Risk, Uncertainty and Discrete Choice Models

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Abstract

This paper examines the cross-fertilizations of random utility models with the study of decision making under risk and uncertainty. We start with a description of the Expected Utility (EU) theory and then consider deviations from the standard EU frameworks, involving the Allais paradox and the Ellsberg paradox, inter alia. We then discuss how the resulting Non-EU framework can be modeled and estimated within the framework of discrete choices in static and dynamic contexts. Our objectives in addressing risk and ambiguity in individual choice contexts are to understand the decision choice process, and to use behavioral information for prediction, prescription and policy analysis.

Keywords: *discrete choice, decision making, risk, uncertainty, (cumulative) prospect theory, ambiguity*

Introduction

The field of decision making under risk (and uncertainty) has a long history, starting with the early mathematical developments of B. Pascal. The first formal model, almost unchallenged for about 60 years, is the expected utility (EU) model, formalized by von Neumann and Morgenstern (VNM). This axiomatic approach is powerful and tractable. However, a long series of well-known experiments has shown that the underlying axioms can be challenged, even in the context of simple choice situations. Notable, inter alia, are the following paradoxes (see section 2): the Allais paradox; the preference reversal paradox (irrelevant reframing of questions drastically affects decisions in irrational ways, challenging all modern theories of choice); and the Ellsberg paradox (concerning choice when the probability distributions are unknown, i.e., when uncertainty prevails). Two types of responses have been provided in the literature:

- Non-expected utility theories that extend the axiomatic approaches in ways that explain these paradoxes and the deviations from the standard choice axioms. The best-known approach is the Cumulative Prospect Theory (CPT) due to Tversky and Kahneman (1992), who emphasize biases in the perception of probabilities and outcomes.
- Non-deterministic approaches to choice under risk and uncertainty that address the EU violations. The most commonly used framework is the Random Utility Model (RUM); see McFadden (2001). Hey and Orme (1994), for instance, found that EU with some additional structure of error terms provides satisfactory predictions of individual choice. (Note that the same approach of adding an error structure, as by Hey and Orme, can also be applied with non-expected utility models.) Conclusions regarding the descriptive power of probabilistic choice models are dependent on the assumed distribution of the error terms. Therefore, experimentalists and other researchers need to find reliable and efficient ways to estimate (possibly heterogeneous) people’s preferences based on theoretically sound specification of a RUM.

This paper addresses the following questions: (a) what is the degree of substitution/overlap and coherence between the random utility models and the non-expected utility models? (b) Under which circumstances is there a need to extend the standard expected utility theory towards non-expected utility theories?

1. Examples of Individual Choice under Risk and Uncertainty

Issues of risk and uncertainty are critical factors in a wide variety of choice contexts. These contexts vary along numerous dimensions, including what is uncertain, how much the decision maker knows about the probability distribution, the importance of the decision (e.g., life changing events versus games), and the severity of the outcomes (e.g., loss of pocket change versus loss of large sums of money, health, or life). In a *choice under risk*, the probability distribution of the potential outcomes is known. Under *uncertainty* (or *ambiguity*), this distribution is unknown to the decision maker. The fields of applications are numerous and include accident and prevention, investment and finance, environmental protection, computation of willingness to pay when risk is involved, statistical value of human life, as well as relation between equity, discount rate, and risk aversion when analyzing saving behavior or optimal taxation.

Such individual decisions often involve trading off costs or benefits now, which are known with certainty, with risky outcomes in the future, sometimes the far future. Two prominent examples concern decisions at a young age: whether to go to college or to start working; or, at a later age, whether to retire or to continue working. For **education**, decisions involve weighing costs including tuition and foregone earnings against higher incomes in the following 40 years or so, notwithstanding changes in lifestyles that might also be attached to this choice. Over the life-cycle, the risk and uncertainty in incomes can be very large, and recent research, summarized in Heckman and Navarro (2007), proposes a robust methodology to estimate these trade-offs. For **retirement**, the decision involves a comparison of two sources of income: additional labor income and the annuities derived from owned assets. Both sources of income are likely to be known with certainty at the time the decision is made. However, longevity and health shocks are the main risks and uncertainties during the post retirement period (Van Soest, Kapteyn and Zissimopoulos, 2007). Furthermore, the expected and discounted social security payments might also be risky and ambiguous (Benitez-Silva and Dwyer, 2005). The choice among alternative financial **investment** strategies with varying levels of expected return and volatility is dependent on the degree of risk the investor is willing and capable to bear. These choice situations involve an uncertainty dimension (since experts' predictions differ) and an inter-temporal dimension.

Other examples include:

Medical plan: In choosing a medical plan, the uncertainty is health and associated medical needs over the lifetime of the plan.

Surgery versus radiation therapy: When dealing with illness, there are often several treatment options, each with multiple potential outcomes (from full recovery, to partial recovery, to additional complication) of varying probabilities.

Fixed/variable mortgage: Uncertainty about interest rates leads to ambiguity in decisions involving mortgages.

TV game shows: In the popular TV game show "Deal or no Deal" contestants are offered a deal of guaranteed money in their pocket versus a chance to win more (and a chance to lose more).

Airline connection: There is ambiguity associated with air travel in that one cannot be certain of making a particular connection, and the probability of missing the connection will vary based on the scheduled layover time and the on-time performance of the flights.

Freeway driving: In merging onto a freeway, one can choose among different tactics, including a normal merge in which there is ample room to change lanes, a courtesy merge in which the lag vehicle yields, and a forced merge in which the lag vehicle is forced to yield. The ambiguity is that one does not know the response or action of the other drivers on the road.

In section 2 we present the main EU and Non-EU theories that have been used to model ambiguity and perception biases. This section, and the subsequent ones, present selective rather than exhaustive surveys. Section 3 discusses various ways to introduce EU and Non-EU theories in the random utility framework (paying special attention to heterogeneity). Estimation issues are discussed in sections 4 and 5, and a detailed example in experimental economics is discussed in section 6. Recommendations to modellers for incorporating risk into their analysis and research perspectives are discussed in section 7.

2. Behavioral Theories

This section provides a selective overview of decision making theories. We first define a prospect and review the basic concepts of EU models. We then discuss key paradoxes of the EU model predictions, in particular the Allais paradox. Next, we introduce the main Non-EU models such as the loss aversion model (involving asymmetry between gain and losses), rank-dependent utility and probability weighting models (involving gains and losses). Finally, we discuss examples and the modelling of ambiguity.

Let $\mathcal{X} = (E_1{:}x_1,\ldots,E_n{:}x_n)$ denote a *prospect*. The E_j, $j = 1,\ldots,n$ denote possible *events,* of which exactly one is true and the others are not true, and it is unknown to us which one is true. The quantity x_j designates an amount of money (or any similar source of utility), which is the *outcome* of the prospect if E_j is true, $j = 1,\ldots,n$. For example, x_1 can represent the mortgage rate when the reference rate goes up, while x_2 can represent the mortgage rate otherwise. In Lam and Small (2001) or in de Palma and Picard (2006), the E_j denote traffic conditions and the x_j refer to travel times.

We write $\mathcal{X} \succeq \mathcal{Y}$ if the decision maker is willing to choose prospect $\mathcal{X}$ from $\{\mathcal{X}, \mathcal{Y}\}$. This revealed binary choice is interpreted as a preference of $\mathcal{X}$ over $\mathcal{Y}$. When choosing from multiple prospects, the decision maker selects one that $\succeq$-dominates all others.

Because we do not know for sure which event is true, we do not know for sure what outcome will result from a prospect. This is reflected by the term *decision under uncertainty*. Sometimes objective probabilities, say p_j, are known for all events E_j, $j = 1, \ldots, n$. Then the prospect generates a probability distribution $(p_1{:}x_1, \ldots, p_n{:}x_n)$ over the outcomes, which is then identified with the prospect. Such situations are designated as *decision under risk*, and they are a special case of decision under uncertainty.

Some decision makers maximize EU: $\sum_{j=1}^{n} \mathbb{P}(E_j)U(x_j)$, where $U(.)$ is the *utility function* and the $\mathbb{P}(E_j)$'s are (subjective) *probabilities*. Then $\mathcal{X} \succeq \mathcal{Y}$ if and only if $\sum_{j=1}^{n} \mathbb{P}(E_j)\left[U(x_j) - U(y_j)\right] \geq 0$. A crucial property of this formula is that probabilities are processed in a linear manner. They need not be objective probabilities, but may instead reflect subjective judgments of the decision maker. If objective probabilities are common knowledge then the subjective probabilities agree with them (under mild assumptions), and we often suppress the events E_j, writing p_j for $\mathbb{P}(E_j)$.

There exists much empirical evidence against EU (Kahneman and Tversky, 1979). For instance, consider the commonly found Allais paradox: $3000 \succ (0.8{:}4000, 0.2{:}0)$ and $(0.25{:}3000, 0.75{:}0) \prec (0.20{:}4000, 0.80{:}0)$. Under EU, and the common scaling $U(0) = 0$, the former preference implies $U(3000) > 0.80U(4000)$ and the latter preference implies $0.25U(3000) < 0.20U(4000)$, that is $U(3000) < 0.80U(4000)$, contradicting the former inequality; EU is falsified. Thus, there is a descriptive interest in alternative models, so-called non-expected utility models. According to some researchers there is also a normative interest in such models. Researchers who consider EU to be normative will be interested in deviations so as to correct for these when determining optimal behavior in prescriptive applications.

Psychologists primarily argued convincingly that descriptive attitudes towards risk and uncertainty should not be (merely) modeled through nonlinear U (the "psychophysics" of money), but also through nonlinear functions depending on the events E_j and the probabilities p_j. This led to an evaluation (E:x, $\bar{E}$:0) $\rightarrow W(E)U(x)$ where, as usual, $U(0) = 0$, and $\bar{E}$ denotes the complementary event, not-E. $W(A \cup B) \neq W(A) + W(B)$ is allowed (even when $A \cap B = \varnothing$), so that W can be nonlinear, and this nonlinearity can reflect a variety of attitudes towards uncertainty and risk. For the special case of risk, W is a transformation w of probabilities, and we get $(p{:}x, 1-p{:}0) \rightarrow w(p)U(x)$ with w nonlinear, strictly increasing, and $w(0) = 0$, $w(1) = 1$.

How to apply W and w to multiple-outcome prospects $(E_1{:}x_1,\ldots,E_n{:}x_n)$ was not clear for a long time. The often-used formula $\sum_{j=1}^{n} w(p_j)U(x_j)$ and its analogue for uncertainty turned out not to be sound because they imply violations of stochastic dominance in preferring less money to more money in manners that are not only normatively but also descriptively unwarranted.

One of the key ideas in risk and uncertainty, the idea of *rank dependence*, was advanced independently by Quiggin (1982) and Schmeidler (1989). It shows a natural way to turn the valuable concepts of w and W from the psychological literature into a theory sound enough for economists to use. First the outcomes and events of a prospect are renumbered so that $x_1 \geq \cdots \geq x_n$, and then its valuation is

(1) $$\sum_{j=1}^{n} \pi_j U(x_j),$$

with the *decision weights*

(2) $$\pi_j = W(E_1 \cup \cdots \cup E_j) - W(E_1 \cup \cdots \cup E_{j-1}),\ j = 2,\ldots,n \text{ and } \pi_1 = W(E_1).$$

For risk, we have $\pi_j = w(p_1 + \cdots + p_j) - w(p_1 + \cdots + p_{j-1})$. Although this formula is more complex than the ones presented above, it can be seen to be a natural way to model attitudes towards probabilities and events and it is the most popular non-expected utility model, in combination with its extension to prospect theory explained next.

Besides attitudes towards uncertainty and risk, the different perception of gains and losses is another major phenomenon deviating from EU. Assume that $x_1, \ldots, x_k$ are gains (≥ 0) and $x_{k+1}, \ldots, x_n$ are losses (< 0). Then the above evaluation becomes

(3) $$\sum_{j=1}^{k} \pi_j U(x_j) + \lambda \sum_{j=k+1}^{n} \pi_j U(x_j),$$

where $\lambda > 1$ (referred to as *loss aversion*) generates bigger sensitivity towards losses than towards gains. (Note that $U(x_j) < 0$ if $j > k$ since $U(0)=0$.) In marketing and many other domains, it is well known that people are especially sensitive to whether outcomes are gains or losses.

Cumulative prospect theory further allows for different probability weighting for gains than for losses, a generalization that we ignore here. The empirical separation of λ from U is a subtle issue, depending on observations with different reference points and assumptions of U at 0, topics that we will not elaborate on here. The effects of loss aversion are strong but volatile, strongly influenced by seemingly minor changes in framing. Empirical studies suggest that often $\lambda \approx 2$. Thus, λ enhances risk aversion, to the effect that the major part of empirically observed risk aversion may be driven by loss aversion.

The sensitivity towards probability and uncertainty, through w and W, constitutes a new and essential component of risk attitude that was missing in classical theories. Empirical studies into the nature of w for risk suggest that w often underweights probabilities ($w(p) \leq p$), which can be seen to enhance pessimism and risk aversion. Another prevailing phenomenon is the inverse-S shape, with w overestimating low probabilities and underestimating high probabilities. This explains the coexistence of gambling (risk seeking for long shots) and insurance, which was a major paradox in classical theories. It suggests no aversion, but rather, lack of understanding and sensitivity towards probabilities.

The phenomena just described for risk also occur for uncertainty, but to a more pronounced degree. Ellsberg considered a known urn with 50 red and 50 black balls, and an *unknown urn* with 100 red and black balls in unknown proportion. People prefer to receive \$100 ("bet") on event R_k of a red color drawn randomly from the known urn than to bet on the similarly defined event R_u. They would also rather bet on B_k than on B_u. Applying equation (1) shows that $W(R_u)\text{U}(100) < w(0.5)\text{U}(100)$, i.e., $W(R_u) < w(0.5)$ and, similarly, $W(B_u) < w(0.5)$. Thus, W is systematically lower than w. Note that this finding rejects EU because, under EU, W is a probability measure and w is the identity, and then $W(B_u) + W(R_u) = 1$ so that at least one must exceed 0.5. Such phenomena, where uncertainty shows characteristics fundamentally different than risk, are described as *ambiguity* attitudes, the most intensively investigated topic in decision under uncertainty today. In general, the less familiar we are with events, the more W deviates from linearity. This underlies the *home* bias, where people invest more in home stocks than in foreign stocks. The most popular alternative to the rank-dependent model for the study of ambiguity is the *multiple-priors model* by Gilboa and Schmeidler (1989).

The phenomena of probability weighting through W or w, and loss aversion through λ, are important also if we are merely interested in measuring U. We can only understand U if we can understand uncertainty attitudes and then we have to know about W, w, and λ. Classical measurements of utility have usually assumed EU, but then the measurements of U are distorted by the existing but ignored effects of W, w, and λ. This has led to a general overestimation of the concavity of utility.

A major problem for the application of non-expected utility models concerns their implementation in dynamic decisions, for which no consistent method seems to exist. This constitutes a strong normative argument in favor of expected utility (Machina, 1989).

3. Incorporating EU and Non-EU in Discrete Choice Models

We next consider the embodiment of the theories from the previous section in the framework of econometric discrete choice models. We discuss estimation issues with (observed and unobserved) heterogeneity, either in preferences or in perceptions, with a special focus on the benefit from using panel data.

We explicitly recognize that preferences and perceptions may vary across individuals, indexed by *i*. Individual preferences are imbedded in the individual-specific utility function $U_i(.)$, and perceptions may also be individual-specific. The expected value of a prospect $\mathcal{X}$ for individual *i* then becomes

(4) $$\sum_{j=1}^{n} \pi_{i,j} U_i\left(x_j\right),$$

where $\pi_{i,j} = w_i\left(p_1 + ... + p_j\right) - w_i\left(p_1 + ... + p_{j-1}\right)$ denotes the decision weight, as perceived by individual *i*. In the EU framework, $w_i(.)$ is the identity and p_j is known.

A review follows of the different ways of modelling differences across individuals, through observed and unobserved heterogeneity in preferences and/or in perceptions, either in parametric or non parametric models. There is generally a trade-off between the flexibility of functional forms for utility and/or probability weighting functions, and the degree of heterogeneity which can be taken into account. In the RUM framework under EU or Non-EU, a decision-maker is assumed to select the prospect $\mathcal{X}_k$ which maximizes the value of the prospect, modelled as

(5) $$\psi\left(X_k; \beta_i\right) + \sigma\varepsilon_{ik},$$

where X_k denotes the vector of attributes of prospect $\mathcal{X}_k$, β_i is a vector of associated parameters reflecting both preferences and probability weighting, ε_{ik} is a residual reflecting unobserved heterogeneity with some standard distribution (e.g., normal or double exponential), and σ^2 is the variance of the residual. The probability that individual i selects prospect $\mathcal{X}_k$ is therefore equal to the probability that $\varepsilon_{ik'} - \varepsilon_{ik} < \frac{\psi(X_k;\beta_i) - \psi(X_{k'};\beta_i)}{\sigma}$ for all k' different from k. In the binary case, the probability that individual i prefers 1 to 0 is

(6) $$\Phi\left(\frac{\psi(X_1;\beta_i) - \psi(X_0;\beta_i)}{\sigma}\right),$$

where $\Phi(.)$ denotes the c.d.f. of $\varepsilon_{i0} - \varepsilon_{i1}$.

In the linear specification under EU, $\psi(X_k;\beta_i) = X_k\beta_i$, and the vector β_i measures marginal utilities. It is indexed by i to take heterogeneity of preferences into account. The observed heterogeneity of preferences can be captured through covariates. In that case, $\beta_i = Z_i\gamma$, so that $X_k\beta_i = (X_kZ_i)\gamma$, where Z_i is a matrix of individual characteristics and γ is the associated coefficient vector to be estimated. In their application to drivers' route choice, Lam and Small (2001) consider the following attributes of prospects: cost, expected travel time, and variability of travel time measured by the variance (or standard deviation) of travel time. (In this example the attributes are also specific to individuals, but this does not affect the econometric analysis.) Risk aversion (with respect to travel time, not to monetary cost) is then measured by the ratio of their respective coefficients. The Mean-Variance model used by Lam and Small (2001) was consistently derived from CARA (Constant Absolute Risk Aversion) and log-normal distribution of outcome by Markowitz. The consistency of this model with behavioral theories presented in section 2 is discussed in de Palma and Picard (2006) in the context of route choice. See also Avineri and Prashker (2005) for the use of Non-EU to model route choice.

An alternative solution, always consistent with behavioral theories, consists of explicitly writing expected utility as in (4), with possibly a more flexible functional form. For example, Holt and Laury (2002) consider a power-exponential utility function under EU, and find a decreasing risk aversion.

The random terms ε_{ik} in (5) are generally assumed independently and identically distributed (i.i.d.) across individuals. They reflect specification errors, omitted factors, non-observable factors, and unobserved heterogeneity of preferences (or heterogeneity not modelled in β_i).

In the simplest and most convenient model for estimating (5), the Multinomial Logit Model, the ε_{ik} are assumed i.i.d. double exponential. The main flaw of this model (when there are more than 2 alternatives) is the Independence from Irrelevant Alternatives property. If this property is not met in a given data set, it is possible to use less restrictive models such as Nested Logit or Multinomial Probit or Ordered Probit (see Small, 1987). In the latter model, consider the choice among increasingly risky alternatives and denote by θ_i individual *i*'s risk aversion (see de Palma and Picard, 2005). In this case, (5) is replaced by

(7) $$F\left(\theta_i\right) = X_k \beta_i + \varepsilon_{ik},$$

where F is an increasing function to be estimated. The estimation of the distribution of θ_i then relies on stochastic dominance and ordinal representation of preferences. The idea is that, whatever their preference and probability weighting functions, all respondents should agree on the ranking of some prospects. Based on this ranking, the most risk-averse individuals choose the least risky prospects. If $\mathcal{X}$ is more risky than $\mathcal{Y}$ and individual *i* is indifferent between $\mathcal{X}$ and $\mathcal{Y}$, then another individual *i'* prefers $\mathcal{X}$ to $\mathcal{Y}$ if and only if *i'* is less risk averse than *i*. The model developed by de Palma and Picard (2005) allows determining both ordinal (consistent with EU and Non-EU) and cardinal representations of individual risk aversion.

When respondents face multiple choice occasions, which is generally the case in experimental economics, one should question the assumption (too often implicit) that the ε_{ik} are i.i.d. across choice occasions for the same individual. Indeed, according to the way they are interpreted, the ε_{ik} may be assumed either individual-specific (and therefore perfectly correlated across choice occasions) or specific to the question (in this case, independence across choice occasions is acceptable). We suggest the use of panel data techniques for dealing simultaneously with both cases.

In the spirit of Hey and Orme (1994), assume that an experimental subject faces a sequence of T independent binary choices, and let d_t = 1(-1) if the subject chooses the first (second) prospect in problem t, t=1…T. The likelihood contribution for a single subject corresponds to the probability of the series of choices made by the subject, and is of the form (see (6))

(8) $$\prod_{t=1}^{T} \Phi\left(d_t \times \frac{\psi\left(X_{1t};\beta_i\right) - \psi\left(X_{0t};\beta_i\right)}{\sigma} \right),$$

where β_i denotes a vector of individual-specific parameters reflecting preferences and/or weighting function. In order to allow the preference parameter and weighting parameters to vary across the population, the likelihood contribution for a single subject must become

(9) $$\int_{\beta} \prod_{t=1}^{T} \Phi\left(d_t \times \frac{\psi\left(X_{1t};\beta\right) - \psi\left(X_{0t};\beta\right)}{\sigma} \right) f\left(\beta\right) d\beta,$$

where $f\left(\beta\right)$ is the assumed joint probability density function of the vector of parameters. Of ultimate interest are the parameters of this joint density function. Examples of these parameters are given in the next section.

The presence of the multivariate integral appearing in (9) clearly requires the use of simulation methods in order to maximize the sample log-likelihood, as described, for example, by Train (2003).

4. Specification and Estimation of Weighting Function

In this section we focus on the parametric specification and on the estimation of the weighting function that are embedded in the β vector, which has to be estimated (see (6) and (9)).

We focus on the case in which outcomes are positive and probabilities are known so that we are considering the weighting function *w(p)*. A prevailing phenomenon is the inverse-S shape in *w(p),* with small probabilities (of the best outcome) overestimated, and large probabilities underestimated. When *w(p) = p*, the function coincides with the 45°-line and we have EU. Three parametric functions that appear in the literature are specified below:

$$(10) \qquad \begin{cases} \text{Power:} \quad w(p) = p^{\gamma} & \text{, with } \gamma > 0 \\ \text{Quiggin:} \quad w(p) = \dfrac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{1/\gamma}} & \text{, with } \gamma > 0.279 \\ \text{Prelec:} \quad w(p) = \exp\left(-\alpha\left(-\ln p\right)^{\gamma}\right) & \text{, with } \alpha > 0; \gamma > 0. \end{cases}$$

The first of these, the power weighting function, might be seen as undesirably restrictive since it does not allow an inverse S-shape; it is either completely above (if γ<1) or completely below (if γ>1) the 45° line. The second function is due to Tversky and Kahneman (1992). While this function only has one parameter, it has the required inverse-S shape (if 0.279<γ <1), crossing the 45°-line at a point that depends on the value of γ. The lower limit of γ is required for monotonicity. The third function is due to Prelec (1998) and has two parameters. When both of these parameters are equal to one, we have EU. The parameter α reflects pessimism and the parameter γ determines the pronouncedness of the inverse-S shape. Econometric work tends to find both parameters to be somewhat less than one.

Note that all of the functions considered above are continuous functions of *p*. Here, we would like to consider a discontinuous weighting function. We conjecture that the discontinuities occur at *p* = 0 and *p* = 1. The simplest possibility would be

(11) $$\begin{cases} w(0)=0 \\ w(p)=b+(1-a-b)\,p \quad (\text{for } 0<p<1, \text{with } a,b\geq 0; a+b<1) \\ w(1)=1. \end{cases}$$

That is, a probability of 0 for the best outcome is correctly interpreted, but as soon as the true probability becomes positive, the perceived probability jumps to *b*. The weighted probability then rises linearly with *p*, until *p* reaches 1, when the weighted probability takes another discrete jump of *a*, resulting in a probability of 1 being correctly interpreted. Note that if $a = b = 0$, we have EU.

The rationale for considering such a weighting function is that much experimental evidence suggests that there is a discrete shift in behavior when the probability of the best outcome (and to a lesser extent the worst outcome) changes from zero to a small positive number. There is also evidence from real life: people take part in public lotteries presumably because they significantly over-weight the miniscule probability of winning the jackpot. Such degrees of overweighting may not be possible for a continuous weighting function passing through the origin. This sort of weighting function has been used in theoretical contexts, for example by Chateauneuf et al. (2007).

To the best of our knowledge, only one paper has tested such a function econometrically: Loomes et al. (2002). They find the parameter *a* to be insignificantly different from zero, but the parameter *b* to be large in magnitude. In fact, they allow this parameter to depend on task experience and find that it is 0.202 at the start of the experiment, decaying to 0.118 by the end. This implies significant over-weighting of low probabilities, even with experience.

One restrictive feature of the model of Loomes et al. (2002) is that all subjects are assumed to have the same weighting parameters *a* and *b*. Realistically, we would wish to allow such parameters to vary across the population and, indeed, to allow a proportion of the population to obey EU. Estimation would proceed using the techniques introduced in section 3.

5. Identification and Dynamics

Experiments in the lab and in the field are not the main source of data in economics. It is much more frequent to construct data from surveys. In these non-experimental or observational data, there are no variables under the control of the observer. As a substitute, economists look for frameworks and assumptions under which parameters governing behavior can be identified. Exogenous variation in some variables or natural experiments is a well known instance that can lead to identification, although it is neither a necessary nor a sufficient condition. Typically, in experimental economics, the number of subjects is very small, but the number of choice occasions is large compared to survey data. If the number of choice occasions is large enough, it may be possible to estimate individual-specific parameters using experimental economics data. This is less easy, or impossible, with survey data, but it is then easy to estimate some distribution of the parameters in the population and the potential dependence of these parameters on individual characteristics.

Data on choices, observed over time in panel surveys, can be used to measure risk attitudes. Arguably, agents are assumed to know with certainty what are the costs and payoffs of their actions today, but have risky, uncertain or ambiguous beliefs about costs and payoffs tomorrow. The identification of agent preferences could be provided by the restriction that agents are rational and forward looking. They decide about choices in the current period, depending on the implications for choices and welfare in the next period in terms of constraints, benefits and costs.

Is this restriction strong enough to identify risk attitudes and behavior vis-à-vis uncertain or risky prospects? In discrete choice frameworks, McFadden (1981), such as choosing lottery A against B, or self-employment versus wage work, the answer is negative in the absence of strong assumptions (Rust, 1994). The framework needs to be tightly constructed before one could get to the parameters of interest. One example is standard dynamic discrete choice. Decisions are assumed to be taken using expected utility. Expectations about future events are perfect and are equal to the probabilities that can be constructed in the data. Subjective probabilities are, in consequence, equal to objective probabilities. Intertemporal utility is additive and the discount rate is fixed. The distribution of errors is known. Then, parameters relative to risk aversion are identified, as shown by Magnac and Thesmar (2002). Other modeling set-ups can be used, for instance, by weighting the probabilities of future events differently, although the assumptions required for identification remains the same. Furthermore, no testing procedure of one decision framework against another is available.

One crucial piece of information distinguishes experimental from observational data. In experiments involving risky situations, knowledge of the probability of events is assumed and these probabilities can vary. In observational data, subjective probabilities about future events are unknown. Some assumptions have been postulated on the relationship between what is observed in reality a few years later (objective probabilities about the future) and what the agents expect (subjective probabilities). It is only recently that survey questions have been asked about probabilities of future events; see Manski (2004) for a review. Heckman and Navarro (2007) have shown that restrictions among subjective probabilities and external information could also be used. Both approaches should bring forth the possibility of constructing tests using observational data to compare EU and Non-EU frameworks.

One should anyhow address the delicate issue of the relationship between subjective and objective probabilities, an example of which is most easily seen using continuous choice data. Indeed, Non-EU theories have been used in consumption studies (for instance, Vissing-Jorgensen and Attanasio, 2005). It is not without complication because of the presence of macroeconomic shocks in most economic data. Specifically, macroeconomic shocks are important in consumption or welfare studies since they are not insurable, even if the structure of markets is complete. The attitudes towards risk with respect to these shocks are, thus, of crucial importance in macroeconomics. If macroeconomic shocks, assumed stationary, do not enter linearly or multiplicatively in preferences, then the researcher needs to have a long period of observation to estimate the parameters of interest (Chamberlain, 1984). Otherwise, one has to assume that the distribution of those shocks is known to agents.

The clarification of the identification of risk attitudes using observational data is thus high on the agenda for future research.

6. Stylized findings in selected contexts

The original experiments that showed violations of expected utility and other choice theories often used large *but hypothetical* payoffs, e.g., Kahneman and Tversky (1979). This naturally led many investigators to wonder how subjects would behave with payoffs based on real cash. In some cases, the use of real money payoffs has not altered the bias, such as in the case of the classic preference reversal experiments done by Grether and Plott (1979) where reversals were at least as clear with financial incentives as without. In other cases, financial incentives in the lab have resulted in more rationality in observed behavior. For example, consider a "probability matching" experiment in which a subject is rewarded for correctly guessing which of two lights will illuminate in a series of random trials. If one of the lights is illuminated with probability 0.75, then the optimal decision is to guess that light 100% of the time as soon as the subject has obtained experience sufficient to ascertain that it is more likely. A common result with hypothetical payoffs is for subjects to select the more likely light three-fourths of the time, when they are not financially motivated, such as when they are told to "do your best." This result perplexed many observers since such probability matching was much less common among rats and other animals making binary choices in random trials. The resolution of this paradox is that it is not possible to tell a rat to "do your best"; in fact, the animals in these experiments were motivated by food or liquids. Non-optimal probability matching is also less common among human subjects who are financially motivated (see the references in Holt, 2006, chapter 27).

This section reports a simple experiment that was done to assess the motivation for cumulative prospect theory which, as noted in section 2, can explain the standard Allais paradoxes, but which uses a probability weighting function that preserves stochastic dominance. The objective was to determine whether violations of stochastic dominance were as common as Allais paradox violations of expected utility. The subjects were University of Virginia students who were also participating in a series of auction experiments in the summer of 2007. After finishing the auction part, they logged onto a web-based interface that presented them with three paired lottery choices (veconlab.econ.virginia.edu/login.htm for subjects and veconlab.econ.virginia.edu/admin.php for administrator setup). The three decisions are shown in table 1; these were actually presented in random order, with one of them being selected ex post to determine actual earnings by playing out the lottery selected in that case.

(Table 1. Three Choice Problems)

Decision 1 is a choice between a certain \$3 and a risky prospect that has a higher expected payoff. Note that the prospects in Decision 2 are obtained from the prospects in Decision 1 by multiplying the probabilities in each option by ¼ and adding a ¾ chance of \$0.00 to each option. (This corresponds to compound lotteries.) According to EU, this linear transformation should not change the choice, i.e., those who choose Option A in Decision 1 should also choose Option A in Decision 2. The normal pattern of violation, A in Decision 1 and B in Decision 2, is explained by the observation that the probability of 0.8 for the \$4 payoff in Decision 1 is underweighted, but in Decision 2 the probabilities for the positive payoffs, 0.25 for Option A and 0.2 for Option B are so close that probability weighting has no real effect. Note that prospects in Decisions 1 and 2 are the same as in the Allais paradox described in section 2, except that amounts are divided by 1000.

Note also that Option A in Decision 3 stochastically dominates Option A in Decision 1, so that a person who chooses Option A in the first decision should also choose Option A in the third decision. The motivation behind this treatment was that replacing a certainty with a lottery would make Option A less attractive, even though the lottery version would stochastically dominate the certainty. Of the 36 subjects, about one third (13) exhibited the standard Allais paradox by selecting A in Decision 1 and B in Decision 2. Only 2 of the 36 exhibited the reverse violation (B in Decision 1 and A in Decision 2), and only 3 of the 19 subjects who chose A in Decision 1 turned around and chose B in Decision 3. A second group of 36 subjects was given virtually the same three choices, but with the $4.00 payoffs replaced by $4.20, and the results were almost identical, with 17 exhibiting a normal Allais paradox, 2 exhibiting the reversed violation, and 3 violating stochastic dominance. All together, 40 of the 72 subjects selected the safe Option A in Decision 1, but only 8 of the 72 selected Option A in Decision 2.

It is useful to view these results in terms of the issues raised in the introduction, i.e., whether behavioral patterns can be explained by random utility models (consideration of "errors" and other un-modelled random shocks) or whether some consideration of Non-EU approaches is needed. If the only relevant factor were the presence of random elements in choice, then the choice proportion in option A would have reduced from 40/72 to something closer to one half, 36/72. The very low incidence of A choices with scaled-down expected payoffs in Decision 2 suggests that a Non-EU approach is called for to explain behavior for a significant fraction of the subjects. The small incidence of violations of stochastic dominance suggests that there are some "errors", in the sense that behavior is inconsistent with both expected-utility and Cumulative Prospect Theory. It is natural to suspect that violations of stochastic dominance are due to errors or "trembling." This view is consistent with the results of another experiment done by Conte, Hey and Moffatt (2007), who estimate a model that allows "trembles". They report a tremble rate of about 2%. This suggests that a behaviorally relevant theory should be based on non-expected theory for a large fraction of the subjects, but random-utility theories and models with trembles have a role to play, as well.

The risk aversion that is apparent for the majority of subjects in Decision 1 is consistent with other results in the literature, but the experiments have shown that the incidence of risk aversions rises dramatically when real cash payoffs are scaled up by factors of 20, 50, and 90 (see Holt and Laury, 2002, and the references therein).

7. Recommendations

Risk and ambiguity are common features of many choice situations. Although this is a fertile area for future research, we have learned a number of things that should be implemented in choice models involving risk. The most obvious recommendation is to recognize that risk aversion is a ubiquitous response to risky choice situations. This implies that just entering the expected value of a risky gamble as a proxy for the certainty equivalent is very likely to be a large misspecification. In some applications (e.g., transportation route choice, Brownstone and Small, 2005) it may be possible to control separately for the mean and variance of the risky gamble. Alternatively, the sample survey can sometimes be supplemented with stated preference questions to directly elicit respondents' risk aversion.

If the probability of the risky outcomes is near zero or one, then choice modelers should also account for probability weighting by survey respondents. The tendency for respondents to overweight small probabilities and underweight large ones can be mitigated by unweighting using a weighting function from other studies (e.g., Tversky and Kahneman, 1992, see section 2), or an individual-specific weighting function elicited using stated preference methods. This latter method is preferred since we do not have much evidence on the stability of weighting functions across individuals or choice situations, but survey time and complexity considerations may preclude this approach.

Although there is compelling evidence that people do not respond to risky prospects as predicted by standard EU theory, it is not at all clear that we should stop using EU theory to evaluate public policy alternatives. When given enough opportunity to learn about the consequences of Non-EU decision making, most people switch to EU behavior. The Non-EU theories should be used to remove biases in responses to unfamiliar choice situations, but these theories should not be used for normative policy analysis.

Although current models used to explain respondent's choices in experimental situations involving risk may yet be falsified by more complex experiments, these models clearly dominate the standard Von-Neumann-Morgenstern EU model. Before these rank-dependent utility models can be recommended for routine use (see sections 2 to 4), more research is needed in two key areas. We know that a large majority of the general population (or at least the population of research university undergraduates) evaluate risky prospects relative to a reference point, but we do not know enough about how these reference points are set and how they change as respondents gain experience with repeated risky choices under similar conditions.

Similarly, we know that most respondents weigh losses more heavily than gains relative to the reference point, but we do not know much about how these relative weights vary across the population or across choice situations for the same individual. Hopefully further research will find ways to characterize the variability of reference points and probability weights as functions of sociodemographic and choice situation attributes.

Ambiguity is a less-studied problem since there are fewer accepted theories to guide empirical work. In many real applications ambiguity aversion may be at least as important as risk aversion. Without strong additional assumptions, ambiguity formally implies that probabilities of risky prospects are only bounded within intervals, so identification of choice models in these situations is problematic. Much more basic theoretical and experimental research is needed in choice situations involving ambiguity.

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Tables

Table 1. Three Choice Problems

	Option A	Option B
Decision 1	\$3.00 with probability 1	\$4.00 with probability 0.8 \$0.00 with probability 0.2
Decision 2	\$3.00 with probability 0.25 \$0.00 with probability 0.75	\$4.00 with probability 0.2 \$0.00 with probability 0.8
Decision 3	\$3.00 with probability 0.50 \$3.20 with probability 0.50	\$4.00 with probability 0.8 \$0.00 with probability 0.2