Risk in Transport investments

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[*] We would like to thank Robin Lindsey as well as an anonymous referee for helpful comments and suggestions.

Abstract

We discuss how the standard Cost-Benefit Analysis should be modified in order to take risk (and uncertainty) into account. We propose different approaches used in finance (Value at Risk, Conditional Value at Risk, Downside Risk Measures, and Efficiency Ratio) as useful tools to model the impact of risk in project evaluation. After introducing the concepts, we show how they could be used in CBA and provide some simple examples to illustrate how such concepts can be applied to evaluate the desirability of a new project infrastructure.

Keywords

Cost-Benefit Analysis, Risk, transportation, large project, Value at Risk, Conditional Value at Risk.

INTRODUCTION

Risk is an important element in all investment decisions. This is particularly true for long lead transport investments.
In recent years, a large number of failures of large projects have been observed. The Suez Canal and Euro Tunnel provide only two among numerous well known examples (see, e.g., Flyvbjerg, Bruzelius and Rothengatter, 2003). One way to avoid such failure is to implement a good and realistic Cost-Benefit Analysis (CBA). Various sources of risk and uncertainty (see Section 1.2 for a detailed distinction between risk and uncertainty) may affect the costs and the benefits of large scale projects. They correspond to, roughly speaking, demand uncertainty, supply uncertainty as well as to various macroeconomic shocks. It is our belief that risk and uncertainty are not well taken into account in current cost-benefit analysis. However, risk and uncertainty affect the major ingredients of the cost-benefit analysis during the whole process of construction, operation and maintenance of a project. Among other things, risk and uncertainty may affect:

- demand (passengers and freight), which is sometimes overestimated (e.g. for urban rail projects in the US)
- production cost (which are often underestimated)
- maintenance costs (which are typically underestimated, especially in developing countries)
- industry structure and regulation (see, e.g., the unexpected crucial impact of the low cost companies in the airline industry)
- market regulation: potential market (i.e. relevant geographical) area and contestability
- execution time (which is typically underestimated to a large extent)
- macroeconomic and regional context (which, of course, have an influence on demand and interest rates)
- interest rate (which -in addition to the macroeconomic and regional context- depends on the level of risk anticipated by the lender and is therefore endogenous)
- other financial variables (which are difficult to predict especially in the case of Public Private Partnerships)
- human resources context (which crucially depends on the quality of the management)
- political environment (which is especially crucial for public infrastructures, both between political parties and between cities or regions)
- evaluation of secondary infrastructures (the main problem is the definition of the area of interest and of the alternative modes taken into account)
- accompanying measures (for example, CBA gives very different results in the railway case for passengers in Europe and in Japan because in Japan the railways draw a large proportion of benefits from accompanying infrastructures such as shopping centers; Japan Railways indeed receive income from rents on land they own and business activity)
- value of time, value of reliability and schedule delay costs (utility is reduced when actual arrival time differs from preferred arrival time)
- value of external costs (accidents, noise, human life, local and global environmental cost)
- scrap value (which is often forgotten).

Since the seminal contribution of von Neuman and Morgenstern (1953), various tools are proposed by a few CBA studies and textbooks (see Pearman, 1983, Drèze and Stern, 1987, Morgan and Henrion, 1990, Boardman et al., 2005 or Mishan and Quah, 2007) to take risk into account. They include the sensitivity analysis (this approach often ignores correlation between the different variables), the expected value of including uncertainty, the analysis of different potential scenarios and Monte-Carlo simulations.

We argue that the Monte Carlo analysis should be used on a more regular basis (see Morgan and Henrion, 1990), and should be based on a careful choice of realistic joint distributions, when the tails matter (which is the case in CBA studies). Indeed, the true distributions generally cannot be approximated in a satisfactory way via parameterized distributions such as Normal or even Extreme value (double exponential). In addition, a limited number of correlation parameters is usually not enough to model the joint distributions. All this means that an analysis of extreme values for the final output of a project is necessary in order to parameterize the relevant figures in CBA analysis.

The first section of this paper discusses the role of risk sharing in the funding of infrastructure and the monetization of risks. The second section deals in more detail with the introduction of risk in cost benefit assessments of transport projects, and with the tools which can be used to evaluate projects in risky environments. The third section presents numerical examples for applying the different tools presented in Section 2, and concludes on the possible divergence of the different criteria.

1. Previous literature and definitions

1.1. Risk sharing and funding of infrastructure

The different dimensions of risk are more or less correlated both from an intertemporal point of view and from a geographical point of view. For example, the demand for freight transportation depends on several factors including macro-economic situation. As a result, demand for transport, which may depend (positively or negatively) on business cycles will obey a complex inter-temporal pattern. Indeed, traffic demand on highway A and highway B, for example, are spatially correlated at a given point in time for two opposing reasons. First, macro-economic fluctuations may affect both highways in a similar way, inducing a positive correlation between the demands on both highways. Second, since some drivers have to choose between the two highways, positive deviations of the demand on highway A (for example as a result of some unobserved marketing campaign) will be associated with negative deviations of the demand on highway B, inducing a negative correlation between the demands on both highways. As a result of these two opposing forces, the correlation between demand on highway A and highway B may be either positive or negative.

The management of the funding of infrastructure (and in particular of infrastructure funds) depends crucially on the way the different facets of risk imbedded in the joint distribution of all relevant variables aggregate in the distribution of the discounted net benefit. We argue that the analysis limited to the first two moments (mean and variance) of this distribution is insufficient to provide useful advice on the best ways to manage infrastructure funds. The optimal (first-best) share of risk requires knowledge of the joint distribution of the discounted net benefits. Two standard arguments have been advocated by several authors and in particular by Laffont and Tirole (1993), to justify the fact that a system with a principal (the fund manager) and one or several agents (operators) may not behave optimally because of asymmetric information, inducing moral hazard (the agents do not perform their task with the optimal effort since its level cannot be observed directly by the principal) and adverse selection (only the “bad guys” are willing to participate). For example, Réseau Ferré de France (RFF), the principal, who owns the rail tracks has much less information about costs and demand, than Société Nationale des Chemins de Fer (SNCF), the agent, who operates the train and manages the tracks. This private information can be used by SNCF in the bargaining process. Moreover, the right incentive (second best) should induce the most efficient operators to be selected (thus solving adverse selection problems) and should have them operate with the optimal level of effort (thus solving moral hazard problems). Therefore, the level of risk that will be borne by each actor will be such that, on the one hand, the “right” operator should be willing to participate (participation constraint condition) and on the other hand that, once he is selected, he will perform his task with the optimal level of effort (incentive constraint condition). The second-best situations are analysed in detail by Laffont and Tirole who use rather simplified assumptions on risk (e.g. the variability is summarized by two states of nature, good and bad). We argue that such simplified assumptions should not be used in the context of infrastructure funding because the distribution tails really matter. In the case of CBA under risk, we therefore wish to use more realistic descriptions of the impacts of risk, taking into account the whole distribution of the net benefits.

1.2. Risk and uncertainty

The distinction between risk and uncertainty is important, although the treatment of this distinction remains very far from the current practice in CBA. A risky environment corresponds to situations in which probability distributions are known, whereas uncertainty corresponds to situations in which probabilities, and possibly the list of potential outcomes, are unknown. In this case, the decision maker should rely on some beliefs, which he can revise once he accumulates new relevant information. The distinction between risk and uncertainty has been extremely well summarized by Keynes (1937):

> *“By ‘uncertain’ knowledge […] I do not merely distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty; nor the prospect of a Victory bond being drawn… Even the weather is only moderately uncertain. The sense in which I am using the term is that in which the prospect of a European war is uncertain, or the price of copper and the rate of interest twenty years hence… About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know*”.

Even though it has been shown that individuals behave differently when they face risk than when they face uncertainty, here we will mainly focus on risk. In the presence of uncertainty, the principle of minimum regret (“MinMax” criterion) introduced by Savage (1954) can be used, since it relies on the set of potential outcomes, but not on their probability distribution. For each scenario, the maximum regret corresponds to the largest difference between the outcome of the better project and the outcome of the worst project. The MinMax criterion then selects the project which minimizes (over all scenarios) the maximum regret (over all projects). A more detailed analysis of choice under uncertainty is beyond the scope of this research.

1.3. Monetization of risk

We will consider here a simple example due to Anand and Nalebuff (1987), which shows that small fluctuations may have potentially a very large impact in the CBA analysis when these shocks are correlated with the macroeconomic context.

We use the following notation: z represents the certainty equivalent of a project with a random yield of Z (per capita). Y represents the (random) GNP (per capita). $U(.)$ denotes the (increasing) utility function of a representative consumer and $\mathbb{E}[.]$ denotes the expectation operator. The certainty equivalent, *z*, is the unique solution of the following equation:

$$\mathbb{E}\left[U\left(Y+Z\right)\right]=\mathbb{E}\left[U\left(Y+z\right)\right].$$

Clearly, when the utility function is linear (risk neutrality), we have: $z=\mathbb{E}[Z]$. When the utility function is concave (risk aversion) and risks are not correlated, we have $z<\mathbb{E}[Z]$.

We now argue that the variances and the covariance (for yield and GNP) play important roles in the measurement of the certainty equivalent. Let $\mathbb{V}[Z]$ denote the variance of the yield and let *Cov(Y,Z)* denote the covariance between yield and GNP. We also introduce the relative risk aversion *RR(x)*, which is defined as *RR(x)=-xU''(x)/U'(x)*. In this case, it can be shown that the certainty equivalent is given by the following relation:

$$z=\mathbb{E}\left[Z\right]-\frac{1}{2}\frac{RR\left(\mathbb{E}\left[Y\right]+\mathbb{E}\left[Z\right]\right)}{\mathbb{E}\left[Y\right]+\mathbb{E}\left[Z\right]}\left(\mathbb{V}\left[Z\right]+2Cov\left[Y,Z\right]\right)$$

Note that, in the standard case, relative risk aversion is positive since agents are risk averse. When *Y* and *Z* are positively correlated (the most likely case), the covariance magnifies the impact of the variability of the yield (negative effect on the certainty equivalent *z*). However, the situation can be qualitatively different (the covariance effect can offset the negative effect of the variability of yield) when *Y* and *Z* are negatively correlated. In that case, if the absolute value of the correlation between *Y* and *Z* is large enough and if the variance of *Y* is large compared to the variance of *Z*, (namely, if $Corr\left[Y,Z\right]<-\sqrt{\mathbb{V}\left[Z\right]/4\mathbb{V}\left[Y\right]}$, we have $z>\mathbb{E}[Z]$.

We provide below a simple numerical example. We assume that the value of the relative risk aversion is *RR(.)=2* and that the per capita income is either *100* with certainty, or *110* or *90* with equal probabilities. We further assume that yield and GNP are negatively correlated (the reader can easily treat the more intuitive case in which yield and GNP are positively correlated). For a negative correlation, we have the following result:

Variable	Scenario 1	Scenario 2
Per capita income Y	100	110 (Prob=½): state H 90 (Prob=½): state L
Yield project Z	0.5 (Prob=½): state H 1.5 (Prob=½): state L	0.5 (Prob=½): state H 1.5 (Prob=½): state L
$(z-\mathbb{E}[Z]) / \mathbb{E}[Z]$	-0.25%	+10%

Table 1 Impact of variance and correlation (RR=2)
Source: Adapted from Anand and Nalebuff (1987)

This example shows that fluctuations in yield are not negligible if they are correlated with the economic performance. When the yield is negatively correlated with the GNP, the value of the project (here measured by z) would be underestimated if the correlation term were omitted. In our numerical example, z is 10% *more* than $\mathbb{E}[Z]$, whereas it would be 0.25% *less* than $\mathbb{E}$ [Z] with deterministic per capita income. With negative correlation, the project has a significant additional social value because it dampens the macroeconomic fluctuations.

2. Risk and investment

2.1. Net Present Value

Before the 1950s, two parameters were reported in the analysis of a project: the time horizon for full refund and the average rate of return of this investment. In the 50's, several authors (see, e.g. Dean, 1951 and, later on, Lesourne, 1972) introduced the concept of Net Present Value (NPV), which measures the discounted value of an investment. It is defined by:

$$NPV\left(S,r\right)=\sum_{t=0}^{n}\frac{S_t}{\left(1+r_t\right)^t}+\frac{R_n}{\left(1+r_n\right)^n},$$

where n denotes the time horizon considered, S_t denotes the net cash flow for year t (S denotes the vector of net cash flows over the time horizon considered), r_t is the interest rate for period t (r denotes the vector of interest rates over the time horizon considered) and R_n is the residual value of investment at period n. The interest rate is denoted by r when it does not vary from period to period. When comparing one project to the *laisser-faire* policy, the project will be valuable if its NPV is positive. When comparing two projects, the project with the largest *NPV* will be preferred. In practice, the cash flow depends on demand and cost, which are uncertain, and there may be some disagreement concerning the value of the interest rate to be chosen. The Commission of European Communities (1992) therefore advises that more risky projects use higher interest rates, although this is not obvious according to Boardman et al (1987). We discuss below better solutions for evaluating risky projects. The idea is to consider the net cash flow as a random variable and define a criterion to compare projects, which explicitly takes into account the randomness of S_t.

In the case of continuous time and constant interest rate, we have:

$$NPV\left(S,r\right)=\int_0^T S_t e^{-rt}dt+R_T e^{-rT},$$

so the derivative is:

$$\frac{\partial NPV\left(S,r\right)}{\partial r}=-\int_0^T tS_t e^{-rt}dt-TR_T e^{-rT}.$$

Usually, this derivative is negative, but it may be positive in some cases, when S_t is negative at some periods. Indeed, in Figure 1, the NPV is locally decreasing at the relevant interest rate r*, but it is increasing between the middle of segment [A,B] and the middle of the segment [B,C]. Note that, in this example, the NPV is positive when the interest rate is less than A or between B and C (and therefore the project is valuable), but it becomes negative when r>C or A<r<B (and the project is not valuable according to this criterion).

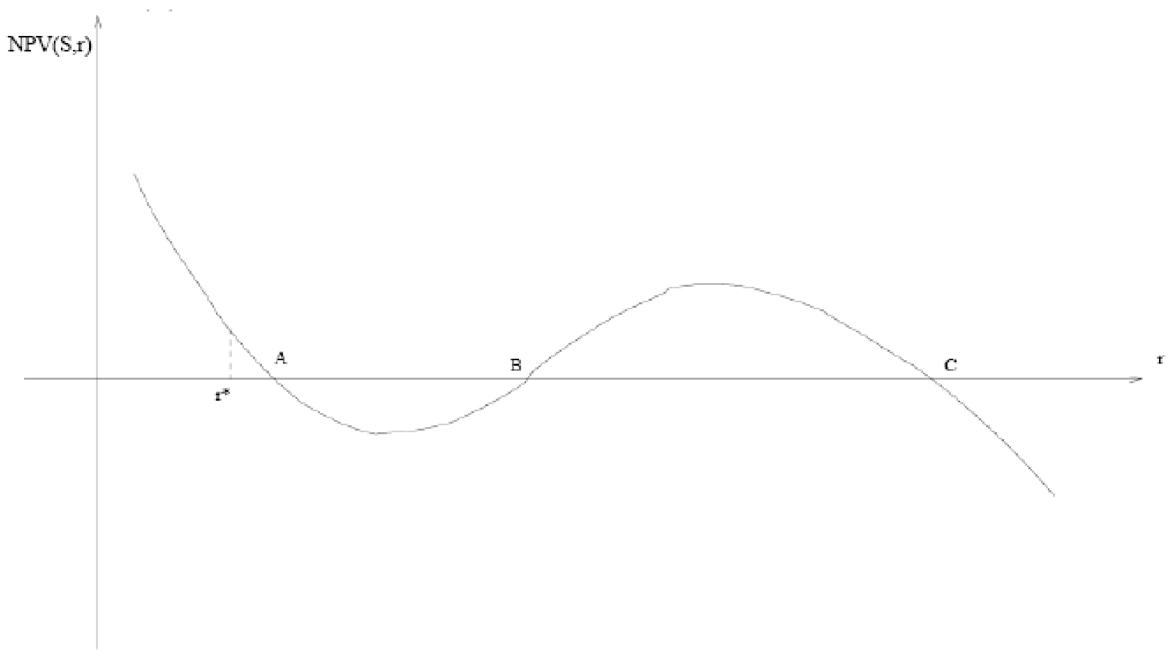

Figure 1 Sensitivity of NPV to interest rate

The (endogenous) interest rate is given by the following formula (see, e.g., Gollier, 2001):

$$r_t = \delta + \gamma m(t) - 0.5\gamma^2\sigma^2, \qquad (1)$$

where δ denotes the impatience rate, γ denotes the *constant* level of the relative risk aversion parameter, *m(t)* denotes the average growth rate of the GNP per capita over the period considered and σ represents the standard deviation of the growth rate of the GNP per capita. In the following example, we assume that $\delta = 2\%$, $\gamma = 4$, $m(t)=2\%$. We also assume that σ is small enough that the last term in (1) can be omitted. Therefore, we get an interest rate of *10%*. Note, however, that the interest rate used in France, for example, in the cost-benefit analysis is lower: the "rapport Boiteux 2" recommended *8%* in 2001 (see Commissariat Général du Plan, 2001). The rate recommended 4 years later is even lower (see Ministère de l'Equipement, des Transports et du Logement, 2005): from 4% for the short term to 3% for the very long term (100 years).

The simplest criterion for comparing two risky projects is the Mean-Variance model. Modern portfolio theory models an asset return as a random variable, and models a portfolio as a weighted combination of assets; the return of a portfolio is thus the weighted combination of the assets returns. Moreover, a portfolio return is a random variable, and consequently has an expected value and a variance. The model assumes that investors are risk averse. This means that given two assets that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. The exact trade-off will differ between investors, based on individual risk aversion level. The implication is that a rational investor who will not invest in a portfolio if a second portfolio exists with a more favourable risk-return profile, i.e. if for that level of risk an alternative portfolio exists which has better expected returns.

In CBA, this criterion can be seen as the expected utility of the *NPV* when *NPV* has a Gaussian distribution and utility is of the CARA (negative exponential) form. The expected value of *NPV* is:

$$\mathbb{E}\left[NPV\left(S,r\right)\right]=\sum_{t=0}^{n}\frac{\mathbb{E}\left[S_t\right]}{\left(1+r_t\right)^t}+\frac{\mathbb{E}\left[R_n\right]}{\left(1+r_n\right)^n},$$

while its variance is:

$$\mathbb{V}\left[NPV\left(S,r\right)\right]=\sum_{t=0}^{n}\frac{\mathbb{V}\left[S_t\right]}{\left(1+r_t\right)^{2t}}+\frac{\mathbb{V}\left[R_n\right]}{\left(1+r_n\right)^{2n}}+2\sum_{t\neq t'}\frac{Cov\left[S_t,S_{t'}\right]}{\left(1+r\right)^{t+t'}}+2\sum_{t=0}^{n}\frac{Cov\left[S_t,R_n\right]}{\left(1+r\right)^{t+n}}.$$

In the Mean-Variance model, the value of a project for a decision-maker is measured by $\mathbb{E}[NPV(S,r)]-\gamma\ \mathbb{V}[NPV(S,r)]$. Recall that γ denotes the absolute risk aversion of the decision-maker. A higher value of γ means that the decision-maker is less willing to take risk.

2.2. Stochastic order and utility functions

When the *NPV* is stochastic, we have seen that the expected value and variance provide valuable information to rank projects. Although the mean-variance model has been widely used in finance, it has been criticized in the transport literature (see, e.g., de Palma and Picard, 2006). Indeed, it can be shown that with the Mean-Variance expected utility, an individual (enough risk averse) may prefer to receive X with probability 0.5 and Y>X with probability 0.5, than receiving X with probability 0.5 and Y+ε with probability 0.5 (ε>0). This violates a fundamental principle of expected utility theory (the monotonic property of the utility functions). More general criteria based on first or second order stochastic dominance can be used to rank projects. Let F_i denote the cumulative distribution function (CDF) of the *NPV* corresponding to project *i* and G_i the integral of F_i.

Project *X* is said to First-Order stochastically dominate project *Y* if its CDF is always lower. Mathematically, this means that:

$$X\geq_{DS_1}Y\Leftrightarrow F_X\left(\eta\right)\leq F_Y\left(\eta\right),\forall\eta\in\mathbb{R}.$$

It can be shown that there is a close relationship between First-Order stochastic dominance and the expected utility criterion. Let *U(NPV)* denote the utility of the decision-maker and $\mathcal{I}$ the set of increasing functions on $\mathbb{R}$. We have (see Mas-Collel et al, 1995):

$$X\geq_{DS_1}Y\Leftrightarrow\mathbb{E}\left[U\left(NPV_X\right)\right]\geq\mathbb{E}\left[U\left(NPV_Y\right)\right],\forall U\in\mathcal{I}.$$

Unfortunately, first order stochastic dominance corresponds to a partial order which does not allow ranking projects when the CDF of their NPV cross.

A stronger result can be obtained for risk averse decision-makers. Denote by $\mathcal{CIC}$ the set of continuous (implied by concavity), increasing and concave functions on $\mathbb{R}$.

Project X is said to Second-Order stochastically dominate project Y if its integrated CDF is always lower. Mathematically, this means that:

$$X \geq_{DS_2} Y \Leftrightarrow G_X(\eta) \leq G_Y(\eta), \forall \eta \in \mathbb{R}.$$

It can be shown that there is a close relationship between Second-Order stochastic dominance and the expected utility criterion. We have equivalence between Second-Order stochastic dominance and expected utility for risk averse decision makers:

$$X \geq_{DS_2} Y \Leftrightarrow \mathbb{E}\left[U\left(NPV_X\right)\right] \geq \mathbb{E}\left[U\left(NPV_Y\right)\right], \forall U \in \mathcal{CIC}.$$

When the *NPV* of two projects have the same expectation and the same variance, it is still possible to rank the two projects. It has been observed empirically that, in this case, decision-makers have a greater dislike of distributions, which are thicker on the left than on the right. More precisely, Menezes, Geiss and Tressler (1980) have shown that a project X is preferred to a project Y for all preferences $U(.)$ such that $U'''(\eta) > 0$ for all η if and only if Y has more downside risk than X. Recall (see Menezes et al., 1980) that Y has more downside risk than X if Y can be obtained from X by a sequence of probability transfers which unambiguously shift dispersion from the right to the left (that is from large to small *NPV*) without changing the mean and the variance (MVPT, or Mean-Variance Preserving Transformations). Note that individuals such that $U'''(\eta) > 0$ dislike downside risk, but may be risk averse or risk lovers.

The above discussion suggests that the expected value and variability are not enough to evaluate the distribution of cost or *NPV*, but that the tails of the distribution also matter. The distribution is here described by a density function with argument x corresponding either to *NPV* or to cost. When the decision-maker is interested in the *NPV*, he will focus on the left tail (very low *NPV*), whereas when he is interested in the cost, he will focus on the right tail (very large cost). We now turn to quantitative indicators explicitly focusing on the relevant tail (the one corresponding to the worst events).

2.3. The Value at Risk

The Value at Risk of a project for a probability p (for example *5%*) corresponds to the smallest value (quantile) q such that no more than $p\%$ of the time NPV_X is less than q. If the CDF of the random variable NPV_X is continuous and strictly increasing, then the Value at Risk $VaR_X(p) = q$ satisfies the following equation:

$$\Pr\left(NPV_X \leq VaR_X(p)\right) = p.$$

Recall that the CDF of NPV_X is $F_X(.)$, so that $VaR_X(p) = (F_X)^{-1}(\text{p})$. More generally, when F_X is only non-decreasing and right-continuous, the Value at Risk is:

$$VaR_X(p) = \min\left\{q \mid \Pr\left(NPV_X \leq q\right) \geq p\right\}.$$

Figure 2 illustrates the three potential cases. When F_X is locally continuous and increasing (p_1 case), $VaR_X(p) = (F_X)^{-1}(\text{p})$. When F_X is locally constant (p_2 case), $VaR_X(p)$ is the lower bound on the interval such that $F_X(q)=p$. When there is a discontinuity in F_X (p_3 case), $F_X(q)=p$ has no solution and $VaR_X(p)$ is the lower bound of the interval on the right side of the discontinuity. $VaR_X(p)$ can be interpreted as the NPV_X of the least favourable outcome among the (1-p)% most favourable ones. When $F_X(.)$ is continuous and strictly increasing, $VaR_X(p)$ can also be interpreted as the NPV_X of the most favourable outcome among the p% least favourable ones.

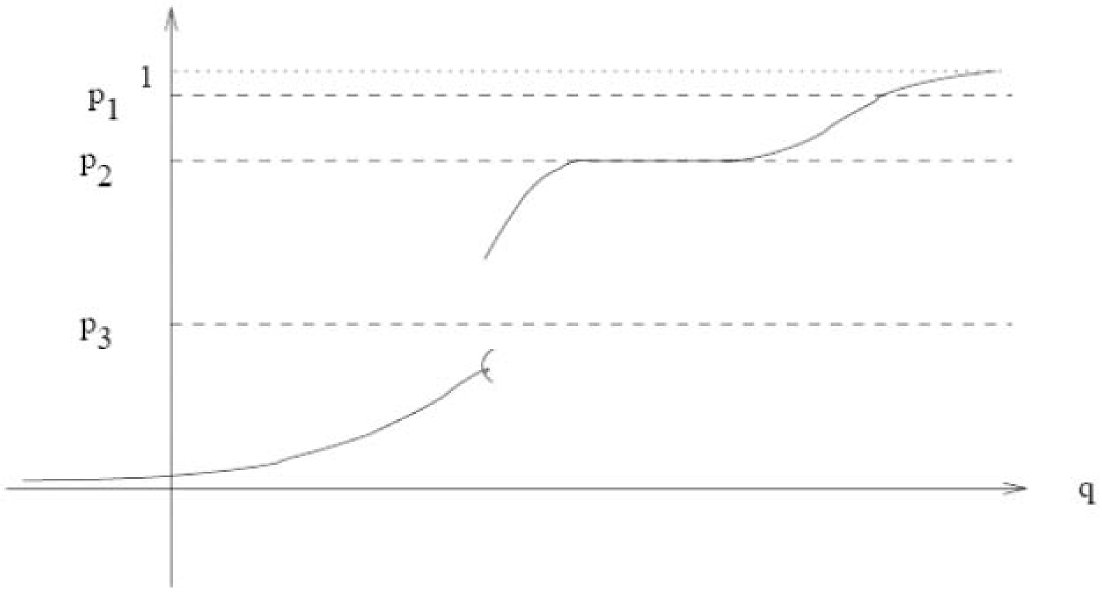

Figure 2 Value at risk and nature of the CDF of NPV

The *VaR* of a project *X* expresses the level of risk acceptable for this project. It depends on two ingredients: the probability distribution of NPV_X and the tolerance level of the decision-maker, which is measured by the probability *p*. Of course, the *VaR* depends on the time horizon of the project as well as on the interest rate which has been chosen. Note that the *VaR* is relevant when one decides to neglect events which are associated with a very low probability (although they may have a dramatic impact on the project) because they correspond to events which cannot be insured and which can therefore be omitted from the current analysis (natural catastrophes, terrorist attack, *coup d'état*).

The computation of the *VaR* requires knowledge of the whole distribution of NPV_X, and therefore of the *joint* distribution of the yearly returns of the project. Three methods can be envisaged for computing the *VaR*:

1. The historical analysis assumes that the distribution of the returns can be inferred from the distribution of the yearly returns observed in the past for similar projects. This does not rely on a parametric distribution of returns. The drawback is that it requires a very large amount of data on similar past projects, so that the tails are significantly represented.
2. The variance-covariance method assumes that the yearly returns are jointly normal, so that the joint distribution of returns can be easily computed. The drawback is that the tails can be very poorly represented under the normality assumption.
3. Monte-Carlo methods involve the computation of a non parametric joint distribution of the returns. It is less restrictive and more realistic than the previous ones. Its drawback is that it requires a very large amount of computations.

Although the *VaR* provides a convenient way of evaluating the risk associated to a project, it has been criticized over several aspects briefly discussed below. First, its value is sensitive to the method used to compute it (see above), and is therefore to some extent subjective. The *VaR* is more popular in finance, since the time scale (daily or hourly returns over many years) allows very large data sets to be used. It is then possible to perform *backtesting*, that is to compare the predicted *VaR* to the actual fraction of very low returns. Second, the information imbedded in the *VaR* is limited in some sense given that it only provides information on the probability of the NPV being lower than some threshold, but does not specify how low the *NPV* is when it is below the threshold. We introduce another measure of risk below, which does not suffer from this second drawback.

2.4. The Conditional Value at Risk

The Conditional Value at Risk of a project *X* for a probability *p* (for example *5%*) corresponds to the expected value of NPV_X conditional on NPV_X being lower than $q=VaR_X(p)$. The Conditional Value at Risk is formally defined by the following equation:

$$CVaR_X\left(p\right)=\mathbb{E}\left\{NPV_X \mid NPV_X \leq VaR_X\left(p\right)\right\}.$$

The *VaR* indicates the *frontier* of the set of the worst events (which may be the most relevant information concerning default risk), whereas the *CVaR* focuses on the *average contents* of the same set. In other words, the *VaR* indicates the limit for an outcome to be considered among the worst ones, whereas the *CVaR* indicates how bad the worst events are (on average).

It is worth noting that the *CVaR* can be obtained as the maximization of a concave one-dimension function. We transpose the result obtained by Rockafellar and Uryasev (2002) from the case of losses to the case of Net Present Value:

Theorem: The *VaR* and *CVaR* of a project *X* for probability *p* can be obtained as:

$$VaR_X\left(p\right)=\arg\max_{\nu\in\mathbb{R}}\left[\frac{1}{1-p}\mathbb{E}\left\{\max\left(NPV_X+\nu,0\right)\right\}-\nu\right].$$

and

$$CVaR_X\left(p\right)=\max_{\nu\in\mathbb{R}}\left[\frac{1}{1-p}\mathbb{E}\left\{\max\left(NPV_X+\nu,0\right)\right\}-\nu\right].$$

Note that the *VaR* and *CVaR* are linked by the following relationship:

$$CVaR_X\left(p\right)=VaR_X\left(p\right)-\frac{1}{1-p}\mathbb{E}\left\{\max\left(NPV_X-VaR_X\left(p\right),0\right)\right\}.$$

The relationship between VaR and CVaR is mathematically clear from this equation. Since VaR and CVaR measure different properties of the distribution (the first one is a quantile and the second one is a conditional tail expectation), we cannot assert for all that that the one is superior to the other one from a risk measures point of view. But CVaR has superior properties for portfolio optimization for instance: since it can be expressed by a minimization formula, it can be incorporated into problems of optimization that are designed to minimize risk or maximize return under risk constraint.

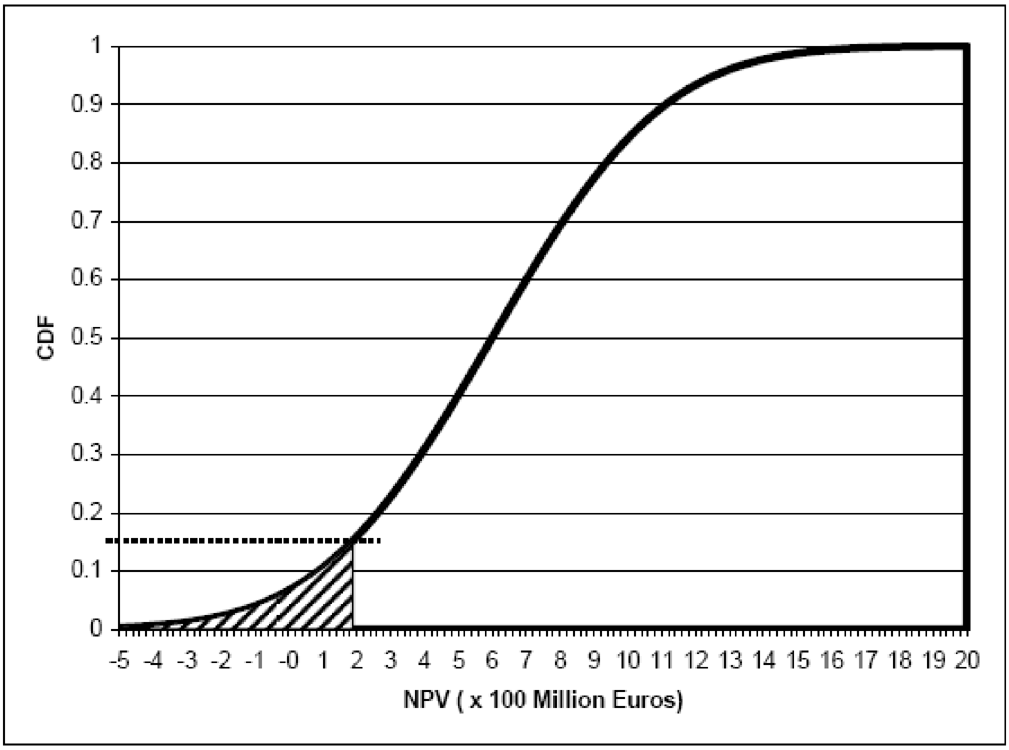

Figure 3 VaR and CVaR of a project X for p=15%

In Figure 3, we represent the CDF of the *NPV* of a project *X* with an expected *NPV* of *600* million € and a standard deviation of *400* million €. We can observe that the *NPV* of this project is smaller than *185* million € *15%* of the time, which means that $VaR_X(0.15) = 1.85$. The *CVaR* corresponds to the expected *NPV* in the *15%* worst cases, that is when *NPV* is less than *1.85*. Assuming a Gaussian distribution, we obtain $CVaR_X(0.15) = -0.22$, corresponding to an average conditional loss of *22* million €.

2.5. Downside risk measures

VaR and CVaR are among the most widely used risk measures for managing financial risk, both by financial institutions and by institutional investors. However, another class of risk measures seems equally important for project management, namely downside risk measures. In this section, $J_A(\xi)$ denotes the random variable measuring the return of a project associated to a decision A and a random variable ξ with density $\phi_A(\xi)$. For each decision A, the distribution of $J_A(\xi)$ is induced by the distribution of ξ. The function $J_A(\xi)$ is assumed to be increasing. The downside risk measures introduced by Bawa and Lindenberg (1977) are based on partial moments:

$$R_\gamma(\theta) = \mathbb{E}\left[\left(\max\left\{\theta - J_A(\xi), 0\right\}\right)^\gamma\right] = \int_{-\infty}^{J_A^{-1}(\theta)} \left(\theta - J_A(\xi)\right)^\gamma \phi_A(\xi)\, d\xi,$$

where $\gamma \geq 0$ and θ denotes a reference return level. When $\gamma \to +\infty$, only the worst outcome matters (see, e.g., Fishburn, 1977). When $\gamma = 0$, $R_0(\theta)$ corresponds to the probability that

$J_A(\xi) < \theta$. The *VaR* therefore corresponds to the case $\gamma = 0$ and $\theta(p)$ such that $R_0(\theta(p)) = p$. When $\gamma = 1$, $R_1(\theta) = \mathbb{E}\left[\left(\max\left\{\theta - J_A(\xi), 0\right\}\right)\right] = \int_{-\infty}^{J_A^{-1}(\theta)} \left(\theta - J_A(\xi)\right)\phi_A(\xi)\,d\xi$ is called *expected shortfall*.

An investor who aims at maximizing his expected utility in the context of downside risk measures will attach a particular importance to the deviations of $J_A(\xi)$ below θ, as measured by $R_\gamma(\theta)$, and he will choose the decision which solves the following problem:

$$R_\gamma(\theta) = \max_A \left\{ \mathbb{E}\left[J_A(\xi)\right] - M\mathbb{E}\left[\left(\left\{\theta - J_A(\xi), 0\right\}\right)^\gamma\right] \right\},$$

where $M \geq 0$ measures the investor's risk aversion. This criterion corresponds to the following class of utility functions:

$$U(x) = \begin{cases} x - M(\theta - x)^\gamma \text{ when } x \leq \theta \\ x \text{ when } x > \theta \end{cases}.$$

For small values of γ, the investor dislikes more small deviations than large deviations below the reference level θ. For large values of γ, the investor mainly dislikes large variations below the reference level, and he is only interested in the minimum of $J_A(\xi)$ when γ tends to infinity. Below the reference level θ, the relative risk aversion is:

$$RR(x) = -x\frac{U''(x)}{U'(x)} = \frac{Mx\gamma(\gamma - 1)(\theta - x)^{\gamma - 2}}{1 + M\gamma(\theta - x)^{\gamma - 1}}.$$

Therefore, $RR(x) < 0$ (and the investor is risk lover) when $\gamma < 1$, whereas he is risk averse when $\gamma > 1$. Note that the relative risk aversion is increasing when $1 < \gamma \leq 2$, whereas it may be either increasing or decreasing (depending on x) when $\gamma > 2$.

2.6. Efficiency ratio

The expected shortfall can be used to define a very simple ratio comparing expected loss and expected gain with respect to a reference level. Let X denote a random variable measuring the return or *NPV* of a project and θ denote a reference level, with which X is to be compared. The efficiency ratio is defined as:

$$R_{eff}(X, \theta) = \frac{\mathbb{E}\left[\max\left\{\theta - X, 0\right\}\right]}{\mathbb{E}\left[\max\left\{X - \theta, 0\right\}\right]}.$$

When comparing different projects, the preferred one is the one which minimizes the efficiency ratio. When comparing a single project X to a reference level θ, the project should not be selected if the efficiency ratio is larger than 1 since, in that case, expected loss exceeds expected gain.

3. Implementation of the different criteria and decision rules

The simplest way to compare projects is to compare their expected *NPV*s, which implicitly assumes risk neutrality. Risk-averse decision-makers with risk aversion θ would prefer the

Mean-Variance criterion, and select the project X with the largest $\mathbb{E}(NPV_X)$- θ $\mathbb{V}(NPV_X)$. However, the above criteria neglect the asymmetry of the distribution of NPV_X and neglect the very poor outcomes of the project, which are the most important ones because of default risk. In addition, the Mean-Variance criterion implies that, with sufficiently high risk aversion, an agent may prefer a distribution that is stochastically dominated by another. To remedy these two last drawbacks, we have introduced the *VaR*. For a given level of probability *p* (chosen by the decision-maker), project *X* is preferred to project *Y* if $VaR_X(p) > VaR_Y(p)$. Recall that the left tail of the distribution of NPV_X (i.e. the extent of the worst events) is not taken into account by the *VaR* criterion, and that the *CVaR* meets this criticism. In this case, for a given level of probability *p* (chosen by the decision-maker), project *X* is preferred to project *Y* if $CVaR_X(p) > CVaR_Y(p)$. However, investors are often mainly interested in comparing the (random) *NPV* of a new project to the (deterministic) *NPV* of a reference project (existing infrastructure). The above criteria do not integrate this fundamental difference between above and below this *NPV* reference level. The downside risk measures, and especially expected shortfall and efficiency ratio, answer this criticism.

Importantly, the different criteria listed above may lead to different conclusions, especially when some projects have a high expected *NPV* but a thick left tail. For example, the tail of the double exponential distribution is thicker than the tail of the Gaussian distribution. Since the double exponential distribution is an extreme value distribution, it is likely to be relevant for some parameters of the model. This is illustrated in the example below.

We consider two investments, *X* and *Y*, with positive net cash flows X_t and Y_t, respectively, during two periods (t=1,2) after the investment period (t=0), and 4 scenarios i=1…4. The rate of return we use is $r=10\%$. Table 2 sums up the characteristics of the two projects, namely for each project *S* and each scenario *i*, the net cash flow $S_{t,i}$ at each period, the net present value $NPV_{S,i} = \sum_{t=0}^{2} \frac{S_{t,i}}{(1+r)^t}$, and the probability of scenario *i*. Figures are in Million €.

	Project S=X				***Project S=Y***			
Scenario	*i=1*	*i=2*	*i=3*	*i=4*	*i=1*	*i=2*	*i=3*	*i=4*
Net cash flow $S_{0,i}$	-10	-10	-10	-10	-10	-10	-10	-10
Net cash flow $S_{1,i}$	10	10	5	5	12	12	7	7
Net cash flow $S_{2,i}$	15	11	9	7	13	5	7	10
$NPV_{S,i}$	13.66	9.93	3.23	1.37	13.73	6.26	3.30	6.10
Probability Pr(S,i)	0.25	0.5	0.15	0.1	0.4	0.3	0.1	0.2

Table 2 Characteristics of the two projects

We assume that the NPV of the existing project (reference) is θ =7. Then the expected shortfall of project *S* is:

$$R_1(S,\theta) = \sum_{i=1}^{4} \Pr(S,i) \max\{\theta - NPV_{S,i}, 0\}. \qquad (2)$$

The efficiency ratio is computed according to (23), in which the numerator corresponds to

(2) and the denominator is given by:

$$\sum_{i=1}^{4} \Pr\left(S,i\right) \max\left\{NPV_{S,i} - \theta, 0\right\}.$$

Table 3 sums up the different risk measures for the two projects.

	Project S=X	***Project S=Y***
Expected NPV	9	8.92
Variance of NPV	16.69	16.13
Standard deviation of NPV	4.09	4.02
Expected Shortfall	1.13	0.77
Efficiency ratio	0.36	0.29
VaR and CVaR at p=0.05	1.37	3.30

Table 3 Different risk measures for the two projects

The expected net present values of the two projects are very close, which means that a risk neutral investor would be indifferent between the two projects. The variances (and therefore standard deviations) of the two projects are also very close, so that the Mean-Variance criterion could not give a clear preference to either project. Note the level of risk of the two projects is high since the standard deviation of *NPV* (4 Million €) represents nearly half the expected *NPV* (9 Million €).

The expected shortfall of project *X* is 1.13, which means that the expected loss of this project compared to the reference project is 1.13 Million €. Note that project *X* leads to a loss with 25% chances, which corresponds to scenarios 3 and 4. The expected shortfall of project *Y* is only 0.77 Million €, which is far better, whereas the probability of loss is far higher (60%, corresponding to scenarios 2, 3 and 4). Project *Y* has a high probability of small loss, whereas project *X* has a small probability of a very large loss. The expected shortfall criterion is therefore clearly in favor of project *Y*. This is confirmed by the efficiency ratio criterion, which means that the expected loss represents only 29% of the expected gain for project *Y*, whereas it represents 36% for project *X*. Finally, the *VaR* and *CVaR* criteria are also clearly in favor of project *Y*, since the expected *NPV* in the 5% worst cases is only 1.37 Million € for project *X*, whereas it goes up to 3.30 Million € for project *Y*, which is more than twice the value for project *X*. Note that, in this example, *VaR*(0.05)=*CVaR*(0.05) for both projects because the *NPV* is constant in the 5% worst cases.

Other examples with asymmetric distributions and/or fat tails may lead to more divergence between the mean, mean-variance, *VaR*, *CVaR*, expected shortfall and efficiency ratio. This implies that all the criteria should be explored when evaluating projects in risky environments, so that the importance of the very worst events (with very small probabilities and very severe consequences) is correctly weighed against the average value of the project in more common events with less severe consequences.

The different criteria necessarily lead to the same conclusion when one project dominates the other in the sense of order 1 stochastic dominance. On the other hand, when the CDF functions cross, the different criteria may lead to different conclusions for some confidence levels, as is the case in the following example, applied to transport investments. We consider three projects of public transport: project A concerns a streetcar, project B concerns a driven bus and project C concerns an articulated bus in dedicated lane. We have the following NPV (in Million €) for the three projects, according to five levels of the demand, D1 being the highest demand and D5 being the lowest demand (p denotes the probability of each demand level).

Project \ Demand	D1 p=0.2	D2 p=0.5	D3 p=0. 15	D4 p=0. 1	D5 p=0. 05
A: Streetcar	100	50	30	11	10
B: Driven bus	65	70	40	12	5
C: Articulated bus in dedicated lane	35	45	50	55	60

We assume that the NPV of the existing project is equal to 45 Million Euros. The following table sums up the different risk criteria for the three projects.

	Project A	Project B	Project C	Projects Ranking
Expected NPV	51.1	55.45	45.5	B>A>C
Expected utility (U(x)=ln(x))	3.74	3.84	3.8	B>C>A
CVaR at p= 0.1	10.5	8.5	35	C>A>B
VaR at p= 0.1	11	12	35	C>B>A
Savage (« minmax », see end of Section 1.2)	50	55	65	A>B>C
Expected shortfall (NPV of the existing project=45)	7.4	6.05	2	C>B>A
Efficiency ratio (NPV of the existing project=45)	0.55	0.37	0.8	B>A>C

On this example, we observe that each of the projects can be potentially selected as the best project or as the worst project, according to the risk criterion which we consider.
The expected NPV and the expected utility criteria lead to select project B, but they do not select the same project as the worst project (project C is the worst project for the expected NPV criterion, whereas project A is given as the worst project by the expected utility criterion). Project B is also given as the best project by the efficiency ratio, given that the NPV of the existing project is equal to 45 Million Euros (the expected loss represents only 37% of the expected gain for project B). Let us notice that for each project, the efficiency ratio is lower than 1, which means that expected gain exceeds expected loss with respect to the reference NPV of the existing infrastructure for each project.
According to the VaR at p=0.1, the CVaR at p=0.1 and the expected shortfall criteria, the best project is project C. The CVaR and the expected shortfall do not classify the projects in the same order as the three previous criteria (expected NPV, expected utility and efficiency ratio), because contrary to the previous criteria, the CVaR and the expected shortfall focus only on the worst scenarios: for each project, the CVaR at p=0.1 gives the expected NPV for the 10% worst scenarios, and the expected shortfall gives the expected deviation of the NPV below the NPV of the existing infrastructure. The VaR at p=0.1 gives for each project the minimum NPV among the 90% best cases.
Finally, the Savage criterion leads to select project A. Note that project A is selected by none of the other risk criteria. The explication of this particular classification is the following one. The Savage criterion selects the project with the smallest maximum regret. Regret is defined as the

difference between the NPV of the project and the amount that could have been obtained if the state of nature could have been correctly forecast. For each project, the maximum possible regret is picked. Then, the project that has the lowest maximum regret is selected. In other words, contrary to the other criteria, the Savage criterion is the only criterion affected by the joint distribution of the NPV.

We argue that, in that case, no project should clearly be preferred to the other, that the investor has to be aware of this ambiguity, and that the various tools described here can help him weigh the different arguments in favour of each project.

4. Conclusions

We proposed in this paper different approaches used in finance as useful tools to model the impact of risk in project evaluation, and we compared them on basic examples. Based on simple examples, we show that the criteria usually used in finance may lead to different conclusions regarding the best or the worst project. We argue that no criterion is a priori better than the others, and that the decision-maker has to envisage different criteria in order to weigh the different arguments in favour of each project, based on his own preferences and objectives.

We are still some way from the situation where the correct tools will be used to take risk into account in the cost-benefit analysis. The next step, in our opinion, will be to use some other sophisticated tools developed in finance: “the theory of real option”, which will play an essential role in the strategic management of investment in a risky situation. These tools will allow a market price to be associated with a project. The idea is to duplicate a project with financial instruments which have a market price. This is rather easy when markets are complete. It is not clear and unfortunately unlikely that this is true in the case of large project investments.

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