that the firm obtains zero profits when it freel chooses its quality level, given this price. We assume this price to exist and be unique.

We also state:

Definition 2. *Let p^R and q^R be the pair of price and quality which solves the following problem:*

$$\begin{aligned} \max_{\{p,q\}} \quad & V(p,q) \\ \text{s.t.} \quad & \pi(p,q) \geq 0 \end{aligned} \tag{1}$$

In words, p^R and q^R are the Ramsey price and quality pair which maximises (static) social welfare subject to a nonnegativity constraint on the firm's profits. It is easy to show that, at p^R and q^R, the following holds:

$$\frac{\partial V/\partial p^R}{\partial V/\partial q^R} = \frac{\partial \pi/\partial p^R}{\partial \pi/\partial q^R}, \tag{2}$$

and that, at p^R and q^R, the nonnegativity constraint holds as an equality. We assume p^R and q^R to exist, to be unique and different from p^M and q^M, where p^M and q^M are the profit maximising price and quality values.

Figure 1 gives an illustration of these Definitions. It depicts the price-quality cartesian plane; the solid curves are the isoprofit lines and the dashed curves are the isowelfare lines, upward sloping because welfare is increasing in quality and decreasing in price.[3] The pair $\{p^M, q^M\}$ is the profit maximising price and quality pair and $\{p^R, q^R\}$ is the second best optimal pair: at this point, the zero-profit isoprofit line is tangent to the isowelfare map. At prices p^0 and p^1, the firm, freely choosing its quality level, selects $\widehat{q}(p^0)$ or, respectively, $\widehat{q}(p^1)$: at the price and quality pair $\{p^0, \widehat{q}(p^0)\}$ the firm makes zero profits.

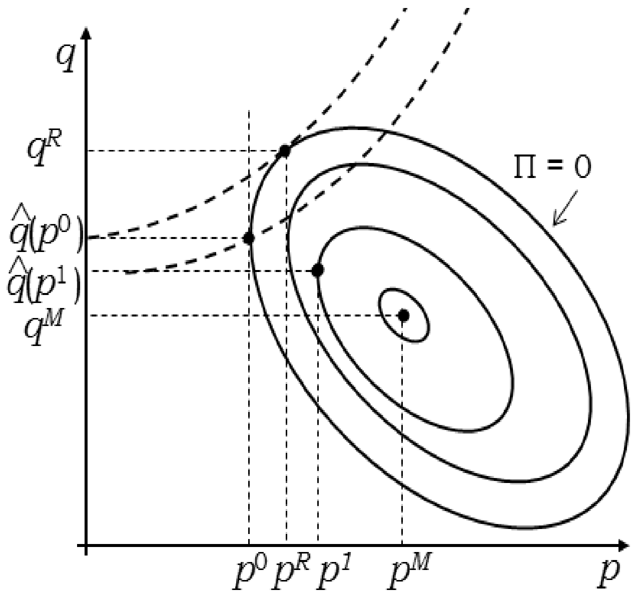


Fig. 1 - The static game

3 Equilibrium

3.1 The static game

We start by noting that, in a static context, unverifiability of the quality provided by the firm implies that the regulator cannot enforce the second best quality level. In other words, since the regulator can only observe but not punish any choice of quality other than the mandated level, we are back

[3]Isowelfare lines are drawn convex coherently with the quasi-convex assumption made for the social welfare function. When the regulator is a benevolent utilitarian and consumers' preferences are quasi-linear, quasi-convexity would simply reflect the quite natural assumption that consumers' willingness to pay for increases in quality is higher when quality is low than when quality is already high; for further discussion, see De Fraja and Iozzi, 2009.

in the context of price regulation with endogenous quality, firstly analysed by Spence (1979). It is then straightforward to characterize the equilibrium of the stage game described above. In the last stage of the game, for any price mandated by the regulator, the firm chooses the profit maximizing quality level $\widehat{q}(p)$. Anticipating this, in the first stage the regulator makes an offer $\mathcal{F}^S \equiv \{p^S, \widehat{q}(p^S)\}$, where p^S comes as the solution of the following problem:[4]

$$\begin{aligned} \max_{p} \quad & V(p,q) \\ \text{s.t.} \quad & \pi(p,q) \geq 0 \\ & q = \widehat{q}(p). \end{aligned} \tag{3}$$

The properties of this equilibrium price are described in the following Proposition:

Proposition 1. *The price p^S offered by the regulator in static equilibrium of the stage game has the following features:*

- $p^S = p^0$ *whenever* $-\frac{\partial V}{\partial p^S} > \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S}$, *which implies* $\widehat{\pi}(p^S) = 0$, *and*
- $p^S > p^0$ *whenever* $-\frac{\partial V}{\partial p^S} = \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S}$, *which implies* $\widehat{\pi}(p^S) > 0$.

Proof. To solve problem (3), set up the Lagrangean incorporating the second constraint

$$L = V(p, \widehat{q}(p)) + \mu \widehat{\pi}(p).$$

FOCs are:

$$\frac{\partial L}{\partial \mu} = \widehat{\pi}(p^S) \geq 0; \quad \mu \geq 0 \quad \mu \widehat{\pi}(p^S) = 0$$

and

$$\frac{\partial L}{\partial p^S} = \frac{\partial V}{\partial p^S} + \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S} + \mu \frac{\partial \widehat{\pi}}{\partial p^S} = 0.$$

If $\mu = 0$, then $\widehat{\pi}(p^S) \geq 0$ and $\frac{\partial V}{\partial p^S} + \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S} = 0$. Instead, if $\mu > 0$, then $\pi(p^S, \widehat{q}(p^S)) = 0$ and $\frac{\partial V}{\partial p^S} + \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S} + \mu \frac{\partial \widehat{\pi}}{\partial p^S} = 0$. Since $\widehat{\pi}(p)$ is monotonically increasing in p whenever $p < p^M$, this establishes the result. $\square$

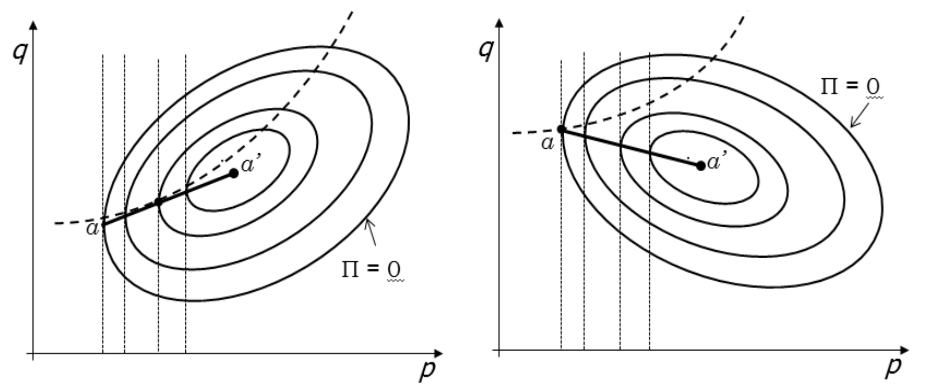
[4] We take $\widehat{q}(p^S)$ as the quality level included in the offer $\mathcal{F}^S$ only for the sake of definiteness; indeed, any quality level could be part of such an offer because, in this static setting, the regulator anticipates that the firm will always choose its profit maximizing quality level and that it cannot prevent or punish this behaviour.

Proposition 1 illustrates that the optimal static offer is such that the firm may obtain strictly positive profits.[5] When the optimal offer implies strictly positive profits, a marginal increase in the price induces an increase in the quality provided by the firm (i.e. $\frac{\partial \widehat{q}(p^S)}{\partial p^S} > 0$). The optimal offer then equalizes the marginal negative direct effect on welfare of a price increase with the marginal positive indirect effect, due to an increase in the quality provision (i.e. $-\frac{\partial V}{\partial p^S} = \frac{\partial V}{\partial \widehat{q}(p^S)} \frac{\partial \widehat{q}(p^S)}{\partial p^S}$). On the other hand, when the optimal offer implies zero profits, the direct positive effect on welfare of a price reduction would outplay the effect going through a change in quality; however, the non-negativity constraint on the firm's profits limits a further price decrease. Notice that in this case, at the equilibrium, the sign of the marginal change in quality due to a marginal price change ($\frac{\partial \widehat{q}(p^S)}{\partial p^S}$) is indeterminate. It should be also noted that, as already pointed out in the existing literature (Spence, 1979; and De Fraja and Iozzi, 2008), at the optimal offer, equilibrium quality is always underprovided, in the sense that there always exists a Pareto improving increase in quality. On the other hand, no clear-cut conclusion can be reached on the magnitude of p^S relatively to the second-best price: it is indeed even possible that the equilibrium price is above the second best price.

This result is illustrated in Figure 2. In both panels, the locus aa' is made of the optimal quality choices for the different price levels, i.e. $\widehat{q}(p)$. Taking this as a constraint, the regulator chooses its optimal one-shot price p^S to maximise social welfare. Depending on the local relative slope of iso-welfare ($-\frac{\partial V}{\partial p^S} / \frac{\partial V}{\partial \widehat{q}(p^S)}$) and the sign of $\frac{\partial \widehat{q}(p^S)}{\partial p^S}$ which determines the two possible situations in Proposition 1, the optimal price may be given by a tangency condition between the isowelfare and the locus aa', as in panel (a), or may be a corner solution, as in panel (b). Clearly, given the many possible shapes the locus aa' can take on, restrictions are necessary to ensure that the solution to the regulator's problem is unique or, more restrictively,

[5] Note that the solution to problem (3) need not be unique. In case of multiple solutions, for reasons that will be clearer thereafter, we select p^S as the solution giving the firm the lowest profit.

exists altogether.



(a) (b)

Fig. 2 - the optimal static contract

3.2 The dynamic game

We now turn to illustrate that, under unverifiable quality, a relational contract may ensure a social welfare higher than the one which would prevail in a static context. In line with Levin (2003), to characterise this relational contract we study the dynamic game, introduced in section 2, given by an infinite repetition of the sequential stage game discussed in the previous section.

A regulatory relational contract under unverifiable quality is a strategy profile such that, given the offer $\mathcal{F}^C \equiv \{p^C, q^C\}$, the parties take the following actions in each period

- the regulator makes the offer $\mathcal{F}^C$;
- the firm chooses q^C.

This regulatory relational contract is self enforcing if the strategy profile is a perfect equilibrium of the repeated game.

The definition leaves undefined two elements of the players' strategies, the offer $\mathcal{F}^C$ and the parties' behaviour off the equilibrium path. We make them precise concentrating on the following grim trigger strategies for the players:

- **regulator**: the regulator begins the game by making the firm an offer $\mathcal{F}^C$ and keeps making this offer if the firm has always chosen quality q^C in previous periods; otherwise, it reverts indefinitely to its equilibrium strategy in the stage game;
- **firm**: the firm chooses the quality q^C whenever the regulator has offered $\mathcal{F}^C$ in the past; otherwise, it reverts indefinitely to its equilibrium strategy in the stage game.

Notice the somehow different nature of the two strategies, due to the sequential nature of the stage game; while a choice of the quality level different from q^C is detected by the regulator only in the following period, an offer different from $\mathcal{F}^C$ by the regulator is immediately observed by the firm and triggers a reaction in the same period it is made.

The Folk theorem ensures the existence of an equilibrium in these trigger strategies, provided that the players are sufficiently patient.[6] Formally, this requirement of "sufficient patience" is equivalent to the following condition:

$$\frac{1}{1-\delta}\pi(p^C, q^C) \geq \widehat{\pi}(p^C) + \frac{\delta}{1-\delta}\widehat{\pi}(p^S) \tag{ICF}$$

Observe also that, in principle, an incentive compatibility constraint need to to hold also for the regulator. However, the regulator's IC is also always satisfied provided it gains from offering $\mathcal{F}^C$, that is $V(p^C, q^C) \geq V(p^S, \widehat{q}(p^S))$. Indeed, there is no short-term gain for the regulator in deviating from its the trigger strategy, because this is observed and punished by the firm in the same period before payoffs are realized.

Condition ICF simply imposes limits on the nature of the offer $\mathcal{F}^C$. However, since it is the regulator to choose the offer, it will select the socially optimal among the ones which ensure that the ICF holds. We now turn to

[6] Sorin (1995) proves that the Folk theorem proved by Fudenberg and Maskin (1986) for simultaneous repeated game also applies to sequential repeated games provided that full dimensionality condition (FDC) holds. This requires that the convex hull of the set of the feasible payoff vectors of the stage game must have dimension equal to the number of players, or equivalently a nonempty interior. FDC is clearly satisfied in our model. Abreu et al. (1994) and Wen (1994, 2002) further weaken this requirements.

studying the characteristics of the regulator's *optimal* offer which ensures the self-enforcing nature of the relational contract. We let $\mathcal{F}^* \equiv \{p^*, q^*\}$ be such an offer; it comes as the solution to the following problem:

$$\max_{\{p,q\}} \sum_{t=0}^{\infty} \delta^t V(p,q) = \frac{1}{1-\delta} V(p,q) \tag{4}$$

$$\text{s.t.} \quad \frac{1}{1-\delta} \pi(p,q) \geq \widehat{\pi}(p) + \frac{\delta}{1-\delta} \widehat{\pi}(p^S) \tag{ICF}$$

We can now state the main result of the paper:

Proposition 2. *The price and quality pair $\{p^*, q^*\}$ solving the problem defined by (4) and (ICF) satisfies the following conditions:*

$$\frac{\partial V/\partial p^*}{\partial V/\partial q^*} = \frac{\partial \pi/\partial p^* - (1-\delta)\,\partial \widehat{\pi}/\partial p^*}{\partial \pi/\partial q^*} \tag{5}$$

and

$$\frac{1}{1-\delta} \pi(p^*, q^*) = \widehat{\pi}(p^*) + \frac{\delta}{1-\delta} \widehat{\pi}(p^S). \tag{6}$$

Proof. The Lagrangian and the FOC's of the problem defined by (4) are the following:

$$L = \frac{1}{1-\delta} V(p^*, q^*) + \lambda \left[\frac{1}{1-\delta} \pi(p^*, q^*) - \widehat{\pi}(p^*) + \frac{\delta}{1-\delta} \widehat{\pi}(p^S) \right] \tag{7}$$

$$\frac{\partial L}{\partial p^*} = \frac{1}{1-\delta} \frac{\partial V(.)}{\partial p^*} + \lambda \left[\frac{1}{1-\delta} \frac{\partial \pi(.)}{\partial p^*} - \frac{\partial \widehat{\pi}(.)}{\partial p^*} \right] = 0 \tag{8}$$

$$\frac{\partial L}{\partial q^*} = \frac{1}{1-\delta} \frac{\partial V(.)}{\partial q^*} + \lambda \left[\frac{1}{1-\delta} \frac{\partial \pi(.)}{\partial q^*} \right] = 0 \tag{9}$$

$$\frac{\partial L}{\partial \lambda} = \frac{1}{1-\delta} \pi(p^*, q^*) - \widehat{\pi}(p^*) - \frac{\delta}{1-\delta} \widehat{\pi}(p^S) \geq 0;$$
$$\lambda \geq 0; \lambda \frac{\partial L}{\partial \lambda} = 0. \tag{10}$$

From (8) and (9), it follows that $\lambda > 0$; if this were not the case, we would have that $\frac{\partial V(.)}{\partial p^*} = \frac{\partial V(.)}{\partial q^*} = 0$, which clearly contradicts the hypothesis that the first best is out of reach. Therefore, $\frac{\partial L}{\partial \lambda} = 0$ in (10), which gives (6). Also, dividing (8) by (9), we get (5). □

Conditions (5) and (6) define the optimal equilibrium price and quality pair and illustrate the way it departs from Ramsey conditions. Condition (5) differs from the standard Ramsey condition for two important factors: first, the role of the firm's intertemporal preferences and, second, the marginal effect on the deviation profits of a change in the contractual price, that is the way a change in the optimal price affects the willingness to deviate of the firm. To interpret (5), note that, in the Ramsey condition of tangency, the marginal rate of substitution between price and quality is equated between the regulator and the firm. On the contrary, here, at the social optimum, the regulator finds it optimal to offer a price lower than the one which would ensure the tangency between isoprofit and isowelfare (at the minimum profit level for the firm). This is because it takes into account the fact that the higher is the price offered, the higher are the firm's profits in case of deviation. The greater is this effect the smaller will be the regulator's willingness to substitute away price with quality, because of the risk in inducing a deviation by the firm. Clearly, these considerations play a role in the regulator's choice of the optimal contract which is more important when the firm is more "tempted" to deviate, that is when the firm is the less patient. This implies that the distortion from a Ramsey tangency condition typical of the optimal offer is greater the smaller is the firm's discount factor. Only if the firm were infinitely patient and/or the effect of the optimal price on the deviation profits were null, the optimal contract would correspond to a tangency condition between the isowelfare and the isoprofit, as with the standard Ramsey condition.

On the other hand, condition (6) illustrates the level of the profits the regulator has to ensure to the firm. These increase not only with the profits the firm obtains by deviating from the regulator's offer, but also with the profits the firm would obtain in the punishment phase. Combining conditions (5) and (**??**), it is also possible to see that, whenever the punishment phase entails strictly positive profits for the firm, even if the firms were infinitely patient, the best possible contract would satisfy the Ramsey tangency condition, though on an isoprofit corresponding to strictly positive profit.

Our result has some interesting implications in terms of optimal length of the regulatory period.[7] We have so far interpreted δ simply as an exogenous parameter expressing the players' intertemporal preferences. Standard textbook analysis illustrates how δ depends instead on a much wider range of circumstances, such as the frequency of interaction, the probability of continuation, and so forth. In particular, δ increases with the frequency of interactions in each given period since it reduces the value of the per-period interest rate. Therefore, as our equilibrium outcome becomes closer to a Ramsey solution as δ increases, our result has the immediate policy prescription to suggest an increase in the frequency of the price revision in regulatory settings in which unverifiability plays a role.

To illustrate further the nature of the optimal offer to be made to the firm and to shed light on the effect on it of the time preferences and of the interplay between the "cooperative" profits and, in contrast, the profits arising during the deviation and the punishment phase, we use two examples with specific functional forms. In particular, the first example includes an optimal punishment with zero-profit while in the second the punishment profit is positive.

3.3 Example 1

Demand function is given by $x(p, q) = (4 + q) - p$. Social welfare, defined as aggregate consumers' surplus, is given by

$$V(p, q) = \frac{1}{2}(4 + q - p)^2. \tag{11}$$

The firm's cost function is $c(q, x) = (1 + q^2)x$, and profits are given by

$$\pi(p, q) = (4 + q - p)(p - (1 + q^2)). \tag{12}$$

[7]See Armstrong et al. (1994) and Armstrong et al. (1995). The basic idea explored by the literature so far is that there is a trade-off in setting the timing between price reviews when the firm's cost is exogenous. On the one hand, the longer is the regulatory lag, the higher is the incentive the firm has to undertake cost-reducing efforts; on the other hand, the longer is the regulatory lag, the higher the probability of allocative inefficiency arising from the excessive profits.

The monopolist unconstrained profit maximising choices are given by $p^M = \frac{23}{8}$ and $q^M = \frac{1}{2}$, which give profits equal to $\pi^M = \frac{169}{64}$. For any given price, the optimal quality choice for the firm is

$$\widehat{q}(p) = -\frac{4}{3} + \frac{1}{3}p + \frac{1}{3}\sqrt{13 - 5p + p^2} \tag{13}$$

The (static) second best price and quality pair, found by solving problem (3), is such that

$$p^R = \frac{5}{4} \text{ and } q^R = \frac{1}{2} \tag{14}$$

which give social welfare equal to $V(p^R, q^R) = \frac{169}{32}$ and profits equal to 0. Notice the the socially optimal quality level is also chosen by the unconstrained monopolist, something which the previous literature has already recognised to be possible (see, for instance, Tirole, 1989). The punishment price and quality pair is $p^P = 1$ and $q^P = 0$ with $\pi(p^P, q^P) = 0$.

Table 1: Optimal contract in Example 1

δ	p^*	q^*	p^P	$\pi(p^*, q^*)$	$\pi(p^*, \widehat{q}(p^*))$	$V(p^*, q^*)$
0.1	1.0344	0.0637	1	0.0918	0.1020	4.5884
0.2	1.0658	0.1233	1	0.1548	0.1935	4.6739
0.3	1.0948	0.1792	1	0.1933	0.2762	4.7568
0.4	1.1216	0.2321	1	0.2108	0.3514	4.8374
0.5	1.1466	0.2821	1	0.2101	0.4201	4.9158
0.6	1.1699	0.3297	1	0.1933	0.4833	4.9922
0.7	1.1917	0.3751	1	0.1625	0.5417	5.0669
0.8	1.2123	0.4185	1	0.1192	0.5958	5.1398
0.9	1.2317	0.4601	1	0.0646	0.6461	5.2113
0.999	1.2498	0.4996	1	0.0007	0.6926	5.2806

Unfortunately, it is not possible to solve for the optimal price and quality pair analitically and we then have to resort to numerical methods. Table 1 provides the values of p^*, q^* and p^P for different values of the discount factor δ; the same Table also provides the equilibrium level of static profits and social welfare.

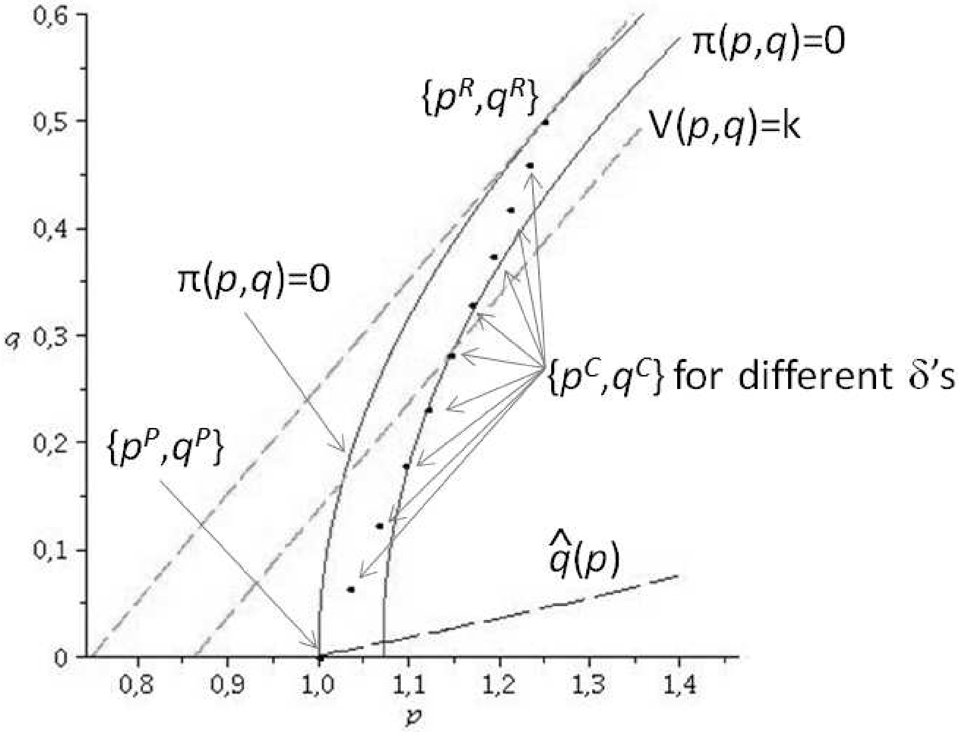


Fig. 3 - Example 1

These same values are illustrated in Figure 3. The points lying southwest to the (static) socially optimal pair $\{p^R, q^R\}$ are the optimal offer, $\{p^*, q^*\}$, drawn for different values of δ. The higher is δ, the closer to the second best are both price and quality included in the optimal contract; on the other hand, the lower is δ, the closer are the contractual pairs to the pair $\{1, 0\}$, the equilibrium of the static game and also the offer by which the regulator may punish any deviation from the optimal contract. Through the optimal offer when $\delta = 0.5$, we draw both the isowelfare and the isoprofit to illustrate the distortion (i.e. the difference between the firm's and regulator's marginal rate of substitution) typical of the optimal contract.

This example neatly illustrates the nature of the optimal offer. This has two important features: first of all, it entails a distortion from the second-best in that the marginal rate of substitution between price and quality is different between the firm and the regulator. As shown in Proposition 2, this distortion is greater the lower is the value of δ and the larger is the marginal effect of a change in the contractual price on the deviation profits. In this example though, both distortions tend to disappear as the value of δ goes to 1: the more patient is the firm, the smaller are the extra profit necessary to convince it to adhere to the optimal contract and, also, the higher is the gain from the deviation. This result however depends on the possibility to punish the firm with a zero-profit contract in case of deviation: the following example illustrates the relevance of the punishment phase for the level of social welfare the optimal contract is able to deliver.

3.4 Example 2

Demand function is given by $x(p,q) = 2q - qp$. This implies that aggregate consumers' surplus is given by

$$V(p,q) = \frac{1}{2}(2-p)^2 q \tag{15}$$

The firm's cost function is $c(q,x) = q^2 + \frac{1}{2}x$, so that its profits are given by

$$\pi(p,q) = p\,(2q - qp) - q + \frac{1}{2}qp - q^2; \tag{16}$$

The monopoly unconstrained price and quality are given by $p^M = \frac{5}{4}$ and $q^M = \frac{9}{32}$, which gives profits equal to $\pi^M = \frac{81}{1024}$. For any given price, the optimal quality choice for the firm is

$$\widehat{q}(p) = \frac{5}{4}p - \frac{1}{2}p^2 - \frac{1}{2} \tag{17}$$

By solving the problem (3), we find that the (static) second best price and quality pair is given by

$$p^R = \frac{7}{8} \text{ and } q^R = \frac{27}{64}, \tag{18}$$

which gives rise to social welfare equal to $V(p^R,q^R) = \frac{2187}{8129}$ and, clearly, profits equal to zero. As in the previous example, we have to resort to numerical methods, whose results are given in Table 2, identical in its nature to the previous one.

Table 2: Optimal contract in Example 2

δ	p^*	q^*	p^P	$\pi(p^*, q^*)$	$\pi(p^*, \widehat{q}(p^*))$	$V(p^*, q^*)$
0.1	0.9492	0.2695	0.8750	0.05458	0.0557	0.1488
0.2	0.9617	0.2906	0.8750	0.05486	0.0575	0.1566
0.3	0.9693	0.3066	0.8750	0.05429	0.0585	0.1629
0.4	0.9747	0.3201	0.8750	0.05333	0.0592	0.1683
0.5	0.9789	0.3319	0.8750	0.05214	0.0598	0.1730
0.6	0.9823	0.3426	0.8750	0.05079	0.0602	0.1774
0.7	0.9852	0.3524	0.8750	0.04933	0.0606	0.1815
0.8	0.9877	0.3615	0.8750	0.04778	0.0609	0.1852
0.9	0.9898	0.3700	0.8750	0.04617	0.0612	0.1888
0.999	0.9917	0.3780	0.8750	0.04451	0.0614	0.1922

These values are illustrated in Figure 4, where the same notation as in the previous Figure is used.

The main result from Table 2 is that, when the "punishment"offer gives positive profits to the firm, the second best is never enforceable; as a matter of fact, for any δ, the optimal price and quality are always distorted respectively upward and downward. In particular, it is possible to see that, even thought the firm was infinitely patient ($\delta = 1$) (or the frequency of the price revision is sufficiently high), the Ramsey condition of tangency is satisfied though with a strictly profit and an isowelfare always lower than the second best value. This means that the punishment is not harsh enough to ensure the second best and time-preference adjustment itself cannot eliminate distortions as in the Example 1. Social welfare indeed is always lower than his second best value.

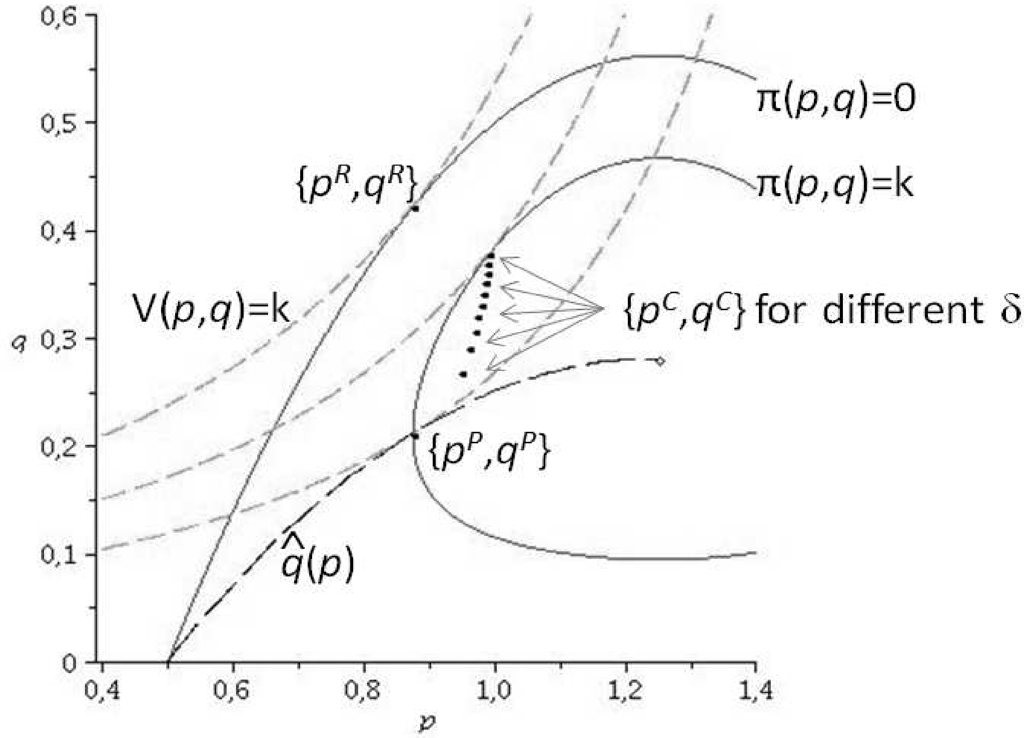


Fig. 4 - Example 2

4 Conclusions

This paper defines the optimal fixed-price contract the regulator needs to offer a regulated firm when the quality is endogenous, observable but not verifiable. We suggest that, using the discretionary powers of the regulator and exploiting the repeated nature of the interaction between the regulator and the firm, there exist self-enforcing agreements which may help overcoming the difficulties due to the unverifiable nature of quality. We show that, in an infinitely repeated contractual relation, if the regulator rewards the firm by means of a high regulated price when it delivers a mandated quality level and punishes it when it deviates from such a level by reducing the regulated price in future periods, the optimal contract improves upon the level of static social optimum. This contract however typically entails distortions from the quality and price of second best, unless the punishment is so harsh to induce zero profit. What this paper predicts in terms of regulation policy is that sufficiently short contracts including harsh punishments could induce the regulated firm to deliver a price and quality combination sufficiently close to the second best.

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