The economics of truck toll lanes

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Abstract

Truck-only lanes and tollways have been promoted as a way to combat road congestion, enhance safety and reduce pavement damage. This paper explores one aspect of truck lanes by considering whether there are advantages in separating cars and trucks. The benefits of vehicle separation are found to depend on several factors: the relative volumes of cars and trucks, the congestion delay and safety hazards that each type of vehicle imposes, values of travel time for cars and trucks, and lane capacity indivisibilities. The optimal assignment of vehicles to lanes can be supported using tolls that are differentiated by vehicle type and route. By contrast, lane access restrictions generally cannot support the optimum and may provide no benefit at all.

Key words: truck-only facilities, segregation, congestion, accidents, marginal-cost pricing

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1 INTRODUCTION

The merits of separating cars and trucks have been debated since the early days of motoring. Recent proposals in the US have focused on establishing truck-only lanes and truck tollways.[1] Although no such facilities have yet been built[2] several US states have conducted studies (see Federal Highway Administration, 2003; Transportation Research Board, 2003; Hedlund, 2004). The Southern California Association of Governments (SCAG) has a plan that calls for an interconnected system of truck-only freeway lanes on four highways that would cost nearly $10 billion. Truck-only lanes were proposed for a North American Free Trade Association (NAFTA) highway between Toronto and Laredo, Texas. The Reason Foundation published a detailed study (Samuel et al., 2002) arguing that an interstate network of private truck tollways could be profitable. More recently, it has formulated a proposal to expand highway capacity in Atlanta with a toll truckway system as one component (Poole, 2006). And Texas has adopted an ambitious plan to build a "Trans Texas Corridor" (Texas Department of Transportation, 2002).[3]

Several potential advantages of dedicated truck-only facilities have been identified in the literature.

Road design: Trucks require higher road-design standards than do light vehicles as far as pavement thickness, lane widths, road curvature, grades, etc. By restricting trucks to a subset of roads or lanes, the rest of the road network can be built to a lower standard and hence more cheaply (Holguin-Veras et al., 2003).

Congestion: Trucks and light vehicles alike may gain from reductions in congestion. If trucks experience reduced traffic volumes on truck-only facilities they will incur less congestion delay and less frequent need for braking, accelerating, and overtaking. Travel time for freight deliveries may also become more predictable (Douglas 2005). Light vehicles may benefit from fewer delays due to slow-moving trucks (Verhoef et al., 1999; Rouwendal et al., 2002) and less interference in fields of vision and changing lanes (Pitfield and Watson, 2001).

Safety: Although the empirical evidence is not clear-cut, it appears that accident rates are higher in mixed (i.e. heavy and light vehicle) traffic than in homogeneous traffic (Middleton and Lord, 2005; Forkenbrock and March, 2005). One reason is lane switching (Pitfield and

[1] Reich et al. (2002) provide an extensive literature review.

[2] Two facilities in the US are designed to accommodate trucks while also permitting passenger vehicles to use them. One is the New Jersey Turnpike, and the other is a segment of Interstate 5 north of Los Angeles (Middleton and Venglar, 2006). The New Jersey Turnpike is a so-called "dual-dual" roadway, defined to be "a system of parallel, grade-separated lanes with trucks restricted from operating in the center, auto-only lanes." (Fischer et al., 2003, p.74)

[3] The Trans Texas Corridor will consist of a 4,000-mile network of corridors up to 1,200 feet wide with three road lanes in each direction for passenger vehicles, and two road lanes in each direction for trucks. Road tollways will be designed for an 80 mph speed limit. In addition, the corridors will have one rail track in each direction for each of three types of rail service (freight, commuter rail, and high-speed passenger rail) as well as corridors for utilities. The first Trans Texas Corridor segment, TTC-35, is under development.

Watson, 2001). If so, safety is promoted by segregating trucks from cars. And surveys indicate that automobile drivers dislike trucks and would be willing to pay to avoid them.[4]

Air quality: By supporting higher and less variable speeds, truck-only facilities contribute to better overall air quality (Douglas, 2005).

Noise: Reductions in noise are a potential benefit if truck facilities are located away from densely populated areas (Douglas, 2005).

Truck type: Truck-only facilities would facilitate use of so-called Long Combination Vehicles (LCVs) that exploit economies of vehicle size.[5] Use of larger trucks reduces not only transport costs per tonne-km, but also congestion delays for a given amount of freight transported because fewer trucks are on the road.

Truck-only facilities are also recognized to have several potential disadvantages.

Capacity indivisibilities: Building truck-only facilities is cost-effective only if truck volumes are sufficiently high (OECD, 1992; Douglas, 2005; Forkenbrock and March, 2005). Lane capacity indivisibilities make it difficult to allocate capacity between vehicle categories in efficient proportions (Small, 1983; Dahlgren, 1998, 2002; Yang and Huang, 1999; Parsons et al., 2005). And to facilitate access in the event of incidents, as well as to provide reliable travel times for truckers, twin truck lanes are required (Fischer et al., 2003) which increases the infrastructure costs. Wilbur Smith Associates (2003) assessed various strategies for dealing with congestion on Interstate 10 (I-10), which runs across the southern US from Florida to California. The study concluded that simply adding more general-purpose lanes to I-10 would be more effective than adding truck-only lanes. Various other studies have reached similar conclusions.

Availability of right-of-way: Some US intercity travel corridors lack sufficient width to accommodate double truck lanes throughout their length (Poole and Samuel, 2004). And many Interstate highways lack an uninterrupted median that would permit an extra lane or lanes to be built in the median (Reich et al., 2002).

Lane access considerations: Complete segregation of heavy vehicles from cars is generally not practical.[6] Truck route restrictions may add to travel distance which militates against using

[4] Using contingent valuation analysis Bambe and McMullen (1996) estimated that motorists would be willing to pay about $35 (1995) annually to remove triple-trailer combination trucks from Oregon's highways. (Information taken from Forkenbrock and March, 2005, p. 8.) However, while automobile drivers generally perceive improvements in safety and operations from lane restrictions on heavy vehicles, truck drivers do not foresee improvements (Koehne et al., 1996; Douglas, 2005).
[5] Samuel et al. (2002) estimate that permitting the largest LCVs could increase productivity by $3.04 per vehicle-mile.
[6] As Wilbur Smith Associates (2003, p.35) remarks: "Even in cases where truck separation is applied, there will have to be some degree of car/truck interaction, especially along segments where local traffic merges on/off the freeway system. This presents significant traffic engineering issues (trucks and cars crossing lanes to merge to and from exclusive lanes)."

them for short-haul trips. And forcing vehicles to use certain lanes may increase the number of lane changes (e.g. if trucks are restricted to left-hand lanes) which contributes to traffic flow turbulence and accident hazards (Pitfield and Watson, 2001; Gan and Jo, 2003).

Temporal segregation: Truckers generally try to avoid traveling during peak commuting periods (Donaghy and Schintler, 1998; Fischer et al., 2003). A majority of their trips are made during mid-day (10:00-15:00) and at night. To the extent that autos and trucks can use the same roads at different times, building separate facilities is unnecessary to separate them.

These lists of pros and cons indicate that both the optimal design and the cost-effectiveness of truck facilities depend on many practical considerations. A large number of facility types have been proposed that differ according to numbers of lanes, conversions vs. additions – including conversion of High Occupancy Vehicle (HOV) lanes, and admitting trucks to High Occupancy Toll (HOT) lanes, and usage restrictions. No formal analytical/economic analysis of truck-only facilities has been conducted to date. Nevertheless, the economics of truck-only facilities resemble in several respects the economics of HOV and HOT lanes which have been studied from an economic perspective. For example, Small (1977, 1983) analyzes whether HOV lanes for buses are cost effective, and how well they perform relative to congestion tolls. He finds that with ideal lane segregation of buses and cars (i.e. when capacity is perfectly divisible), bus priority lanes yield about half the benefits of marginal cost pricing. But when the indivisibility of HOV lanes is accounted for, auto congestion becomes dramatically worse because a full lane has to be allocated to HOV traffic. Only at high volumes of passengers per lane-hour do the positive benefits for HOV lane users outweigh the negative effects of increased delay for travelers using the remaining lanes. Mannering and Hamed (1990) obtain similar results for HOV lanes designed for cars rather than buses.

These and other studies convey two important lessons that carry over to truck-only facilities. One is that the benefits from dedicated facilities depend critically on the volume of traffic that will use them. The second lesson is that lane- or route-access restrictions are second-best policies compared to efficient pricing.[7]

This paper focuses on one aspect of the economics of truck-only facilities: how existing road space should be allocated between cars and trucks. This question has been partially addressed by Berglas et al. (1984) and Arnott et al. (1992) *inter alios*, and the model developed in Arnott et al. (1992) is adopted and extended here. Section 2 describes the general model and derives some results concerning traffic allocation in the unregulated user equilibrium and the social/system optimum. Section 3 develops a specific version of the general model for the numerical examples that are presented in Section 4. Finally, Section 5 summarizes the main findings and identifies various directions in which the analysis could be extended.

[7] This lesson clearly emerges in Mohring (1979).

2 THE GENERAL MODEL

2.1 Model specification

The general model is adopted from Arnott et al. (1992). There are two routes or sets of traffic lanes indexed by r, r=1,2. And there are two types of vehicles, indexed by g, g=L,H, where subscripts L and H refer respectively to light vehicles (henceforth *Lights*) and heavy vehicles (henceforth *Heavies*). [8] The number of trips by *Lights* is N_L, and the number of trips by *Heavies* is N_H. N_L and N_H are fixed; that is independent of the cost of a trip. Trips on the two routes are perfect substitutes. Let $N_{gr} \geq 0$ denote the number of trips by type g on route r. Since trips must be made by one route or the other $N_{g1} + N_{g2} = N_g$, $g = L, H$. The cost incurred by type g for a trip on route r is a linear[9] increasing function of the number of vehicles of each type that take the same route:

$$C_r^L = \underbrace{F_r^L}_{(1)} + \underbrace{c_{Lr}^L N_{Lr}}_{(2)} + \underbrace{c_{Hr}^L N_{Hr}}_{(3)} + \underbrace{\tau_r^L}_{(4)}, \; r = 1, 2, \tag{1a}$$

$$C_r^H = \underbrace{F_r^H}_{(1)} + \underbrace{c_{Lr}^H N_{Lr}}_{(2)} + \underbrace{c_{Hr}^H N_{Hr}}_{(3)} + \underbrace{\tau_r^H}_{(4)}, \; r = 1, 2. \tag{1b}$$

Formulae (1a,1b) incorporate the convention that a superscript denotes the user type that incurs the cost, and a subscript denotes the type that imposes the cost. Term (1) in each expression comprises costs, F_r^g , that are independent of usage. These costs include vehicle operating costs, free-flow travel time costs, and the portion of single-vehicle accident costs that is borne by the individual users. Term (2) is the cost imposed by *Lights* that use the same route. Term (3) is the analogous cost imposed by *Heavies*. Following Arnott et al. (1992) the coefficients c_{Lr}^L and c_{Hr}^H, r = 1, 2, are called *own-cost coefficients* since they describe the costs imposed on users of the same type. Analogously, the coefficients c_{Hr}^L and c_{Lr}^H, r = 1, 2, are called *cross-cost coefficients*. Finally, term (4) is the toll, τ_r^g . It is assumed that tolls can be differentiated by vehicle type and route – as is general practice on existing tolled facilities.

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[8] With two discrete types it is necessary to contend with a number of possible type-to-route allocations (Arnott et al., 1992; Small and Yan, 2001). This complication could be avoided by using a model with a continuum of types (e.g. as in Verhoef and Small, 2004). The discrete typology is adopted here for two reasons. First, it is suitable for a study of truck toll lanes in which there is a natural dichotomy of types. Second, *Heavies* differ from *Lights* not only in size and maneuverability, but also in terms of accident frequencies and costs, emissions, road damage costs, values of time and other characteristics. Constructing an empirically accurate and tractable joint frequency distribution of these dimensions would be a challenge, and it would make analytical results difficult to derive.

[9] Linear functions are chosen for tractability. Most road traffic studies assume that travel time (and travel time cost) is a strictly convex function of usage. These functions are typically specified in terms of instantaneous flows. When specified in terms of trips the functional relationship can be approximately linear. In the case of Vickrey's bottleneck queuing model with identical travelers and linear schedule delay cost functions, the relationship is exactly linear; see Arnott et al. (1998) and Small and Verhoef (2006, Chapter 3).

2.2 User equilibrium

In the absence of tolls or access restrictions, three types of user equilibrium (UE) route allocations are possible (Arnott et al., 1992; Small and Yan, 2001): integrated equilibria, partially separated equilibria and segregated equilibria. In an *integrated* equilibrium both *Lights* and *Heavies* use each route. In a *partially separated* equilibrium, one type uses both routes and the other type uses only one route. Finally, in a *segregated* equilibrium each type uses only one route.

If type *g* uses both routes then, by Wardrop's first principle, the costs for type *g* must be equal on the two routes:

$$C_1^g = C_2^g,\ g = L, H\ . \tag{2}$$

The necessary and sufficient conditions for an integrated UE are therefore:

$$C_1^L = C_2^L\ , \tag{3a}$$

$$C_1^H = C_2^H\ , \tag{3b}$$

$$N_{L1} > 0,\ N_{L2} > 0,\ N_{H1} > 0,\ N_{H2} > 0\ . \tag{4}$$

Substitution of (1a) into (3a), and (1b) into (3b), yields a pair of linear reaction functions of the form $N_{L1} = f^L\left(N_{H1}\right)$ and $N_{H1} = f^H\left(N_{L1}\right)$. By the usual stability criterion an integrated UE can exist only if $\left(\partial f^L / \partial N_{H1}\right)\left(\partial f^H / \partial N_{L1}\right) < 1$. Given eqns. (1a) and (1b) the stability condition works out to

$$\left(c_{L1}^L + c_{L2}^L\right)\left(c_{H1}^H + c_{H2}^H\right) > \left(c_{H1}^L + c_{H2}^L\right)\left(c_{L1}^H + c_{L2}^H\right),$$

or

$$c_{L\bullet}^L c_{H\bullet}^H > c_{H\bullet}^L c_{L\bullet}^H\ , \tag{5}$$

where $c_{h\bullet}^g \equiv c_{h1}^g + c_{h2}^g,\ g = L, H,\ h = L, H$. Stability requires that the product of the own-cost coefficients on the two routes be larger than the product of the cross-cost coefficients. One might expect Condition (5) to hold if the inequality holds separately on each route; i.e. if:

$$c_{Lr}^L c_{Hr}^H > c_{Hr}^L c_{Lr}^H\ ,\ r=1,2. \tag{6}$$

In fact, the two conditions in (6) are neither necessary nor sufficient for (5) as is shown in Appendix A with a generic algebraic proof. This is because stability is a property of pairs of routes rather than routes in isolation. However, the two conditions in (6) do imply Condition (5) (see Appendix A) if the routes satisfy a *similarity property*[10]:

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[10] This term is introduced in Arnott et al. (1992, §2.1).

(Similarity property) $$\frac{c_{L2}^{L}}{c_{L1}^{L}}=\frac{c_{H2}^{L}}{c_{H1}^{L}}=\frac{c_{L2}^{H}}{c_{L1}^{H}}=\frac{c_{H2}^{H}}{c_{H1}^{H}}. \tag{7}$$

The similarity property holds if the cost coefficients are inversely proportional to route capacities; i.e. have the functional form

$$c_{hr}^{g}=\frac{\gamma_{h}^{g}}{s_r},\ g=L,H,\ h=L,H,\ r=1,2,$$

where s_r is the flow capacity of Route *r* and the γ_h^g parameters are independent of *r*.

It is important to note that Condition (5) is a necessary, but not sufficient, condition for an integrated UE. In addition, the nonnegativity conditions (4) must be satisfied. As Arnott et al. (1992, pp. 83-84) explain, there are four cases to consider in all:

1. Stability and nonnegativity conditions satisfied. In this case there is a unique integrated UE.
2. Stability condition satisfied, but nonnegativity conditions violated. There is a unique UE that is either partially separated or segregated.
3. Nonnegativity conditions satisfied, but stability condition violated. There are two stable UE, each of which is either partially separated or segregated.
4. Stability and nonnegativity conditions violated. There is a unique UE that is either partially separated or segregated.

2.3 The social or system optimum

Following usual practice, welfare will be measured by social surplus. Since travel demand is assumed fixed, welfare can be measured by the negative of total social costs, *TSC*. Let e_{gr} denote the external cost of a trip by type *g* on route *r* that is not borne by users of the route collectively. Parameter e_{gr} includes the costs of emissions, noise and pavement damage; for brevity it will be called the *environmental cost*. Since tolls are a transfer from users to the toll-road authority, toll revenues net out of social costs and *TSC* is given by the formula

$$TSC=\sum_{r=1,2}\left(F_r^L+c_{Lr}^{L}N_{Lr}+c_{Hr}^{L}N_{Hr}+e_{Lr}\right)N_{Lr}+\left(F_r^H+c_{Lr}^{H}N_{Lr}+c_{Hr}^{H}N_{Hr}+e_{Hr}\right)N_{Hr}. \tag{8}$$

The social optimum (SO) achieves a minimum of *TSC*. Similar to the UE, the SO may entail integration, partial separation or segregation. For integration to be optimal the second-order condition for an interior cost minimum (see Appendix B) must be satisfied. This condition is stated as:

> **Lemma 1**: The SO is integrated only if the second-order condition for an interior SO is satisfied:

$$c_{L\bullet}^{L}c_{H\bullet}^{H}>c_{H\bullet}^{L}c_{L\bullet}^{H}+\frac{1}{4}\left(c_{H\bullet}^{L}-c_{L\bullet}^{H}\right)^2. \tag{9}$$

In an integrated SO the marginal social cost of a trip is equal across routes for both types. Let $-g$ be the index for the type other than g; i.e. if g=L then $-g = H$, and if g=H then $-g = L$. The marginal social cost of a trip by type g on route r can be written:

$$MSC_r^g = \frac{\partial TSC}{\partial N_{gr}} = F_r^g + 2c_{gr}^g N_{gr} + \left(c_{-gr}^g + c_{gr}^{-g}\right) N_{-gr} + e_{gr} . \tag{10}$$

A necessary condition for type g to use both routes at the SO is that the marginal social costs are equal on the two routes:

$$MSC_1^g = MSC_2^g ,\ g = L, H . \tag{11}$$

Condition (11) is the counterpart to Condition (2) for the UE.

2.4 Comparison of the user equilibrium and system optimum

Despite the simple linear structure of the model, it is not trivial to compare the UE and SO. The general formulae for the route allocations are unwieldy, and the route allocation patterns of the UE and SO may differ. The procedure followed here is to compare the stability and second-order conditions for an integrated equilibrium, and then compare the traffic allocations in the UE and SO under some simplifying, but plausible, assumptions about trip costs on the two routes.

The second-order condition for an integrated SO given in Lemma 1 (inequality (9)) is more stringent than the stability condition for the UE (inequality (5)) unless $c_{H\bullet}^L = c_{L\bullet}^H$; i.e. unless the cross-cost coefficients (summed over the two routes) are equal for *Lights* and *Heavies*. Consequently, the UE may be integrated, but the SO partially separated or segregated. This result is formalized as:

> **Proposition 1**: The stability condition for an interior UE is satisfied if, but not only if, the second-order condition for an interior SO is satisfied. Consequently, it may be optimal to partially separate or segregate *Light* and *Heavy* vehicles even if they are integrated in the unregulated user equilibrium.

To understand Proposition 1, note that the second-order condition can be violated when the stability condition is met if the quadratic term in $\left\| c_{H\bullet}^L - c_{L\bullet}^H \right\|$ on the right-hand side of condition (9) is appreciable. This will be the case if the cross-cost coefficients are very different. Suppose, for example, that *Heavies* severely delay or endanger *Lights* so that $c_{H\bullet}^L >> c_{L\bullet}^H$. If so, it is optimal to keep *Heavies* away from *Lights* to avoid this interference. Suppose, alternatively, that *Heavies* have a much higher value of time than *Lights* so that $c_{L\bullet}^H >> c_{H\bullet}^L$. If so, it is desirable to provide *Heavies* with a high-quality service by giving them exclusive access to one of the routes. In either case integration may occur in the UE when separation is more efficient.

To compare traffic allocations in the UE and SO it is useful to rewrite eqn. (10):

$$MSC_r^g = \underbrace{C_r^g - \tau_r^g}_{(1)} + \underbrace{c_{gr}^g N_{gr}}_{(2)} + \underbrace{c_{gr}^{-g} N_{-gr}}_{(3)} + \underbrace{e_{gr}}_{(4)}, \; g = L, H, \; r = 1, 2. \qquad (12)$$

Term (1) in eqn. (12) is the private cost net of toll borne by a user of type *g* on route *r* as given in eqn. (1). Term (2) is the external cost imposed on users of the same type on route *r*, and Term (3) is the external cost imposed on users of the other type. Finally, Term (4) is the environmental cost imposed on non-users. Terms (2-4) all correspond to externalities, and hence are a potential source of inefficiency in the UE.

The influence of the environmental externality is transparent. If the environmental cost of a trip is higher, say, for *Lights* on Route 1 than on Route 2, then usage of Route 1 by *Lights* will tend to be excessive in the UE. The fixed costs of a trip, F_r^g, contribute to the private cost of a trip (Term (1) in eqn. (12)) but not to the external costs. Consequently, users place undue weight on fixed costs when choosing between routes, and tend to use excessively the lower-cost (e.g. shorter or quicker) route.[11]

If environmental costs and fixed costs are the same on the two routes for each user type, then these two potential sources of inefficiency will be absent. However, the external costs in Terms (2) and (3) of eqn. (12) will still not balance out without a further assumption. A sufficient assumption is that the similarity property, (7), holds. This is formalized in the following proposition:

> **Proposition 2**: Assume that the second-order condition (9) holds and the following three assumptions are satisfied:
>
> Assumption 1: Free-flow travel costs are the same on the two routes: $F_1^g = F_2^g$, $g = L, H$.
>
> Assumption 2: Environmental costs are the same on the two routes: $e_{g1} = e_{g2}$, $g = L, H$.
>
> Assumption 3: Property (7) holds.
>
> Then the UE and SO are both integrated with identical numbers of each type using each route.

Proposition 2 is proved in Appendix C.

2.5 Intervention

Unless the assumptions of Proposition 2 are satisfied, the unregulated equilibrium will generally not be optimal. As §2.5.1 below explains, a first-best optimum can be supported using tolls. Lane access restrictions clearly can support the optimum if it is segregated, but clearly not if it is

[11] This bias is well-known in the literature on two parallel routes with homogeneous travelers; see Barro and Romer (1987) and Verhoef et al. (1996) *inter alios*.

integrated. As §2.5.2 demonstrates, lane access restrictions also generally fail to sustain a social optimum if it is partially separated.

2.5.1 Tolls

The social optimum can be decentralized using suitably differentiated tolls. This is formalized in:

> **Proposition 3**: The system optimum can be decentralized using tolls that are differentiated by user type and route.

Proposition 3 is proved in Appendix D. Because travel demand is assumed to be inelastic, the allocations of users across routes depend only on the differences in tolls between routes. Consequently, it is enough to toll the route with the higher optimal toll and leave the other route toll-free. However, care is required in setting toll levels. If the system optimum is integrated, then the second-order condition and stability conditions are satisfied and the tolled equilibrium is stable. But if the system optimum is partially separated or segregated, the stability condition may not hold. And if it does not hold the tolled equilibrium is unstable. As Appendix D explains, this problem can be circumvented by setting a sufficiently high toll on the route that a user type is not supposed to use.

2.5.2 Lane access restrictions

If the SO is partially separated or segregated the question arises whether it can be supported by lane-access restrictions alone without resorting to tolls. A segregated optimum clearly can be sustained simply by restricting each user type to its designated route. However, if the SO is partially separated it generally cannot be sustained with lane-access restrictions alone. Indeed, if the UE is integrated, then restricting one user type to one route can actually be welfare-reducing under the conditions identified in the following proposition:

> **Proposition 4**: Assume that the stability condition (5) holds and Assumptions 1-3 of Proposition 2 are satisfied. Then, even if the SO is partially separated or segregated, restricting one user type raises total social costs if the other user type uses both routes.

Proposition 4 can be proved with routine algebra. An intuitive explanation for this rather surprising result can be offered by comparing UE and SO usage in the same partially separated configuration. Consider the case in which *Lights* use both routes while *Heavies* are restricted to Route 2. In Appendix E it is shown that

$$N_{L1}^{e} - N_{L1}^{o} \propto c_{H\bullet}^{L} - c_{L\bullet}^{H}. \tag{13}$$

Expression (13) indicates that the no-toll equilibrium route split of *Lights* (who use both routes) is not optimal unless $c_{H\bullet}^{L} = c_{L\bullet}^{H}$; that is unless the external cost that *Lights* incur from *Heavies* matches the external cost that *Lights* impose on *Heavies*. If $c_{H\bullet}^{L} > c_{L\bullet}^{H}$, then *Lights* try too hard to avoid *Heavies* and too many *Lights* use Route 1. Correspondingly, if $c_{H\bullet}^{L} < c_{L\bullet}^{H}$, then too few *Lights* use Route 1. The larger the absolute difference between $c_{H\bullet}^{L}$ and $c_{L\bullet}^{H}$, the greater is the distortion in route split.

Now observe that partial separation of types is desirable only if the second-order condition is violated; i.e. $c^L_{L\bullet}c^H_{H\bullet} < c^L_{H\bullet}c^H_{L\bullet} + \frac{1}{4}\left(c^L_{H\bullet} - c^H_{L\bullet}\right)^2$. Since the stability condition, $c^L_{L\bullet}c^H_{H\bullet} > c^L_{H\bullet}c^H_{L\bullet}$, is satisfied by assumption, this requires that $\left\|c^L_{H\bullet} - c^H_{L\bullet}\right\| > 0$. The larger is $\left\|c^L_{H\bullet} - c^H_{L\bullet}\right\|$, the larger is the potential benefit from restriction, but the larger also is the distortion in the route split of *Lights* as just explained. It turns out that the distortion always outweighs the potential benefit, so that restriction is never optimal without tolls. This is the case regardless of which user type is restricted, or to which route, as long as the lane access constraint results in a partially separated equilibrium rather than a segregated equilibrium. Naturally, if tolls can be levied then the route split of the unrestricted group can be optimized, and restriction will be welfare-enhancing whenever the second-order condition is violated. (However, as Proposition 3 establishes, lane access restrictions are redundant if tolls can be freely imposed.)

To compare the UE and SO further, and to get an idea of the magnitude of the potential welfare differences, a specific version of the general model that focuses on congestion and accident externalities is developed in the next section.

3 THE SPECIFIC MODEL

3.1 Model specification

The general model features own-cost and cross-cost coefficients without specifying their underlying determinants. Congestion and accidents are the two main road transport costs that are external to individual users (partially in the case of accidents), but internal to users in aggregate. To permit separate roles for the congestion and accident externalities the cost coefficients are now written:

$$c^g_{hr} = cong^g_{hr} + acc^g_{hr},\ g = L,H;\ h = L,H;\ r = 1,2,$$

where $cong^g_{hr}$ and acc^g_{hr} are congestion and accident cost coefficients respectively. Define

$$cong^g_{h\bullet} \equiv cong^g_{h1} + cong^g_{h2},\ g = L,H;\ h = L,H,$$

and

$$acc^g_{h\bullet} \equiv acc^g_{h1} + acc^g_{h2},\ g = L,H;\ h = L,H.$$

The stability condition (5) can then be written

$$\left(cong^L_{L\bullet} + acc^L_{L\bullet}\right)\left(cong^H_{H\bullet} + acc^H_{H\bullet}\right) > \left(cong^L_{H\bullet} + acc^L_{H\bullet}\right)\left(cong^H_{L\bullet} + acc^H_{L\bullet}\right). \tag{14}$$

By reasoning parallel to that in Section 2.2, one might expect Condition (14) to be satisfied if it holds for the congestion and accident coefficients separately; i.e. if:

$$cong_{L\bullet}^{L} cong_{H\bullet}^{H} > cong_{H\bullet}^{L} cong_{L\bullet}^{H}, \tag{15a}$$

and

$$acc_{L\bullet}^{L} acc_{H\bullet}^{H} > acc_{H\bullet}^{L} acc_{L\bullet}^{H}. \tag{15b}$$

But for the same reason as for the route decomposition in equations (5) and (6), Conditions (15a) and (15b) are neither necessary nor sufficient for stability. (See Appendix A for the generic proof.) Whether or not the stability condition (14) holds depends in part on the relative magnitudes of the own-cost and cross-cost coefficients. These are considered in the following two subsections.

3.2 Relative congestion costs

For several reasons *Heavies* have a greater impact than *Lights* on highway congestion: they occupy more road space, they take longer to accelerate and decelerate, and they obscure visibility more. These differences are usually accounted for by using a Passenger Car Equivalent (PCE) factor (Transportation Research Board, 2000). Typical PCE values are 1.5-2 for buses and single-unit trucks, and 2-3 for combination vehicles. It is common practice to adjust the PCE factor upwards with the percentage grade and the fraction of road length that is hilly (Middleton and Lord, 2005). And some studies have found that the PCE factor is an increasing function of the fraction of *Heavies* in the traffic stream (e.g. Janson and Rathi, 1991; Yun et al., 2005): a consideration that cannot be assimilated with the linear functions in (1a,1b).

The PCE factor measures the average impedance created by larger vehicles, but not their impedance of individual vehicle types as embodied in the own- and cross-congestion coefficients in eqns. (1a,1b). These individual effects have not been extensively studied although Kockelman and Shabih (2000) have estimated the relative delays imposed by light trucks and by cars. To provide some flexibility in the specification, let PCE_{cong} be a generic PCE factor for *Heavies* and suppose that the relative magnitudes of the congestion-cost parameters $cong_{Hr}^{L}$ and $cong_{Lr}^{L}$ are the same on the two routes and given by:

$$\frac{cong_{Hr}^{L}}{cong_{Lr}^{L}} = \lambda_{H}^{L} PCE_{cong}, \; r = 1,2. \tag{16a}$$

Parameter $\lambda_{H}^{L} \geq 1$ in eqn. (16a) is a scale factor to account for the possibility that – perhaps because of their lower speed and maneuverability, and greater height – *Heavies* impose a disproportionately large delay on *Lights*. The costs of congestion delay are proportional to the value of time (VOT) which is different for *Lights* and *Heavies*. Let v^{g} denote the value of time for type g, $g = L, H$. If a *Light* is assumed to impose the same congestion delay on a *Heavy* as on another *Light* then the relative magnitudes of parameters $cong_{Lr}^{H}$ and $cong_{Lr}^{L}$ are

$$\frac{cong_{Lr}^{H}}{cong_{Lr}^{L}} = \frac{v^{H}}{v^{L}}, \; r = 1,2. \tag{16b}$$

Finally, if is assumed that the standard PCE for congestion is applicable to the delay that *Heavies* impose on each other, then the relative magnitudes of parameters $cong_{Hr}^{H}$ and $cong_{Lr}^{L}$ are

$$\frac{cong_{Hr}^{H}}{cong_{Lr}^{L}} = \frac{v^{H}}{v^{L}} PCE_{cong}, \ r = 1,2 . \tag{16c}$$

3.3 Relative accident costs

Relative accident costs are treated in the same way as relative congestion costs. Let PCE_{acc} be a generic Passenger Car Equivalent for *Heavies* that measures the expected accident cost imposed by a *Heavy* as a multiple of the cost imposed by a *Light*. And let parameter $\phi_{H}^{L} \geq 1$ be a scale factor to account for the disproportionate hazard that *Lights* may experience from sharing the road with *Heavies*.[12] The relative magnitudes of parameters acc_{Hr}^{L} and acc_{Lr}^{L} are then:

$$\frac{acc_{Hr}^{L}}{acc_{Lr}^{L}} = \phi_{H}^{L} PCE_{acc}, \ r = 1,2 . \tag{17a}$$

Let μ^{H} denote the cost borne by a *Heavy* in an accident with a *Light* as a multiple of the cost borne by a *Light* in an accident with a *Light*. It follows that

$$\frac{acc_{Lr}^{H}}{acc_{Lr}^{L}} = \mu^{H}, \ r = 1,2 . \tag{17b}$$

The empirical magnitude of μ^{H} is not obvious. On the one hand a *Heavy* vehicle and its driver may suffer little damage or injury in a collision with a *Light* vehicle. On the other hand the value of the vehicle and cargo at risk is often much greater for a *Heavy*, and the opportunity cost of time spent dealing with an accident is also likely to be higher. Finally, if the factors ϕ_{H}^{L} and PCE_{acc} are assumed to act multiplicatively in determining the costs of accidents between *Heavies* then

$$\frac{acc_{Hr}^{H}}{acc_{Lr}^{L}} = \mu^{H} PCE_{acc}, \ r = 1,2 . \tag{17c}$$

3.4 Implications for the stability condition

As noted in Section 3.1 it is not possible to check the stability condition (14) by examining the congestion-cost and accident-cost coefficient conditions, (15a, 15b), independently. But an examination of the two conditions is nevertheless instructive. Given the relative congestion cost coefficients in (16) and (17),

$$cong_{L\bullet}^{L} cong_{H\bullet}^{H} - cong_{H\bullet}^{L} cong_{L\bullet}^{H} \overset{s}{=} 1 - \lambda_{H}^{L} ,$$

[12] The scale factor ϕ_{H}^{L} can include unreasonable "fear" of *Heavies* to the extent that the fear creates a real psychological cost or distress for drivers of *Light* vehicles.

and

$$acc^L_{L\bullet}acc^H_{H\bullet} - acc^L_{H\bullet}acc^H_{L\bullet} \overset{s}{=} 1 - \phi^L_H ,$$

where $\overset{s}{=}$ means identical in sign. Congestion therefore tends to be destabilizing of an integrated equilibrium if $\lambda^L_H > 1$; i.e. if *Heavies* impede *Lights* more than what is indicated by the standard PCE factor. Similarly, accident costs tend to be destabilizing if $\phi^L_H > 1$; i.e. if *Heavies* impose a disproportionate accident risk on *Lights*.

Note that the magnitudes of the Passenger Car Equivalent factors, PCE_{cong} and PCE_{acc}, and the values of time, v^L and v^H, do not affect whether Conditions (15a, 15b) are satisfied. What does matter are the relative magnitudes of the own-cost and cross-cost coefficients.

4 NUMERICAL EXAMPLES

4.1 Parameterization

The travel corridor featured in the numerical examples is intended to be representative of limited-access highways – which serve most of the medium-to-long urban truck trips in the US. Base-case parameter values are listed in Table 1.

4.1.1 Routes

The corridor has three traffic lanes in each direction. Consideration is limited to one direction. Route 1 comprises two lanes with an aggregate capacity of 4,000 (standard) PCEs per hour, and Route 2 comprises one lane with a capacity of 2,000 (standard) PCEs per hour. Free-flow travel speed on each route is 65 mph. Both routes are 32.5 miles long so that free-flow travel time is 30 mins.

4.1.2 Travel demand

Total trip demand, $N \equiv N_L + N_H$, is fixed at 40,000 trips per day. The proportion of *Heavies*, N_H / N, is varied parametrically from 0% to 100%.[13]

4.1.3 Volume-independent user costs

Numerous studies have estimated the VOT for automobile travel. Small and Verhoef (2006, p. 3-56) identify a typical VOT for work trips of $9.14/h for US metropolitan areas in 2003. Some recent studies assume rather higher values. For heavy vehicles a wide range of VOT have been estimated or assumed – in part because VOT depends on the type of vehicle and its load, drivers' wage rates, the importance of punctual delivery, whether the truck is operated in-house or for-hire, and other factors (De Jong, 2000; Kawamura, 2003). Wilbur Smith Associates (2003)

[13] Typical truck percentages are 20% or lower, but it is instructive to consider the full potential range.

assumes a VOT of \$25/h for trucks while acknowledging that this is a very conservative value. According to Forkenbrock and March (2005, p.7):

> "The value of time used by FHWA is \$25.24 per vehicle-hour for large trucks, compared to \$15.71 for small cars. In other studies in the United States and Europe, estimated values of time for trucking range as high as \$193.80, with a median value among the studies of \$40 and a mean of \$51.80. The value of reliability (that is, the cost of unexpected delay) is another 50 to 250 percent higher than these values of time."

These figures suggest that the average VOT for *Heavies* is several times the average VOT for *Lights*. For the base-case values it is assumed that $v^L = \$12$/h and $v^H = \$50$/h.

4.1.4 Congestion cost coefficients

Based on information in FHWA (1997), Parry (2006, Table 1) assumes a PCE for congestion of 1.9 for single-unit trucks and 2.2 for combination trucks. The value for PCE_{cong} of 2.0 used here is an (approximate) weighted average for the two truck types. The *Light-Light* congestion cost parameters, $cong_{Lr}^L$, $r = 1,2$, are chosen so that the marginal external congestion cost of a *Light* is about \$0.10/mile on each route in the base case. The value of $\lambda_H^L = 1$ is chosen as a benchmark; it is quite plausible that λ_H^L exceeds one and this is explored in the sensitivity analysis.

4.1.5 Accident cost coefficients

Based on information in FHWA (1997), Parry (2006, Table 1) assumes external accident costs of \$0.020/mile for *Lights* and \$0.015/mile for *Heavies*. Given widespread concern about truck accidents, and the perceived dangers that trucks impose on light vehicles, the relatively small value for *Heavies* is surprising. One possible explanation is that truck drivers are better drivers on average than automobile drivers, and less prone to causing accidents.[14] The *Light-Light* accident cost parameters, acc_{Lr}^L, $r = 1,2$, are chosen so that the marginal external accident cost of a *Light* is about \$0.020/mile on each route in the base case. As for parameter λ_H^L, the value of $\phi_H^L = 1$ is chosen as a benchmark and larger values are considered in the sensitivity analysis.

4.1.6 External costs

Road damage, local pollution, global pollution and noise costs are included in the environmental cost parameters, e_{gr}. All are assumed to be proportional to distance. Parry (2006, Table 1) reports pollution costs per gallon. The values in Table 1 are converted to costs per mile by dividing by Parry's assumed values for vehicle fuel economy.

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[14] Consistent with this view, Forkenbrock and March (2005, p.6) write: "According to FHWA [Federal Highway Administration], in 71 percent of two-vehicle fatal crashes involving a large truck and another vehicle, police reported 'one or more errors or other factors' related to the behavior of the passenger vehicle driver and none for the truck driver." Similar statistics are reported in US DOT (2006).

4.2 Results

4.2.1 Base case

With the base-case parameter values, the second-order condition holds and both the UE and the SO are integrated. *Lights* and *Heavies* both split 2:1 between Route 1 and Route 2 in proportion to the route capacities. The second-order condition holds as a strict inequality despite the fact that Conditions (15a) and (15b) for the congestion-cost and accident-cost coefficients are violated (the left-hand and right-hand sides of each condition are equal). The reason for this is that relative to *Heavies*, *Lights* inflict more accident than congestion costs, whereas relative to *Lights*, *Heavies* are more averse to congestion than to accidents. Consequently, it is efficient for *Heavies* to travel with *Lights* and for *Lights* to travel with *Heavies*. Since Assumptions 1-3 in Proposition 2 are all satisfied, the UE and SO coincide and nothing can be gained from either tolling or lane access restrictions.[15]

4.2.2 Sensitivity analysis with variations in the traffic mix

Some parameter values, such as the average VOT for trucks, vary widely from location to location. There is also appreciable uncertainty about the values of other parameters such as the scale parameters, λ_H^L and ϕ_H^L, and the relative cost of accidents for heavy vehicles, μ^H. Sensitivity analysis is therefore warranted. Table 2 summarizes the results for four parameters or sets of parameters: (a) VOT for heavy vehicles, (b) congestion-cost parameters, (c) accident-cost parameters, and (d) route characteristics.

(a) Value of time for heavy vehicles

The base-case VOT for *Heavies* is $v^H = \$50$/h. In Variant 1 (see Table 2) this is reduced to just \$15/h. The stability condition is still satisfied, but the second-order condition is violated. Consequently, the UE remains integrated, but in the SO *Lights* and/or *Heavies* are confined to one route throughout the range of traffic mix as shown in Figure 1(a). Restriction is beneficial because *Heavies* create much more congestion than do *Lights* while valuing travel time only slightly more. Hence $c_{H\bullet}^L > c_{L\bullet}^H$, and it is beneficial to keep *Heavies* away from *Lights*. If the proportion of *Heavies* is small, the *Heavies* are confined to the lower capacity route, Route 2. When the proportion of *Heavies* reaches 41%, all the *Heavies* are moved onto Route 1, and the majority of *Lights* are shifted onto Route 2. As Figure 1(b) shows, the Pigouvian tolls that support the SO take a small downward jump on Route 1 and a small upward jump on Route 2, and the toll differentials reverse sign. (The middle columns of Table 2 report the maximum and minimum toll differentials for each type over the range $N_H / N \in [0,1]$.) The welfare gain from tolling exhibits a double peak (Figure 1(c)) with a local minimum at the point where *Lights* and *Heavies* switch routes. The two peaks occur with *Heavy* proportions of 25% and 57% for which segregation is optimal (cf. Figure 1(a)), and consequently the benefits of keeping the types apart

[15] Naturally, this would not be true if travel demand were elastic. With 20% *Heavies* in the vehicle mix (a representative fraction for urban portions of the US Interstate Highway System) the SO tolls are \$4.62 for *Lights* and \$13.98 for *Heavies*: likely high enough to induce diversion of some trips to alternative routes or modes.

is greatest. At these points a lane-access rule to segregate the two types is as effective as is tolling – as shown by the curve labeled “Segregation” in Figure 1(c). However, segregation is beneficial only within a narrow range of traffic mix about each peak, whereas tolling yields appreciable benefits over much of the range. Consistent with Proposition 3, restrictions on either *Lights* or *Heavies* are not effective for any traffic mix.

In Variant 2, the VOT for *Heavies* is raised 50% above the base-case value to \$75/h. The results are qualitatively similar to those of Variant 1, but the welfare gain is much higher. Now, $c_{H\bullet}^{L} < c_{L\bullet}^{H}$ and the primary motivation for separating types is to minimize congestion for *Heavies* by giving them lots of road space. Because of the high VOT for *Heavies*, the two segregation points occur at much lower *Heavy* proportions than in Variant 1 (14% and 39% vs. 25% and 57%).

(b) Congestion-cost parameters

For Variant 3, the congestion PCE of *Heavies* is reduced from 2 to 1.5. The effects of doing so are qualitatively and quantitatively similar to the effects of raising v^H in Variant 2. By contrast, if the PCE of *Heavies* is raised from 2 to 3 (Variant 4) the second-order condition is still satisfied, and the results remain the same as in the base case.

In Variant 5, parameter λ_H^L is doubled from 1 to 2 to capture greater interference of *Lights* by *Heavies*. This upsets the stability condition, and the UE becomes partially separated or segregated as the *Lights* try to avoid the *Heavies*.[16] Simultaneously doubling λ_H^L and halving v^H (Variant 6) has a much more pronounced effect. Significant differences between the UE and SO route allocations are apparent (Figure 2(a)) and the tolls on the two routes vary in a rather complex way as the traffic mix changes (Figure 2(b)). The welfare gain is substantial (Figure 2(c)), and segregation is welfare-improving for an appreciable range of traffic mix. However, restrictions on either *Lights* or *Heavies* alone are still unproductive.

(c) Accident-cost parameters

Doubling the accident externality cost of *Heavies* (parameter PCE_{acc}) as in Variant 7 does not upset the stability condition. But the SO becomes partially separated or segregated. Halving the costs of accidents for *Heavies* does not affect results of interest, but doubling the costs (Variant 8) has a similar effect to raising the VOT of *Heavies* as in Variant 2.

In Variant 9, parameter ϕ_H^L is doubled from 1 to 2 to reflect a greater danger to *Lights* from accidents with *Heavies*. The effect of this change is nearly identical to doubling the costs of accidents for *Heavies* (Variant 8). Surprisingly, doubling ϕ_H^L again from 2 to 4 (Variant 10)

[16] As explained in Section 2, if the stability condition is violated and the nonnegativity conditions are satisfied (as they are for certain values of the traffic mix), there are two UE. The UE with the lower total social costs is assumed to prevail in Variant 5 and the other variants (Variants 6 and 10) in which the stability condition fails. This assumption biases downwards the inefficiency of the UE and the potential benefits from intervention.

dampens the welfare gain. The reason for this is that the stability condition is now violated, and *Lights* tend to separate themselves from *Heavies* in the UE – thereby leaving less scope for welfare-enhancing intervention. Hence, as is true of the VOT for *Heavies*, parameter ϕ_H^L has a non-monotonic effect on the welfare results.

(d) Route characteristics

For Variants 11 and 12 the two routes are assumed to differ in length, and consequently in free-flow travel times.[17] As discussed in Section 2, the shorter route is used excessively in the UE, and differential route tolls can be levied to correct the bias.[18] As Table 2 indicates, segregation is beneficial for a modest range of traffic mix. Interestingly, restricting *Lights* alone is also beneficial in Variant 11 when the proportion of *Heavies* is relatively large, and restricting *Heavies* is beneficial in Variant 12 when the proportion of *Heavies* falls in a narrow range. These results do not contradict Proposition 4 because Assumptions 1 and 2 of Proposition 2 are violated.

Finally, for Variant 13 the two routes are assumed to have equal capacities of 3,000 vehicles/h, and the VOT for *Heavies* is raised to $75/h as in Variant 2.[19] Despite the fact that the total capacity of the two routes is the same as in Variant 2, the maximum welfare gain from tolling in Variant 13 is about 20% lower. This is because it is efficient to devote the lion's share of road space to the group with the higher travel costs (*Heavies* here) – an option that is not available if the two routes have the same capacities. This illustrates the lesson that the gains from road pricing depend not only on the flexibility of tolls, but also on the scope for allocating road space between vehicle types in efficient proportions.

4.2.3 Sensitivity analysis with a given traffic mix

Figures 1 and 2 display only the effects of varying the traffic mix. To explore the comparative static properties of the numerical example further, Figures 3 and 4 show contour diagrams of the welfare changes from varying two parameters at once while holding the proportion of *Heavies* fixed at 20%. Figure 3 does so for parameters v^H and λ_H^L. A first observation is that the welfare gain from intervention is a convex function of v^H (the value of time for *Heavies*). This is a consequence of the fact that — as noted earlier — the benefits from separation vary with $\left\| c_{H\bullet}^L - c_{L\bullet}^H \right\|$. For low values of v^H, $c_{H\bullet}^L > c_{L\bullet}^H$, and it is beneficial to keep *Heavies* away from *Lights*. Correspondingly, for large values of v^H, $c_{H\bullet}^L < c_{L\bullet}^H$, and it is advantageous to keep *Lights* away from *Heavies*.

[17] Differences in travel time also arise because of differences in speed limits. However, since vehicle operating costs and externality costs are assumed to be proportional to distance, these costs change if the route lengths change. Differences in speed limits are therefore not equivalent to differences in route length that result in the same travel times.

[18] However, the environmental costs of travel are lower on the shorter route, and this dilutes the welfare gain from tolling.

[19] If only the capacities are changed the SO remains identical to the UE.

Figure 3 also reveals that the welfare gain is a concave function of λ_H^L (the scale factor for truck congestion) and reaches a maximum value with λ_H^L slightly above one. To see why, note that as λ_H^L rises the cross-congestion-cost coefficient, $c_{H\square}^L$, increases and with it the benefit from separating *Heavies* from *Lights*. But when λ_H^L exceeds a threshold value, the unregulated equilibrium becomes separated, and the scope for welfare improvement diminishes (although not to zero because the unregulated equilibrium route split is not optimal as explained regarding Proposition 4).

Figure 4 provides an analogous diagram for parameters v^H and ϕ_H^L (the scale factor for the accident hazard created by trucks). The main difference from Figure 3 is that the highest welfare gains accrue for a much higher value of ϕ_H^L than of λ_H^L. This is because the accident externality is much smaller than the congestion externality, so that greater percentage changes in the cross-accident-cost coefficient are required to have maximum effect.

Figure 5 displays the route allocations in the UE and SO that underlie Figure 4. The picture is quite complex — with no less than nine distinct regions. In Regions 1-3 the UE is integrated, in Regions 4-6 it is partially separated and in Regions 7-9 it is segregated. For the three regions depicted with solid patterns the SO configuration is the same as the UE configuration, and for the six regions with shaded patterns the configurations differ. The relative positions of the regions are intuitive. Integration in the UE occurs on the left of Figure 3 in Regions 1-3 where ϕ_H^L is small. Segregation occurs on the right in Regions 7-9. In Region 1, where ϕ_H^L is small and v^H takes on an intermediate value, integration is optimal. And in Regions 3, 5 and 8 where v^H is large, it is optimal to allocate more capacity to *Heavies* than they receive in the unregulated equilibrium.

5 CONCLUDING REMARKS

Truck-only facilities have been promoted as a way to combat road congestion, enhance safety and reduce pavement damage. This paper has focused on the first two factors by analyzing how light and heavy vehicles choose between traffic lanes or routes and whether the allocation can be improved by access regulations or tolling. Several factors were identified as important: the relative volumes of *Lights* and *Heavies*, the congestion delay and safety hazards that each type of vehicle imposes on each type, values of travel time and lane capacity indivisibilities.

One, perhaps unexpected, conclusion is that there is no presumption that *Lights* and *Heavies* should be segregated. Indeed, with the base-case parameter values the two types are integrated in both the unregulated user equilibrium and the system optimum, and neither tolling nor lane-access rules can improve the outcome. Nevertheless, for many plausible alternative parameter values partial separation or segregation is desirable, and it can be achieved using tolls that are differentiated by route and vehicle type. In general partial separation or segregation is warranted if one type suffers higher costs than it imposes. For example, this is the case for *Heavies* if *Heavies* have a particularly high value of time or a low PCE for congestion. And it is the case for *Lights* if *Heavies* impose on *Lights* either a disproportionately high congestion delay or safety hazard. Another finding is that the welfare gains from tolling vary non-monotonically with some key parameters – including the proportion of heavy vehicles in the traffic mix, the value of time for heavy vehicles, and the degree to which light vehicles suffer a disproportionate congestion or safety hazard from heavy vehicles.

This paper provides a preliminary and partial analysis of heavy vehicle facilities. A number of extensions deserve high priority.

1. *Elastic demand*: The numbers of light vehicles and heavy vehicles using the corridor were treated as given. With price elastic demand the numbers of vehicles and the vehicle mix would be endogenous. In addition to route or lane choices, tolls and access regulations would affect trip generation and mode choice decisions. A potential drawback of imposing high tolls on truck-only lanes or tollways is that they will induce truckers to use alternative untolled corridors that may not be designed to handle heavy vehicles, and that suffer severe congestion or accident hazards.
2. *Trip-timing preferences*. As noted in the Introduction, light and heavy vehicles tend to make trips at different times of day. Arnott et al. (1992) provide a simple theoretical analysis of when temporal segregation is a cost-effective alternative to spatial segregation. To examine this question empirically in the case of truck facilities it will be necessary to obtain data on trip-timing preferences for light and heavy vehicles. A complicating factor is that accident rates tend to vary by time of day and are higher at night (Shefer and Rietveld, 1997).
3. *Vehicle characteristics*: The model features just two vehicle categories. In reality, of course, both light vehicles and heavy vehicles differ in numerous characteristics such as size, safety, operating costs, emissions and so on, that are relevant to whether they choose to be, or should be, integrated or segregated on the road network.[20]
4. *Choice of vehicle type*: Vehicle characteristics are exogenous in the model. This is a reasonable assumption for analysis of a single travel corridor since most trucking firms would have little incentive to modify their vehicle fleets. The assumption sits less well for regional or national road networks – particularly since substantial productivity gains may be possible from using large combination vehicles (Samuel et al. 2002). Choice of passenger vehicle type also depends on perceived safety hazards (Brozović and Ando, 2005).
5. *Road design*: Finally, and perhaps most important, it is desirable to account for the effects of building dedicated truck-only facilities on road infrastructure construction and maintenance costs. According to Forkenbrock and March (2005) the costs of truck-only lanes depend on various factors including right-of-way availability, topography, the need to reconstruct overpasses to accommodate heavy vehicles, numbers of entrance and exit ramps required, and so on. Consequently, construction cost per lane-km. will have to be assessed on a case-by-case basis. Truck-only lanes tend to cost more per mile than do car lanes, but by concentrating heavy vehicles on part of the road infrastructure they allow the remainder to be built and maintained more cheaply.

[20] For example, White (2004) shows that amongst passenger vehicles lights trucks (i.e. Sports Utility Vehicles, vans and pickup trucks) impose a much higher danger than do cars to occupants of cars, motorcyclists and people using non-motorized forms of transport. Gayer (2004) finds that light trucks are 2.6 to 4 times as likely than cars to be involved in an accident per year. Accident rates also vary across truck types (US DOT, 2006).

6 REFERENCES

Arnott, R., A. de Palma and R. Lindsey (1992), "Route choice with heterogeneous drivers and group-specific congestion costs", Regional Science and Urban Economics 22(1), 71-102.

Arnott, R., A. de Palma and R. Lindsey (1998), "Recent developments in the bottleneck model", in K.J. Button and E.T. Verhoef (eds.), Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility, Aldershot: Edward Elgar, 79-110.

Bambe, A. and B.S. McMullen (1996), "Measuring the non-pecuniary costs of triple trailer operation in Oregon; A Contingent Valuation Approach", Journal of the Transportation Research Forum.

Barro, R. and P.M. Romer (1987), "Ski-lift pricing, with applications to labor and other markets", American Economic Review 77(5), 875-890.

Berglas, E., D. Fresko and D. Pines (1984), "Right of way and congestion toll", Journal of Transport Economics and Policy 18(2), 165-187.

Brozović, N. and A.W. Ando (2005), "Defensive purchasing, the safety (dis)advantage of light trucks, and motor-vehicle policy effectiveness", University of Illinois at Urbana-Champaign, October 28.

Dahlgren, J. (1998), "High occupancy vehicle lanes: Not always more effective than general purpose lanes", Transportation Research A 32A(2), 99-114.

Dahlgren, J. (2002), "High-occupancy/toll lanes: where should they be implemented?", Transportation Research A 36A, 239-255.

De Jong, G. (2000), "Value of freight travel-time savings", in D.A. Hensher and K.J. Button (eds.), Handbook of Transport Modelling 1, Oxford: Elsevier Science, 553-564.

Donaghy, K.P. and L.A. Schintler (1998), "Managing congestion, pollution, and pavement conditions in a dynamic transportation network model", Transportation Research D 3D(2), 59-80.

Douglas, J.G. (2005), "Planning truck facilities on urban highways", 84th Annual Meeting of the Transportation Research Board, Conference CD Paper No. 05-0863.

Federal Highway Administration (1997), 1997 Federal Highway Cost Allocation study. Final Report to the United States Congress HPP-10/9-98(3M)E, Washington, D.C.: Federal Highway Administration, US Department of Transportation.

Federal Highway Administration (2003), "The Freight story: A national perspective on enhancing freight transportation", FHWA-OP-03-004.

Fischer, M.J., D.N. Ahanotu and J.M. Waliszewski (2003), "Planning truck-only lanes: Emerging lessons from the Southern California Experience", 82nd Annual Meeting of the Transportation Research Board, Conference CD Paper No. 002048.

Forkenbrock, D.J. and J. March (2005), "Issues in the financing of truck-only lanes", Public Roads 69(2).

Gan, A. and S. Jo (2003), "Final Report: Operational performance models for freeway truck-lane restrictions", Project No. BD-015-01, Prepared for Office of the State Transportation Planner, Systems Planning Office, State of Florida Department of Transportation, April.

Gayer, T. (2004), "The fatality risks of sport-utility vehicles, vans, and pickups relative to cars", Journal of Risk and Uncertainty 28, 103-133.

Hedlund, K. (2004), "Truck tollways", 83rd Annual Meeting of the Transportation Research Board, presented at Session 546: Future role of road pricing and tolls in transportation finance.

Holguin-Veras, J., D. Sackey, S. Hussain and V. Ochieng (2003), “Economic and financial feasibility of truck toll lanes”, Transportation Research Record 1833, 66-72.
Janson, B.N. and A. Rathi (1991), “Economic feasibility of exclusive vehicle facilities”, Transportation Research Record 1305, 201-215.
Kawamura, K. (2003), “Perceived benefits of congestion pricing for trucks”, Transportation Research Record 1833, 59-65.
Kockelman, K. and R. Shabih (2000), “Effect of light-duty trucks on the capacity of signalized intersections”, Journal of Transportation Engineering 126(6), 506-512.
Koehne, J., F. Mannering and M. Hallenbeck (1996), “Analysis of trucker and motorist lane evaluation: an exploratory analysis”, Transportation Research A 24A(5), 371-379.
Mannering, F. and M. Hamed (1990), “Commuter welfare approach to high occupancy vehicle opinions toward truck-lane restrictions”, Transportation Research Record 1560, 73-82.
Middleton, D. and D. Lord (2005), "Safety and operational aspects of exclusive truck facilities", http://ceprofs.tamu.edu/dlord/Papers/Safety_Truck_Roadways_Middleton-Lord.pdf (Accessed April 1, 2006.)
Middleton, D. and S. Venglar (2006), "Operational aspects of exclusive truck roadways", 85th Annual Meeting of the Transportation Research Board, Washington, D.C., Conference CD Paper No. 06-0928.
Mohring, H. (1979), "The benefits of reserved bus lanes, mass transit subsidies, and marginal cost pricing in alleviating traffic congestion", in P. Mieszkowski and M. Straszheim (eds.), Current Issues in Urban Economics, Baltimore: John Hopkins Press.
Organization for Economic Cooperation and Development (OECD) (1992), Scientific Expert Group. Cargo Routes: Truck Roads and Networks. Road Transport Research, Paris.
Parry, I.W.H. (2006), “How should heavy-duty trucks be taxed?”, Resources for the Future Discussion Paper 06-23, http://www.rff.org/Documents/RFF-DP-06-23.pdf (Accessed July 3, 2006.)
Parsons, Brinckerhoff, Quade & Douglas (2005), “Truck only toll facilities: Potential for implementation in the Atlanta Region”. http://www.i10freightstudy.org/ (Accessed March 31, 2006.)
Pitfield, D. and R. Watson (2001), “High-occupancy routes and truck lanes”, in D.A. Hensher and K.J. Button (eds.), Handbook of Transport Systems and Traffic Control 3, Oxford: Elsevier Science, 413-423.
Poole, R.W., Jr. (2006), “Reducing congestion in Atlanta: A bold new approach to increasing mobility”, Reason Public Policy Institute Policy Study 351, November (www.reason.org/ps351.pdf, accessed December 10).
Poole, R.W., Jr. and P. Samuel (2004), “Corridors for toll truckways: Suggested locations for pilot projects”. Reason Public Policy Institute Policy Study 316.
Reich, S., J. Davis, M. Catalá, A. Ferraro and S. Concas (2002), “The potential for reserved truck lanes and truckways in Florida”, Center for Urban Transportation Research, University of South Florida.
Rouwendal, J., E.T. Verhoef, P. Rietveld and B. Zwart (2002), “A stochastic model of congestion caused by speed differences", Journal of Transport Economics and Policy 36(3), 407-445.
Samuel, P., R.W. Poole, Jr. and J. Holguin-Veras (2002), "Toll truckways: A new path toward safer and more efficient freight transportation", Reason Public Policy Institute http://www.rppi.org.

Shefer, D. and P. Rietveld (1997), "Congestion and safety on highways", Urban Studies 34(4), 679-692.
Small, K.A. (1977), "Priority lanes on urban radial freeways: An economic-simulation model", Transportation Research Record 637, 8-13.
Small, K.A. (1983), "Bus priority and congestion pricing on urban expressways", in T.E. Keeler (ed.), Research in Transportation Economics 1, Greenwich, Conn.: JAI Press, 27-74.
Small, K.A. and E.T. Verhoef (2006), The Economics of Urban Transportation, London and New York: Routledge. Chapter 3 accessed May 3, 2006 at http://www.socsci.uci.edu/~ksmall/research_page.htm.
Small, K.A. and J. Yan (2001), "The value of "value pricing" of roads: Second-best pricing and product differentiation", Journal of Urban Economics 49(2), 310-336.
Texas Department of Transportation (2002), Crossroads of the Americas: Trans Texas Corridor Plan, www.dot.state.tx.us/ttc/ttc_home.htm (Accessed March 31, 2006.)
Transportation Research Board (2000), Highway Capacity Manual, Transportation Research Board, National Research Council, Washington D.C.
Transportation Research Board (2003), “Freight capacity for the 21st century”.
US Department of Transport (2006), “The 100-car naturalistic driving study: A descriptive analysis of light vehicle-heavy vehicle interactions from the light vehicle driver's perspective”, Federal Motor Carrier Safety Administration, March (http://www.trb.org/news/blurb_detail.asp?id=7227, accessed April 27, 2007).
Verhoef, E.T., P. Nijkamp and P. Rietveld (1996), “Second-best congestion pricing”, Journal of Urban Economics 40(3), 279-302.
Verhoef, E.T., J. Rouwendal and P. Rietveld (1999), "Congestion caused by speed differences", Journal of Urban Economics 45(3), 533-556.
Verhoef, E.T. and K.A. Small (2004), "Product differentiation on roads: Constrained congestion pricing with heterogeneous users", Journal of Transport Economics and Policy 38(1), 127-156.
White, M.J. (2004), "The “arms race” on American roads: The effect of sport utility vehicles and pickup trucks on traffic safety", Journal of Law and Economics XLVII, 333-355.
Wilbur Smith Associates (2003), The National I-10 Freight Corridor Study: Summary of Findings, Strategies and Solutions http://www.i10freightstudy.org/ (Accessed July 3, 2006.)
Yang, H. and H.-J. Huang (1999), "Carpooling and congestion pricing in a multilane highway with high-occupancy-vehicle lanes", Transportation Research A 33A(2), 139-155.
Yun, S., W.W. White, D.R. Lamb and Y. Wu (2005), "Accounting for the impact of heavy truck traffic in volume/delay functions within transportation planning models", 84th Annual Meeting of the Transportation Research Board, Conference CD Paper No. 05-2163.

7 APPENDIXES

7.1 Appendix A: An algebraic property of products of sums

Proposition A1: Let *A*, *B*, *C*, *D*, *a*, *b*, *c* and *d* be arbitrary positive numbers and define

$$Z \equiv (A+a)(B+b)-(C+c)(D+d).$$

Assume that

$$AB = CD \quad \text{and} \quad ab = cd. \tag{A1}$$

Then *Z* can be positive, negative or zero.

Proof: Given (A1), $Z = Ab + aB - (Cd + cD) \overset{s}{=} \frac{a}{A} + \frac{b}{B} - \frac{CD}{AB}\left(\frac{c}{C} + \frac{d}{D}\right) = \frac{a}{A} + \frac{b}{B} - \left(\frac{c}{C} + \frac{d}{D}\right)$,

where $\overset{s}{=}$ means identical in sign, and the last equality follows from the first equality in (A1). The two equalities in (A1) imply that $\frac{ab}{AB} = \frac{cd}{CD}$, but this does not restrict the sign of $\frac{a}{A} + \frac{b}{B} - \left(\frac{c}{C} + \frac{d}{D}\right)$. QED

By making the substitutions $A = c_{L1}^{L}$, $a = c_{L2}^{L}$, $B = c_{H1}^{H}$, $b = c_{H2}^{H}$, $C = c_{H1}^{L}$, $c = c_{H2}^{L}$, $D = c_{L1}^{H}$ and $d = c_{L2}^{H}$, it follows from Proposition A1 that the two inequalities in eqn. (6) of the text do not assure that the stability condition (5) holds. And by making the substitutions $A = cong_{L\bullet}^{L}$, $a = acc_{L\bullet}^{L}$, $B = cong_{H\bullet}^{H}$, $b = acc_{H\bullet}^{H}$, $C = cong_{H\bullet}^{L}$, $c = acc_{H\bullet}^{L}$, $D = cong_{L\bullet}^{H}$ and $d = acc_{L\bullet}^{H}$, it follows that inequalities (15a) and (15b) do not assure that stability condition (14) holds.

7.2 Appendix B: Second-order conditions for a total cost minimum

A total cost minimum can be achieved by minimizing costs with respect to N_{L1} and N_{H1} subject to the constraints $N_{L2} = N_L - N_{L1}$ and $N_{H2} = N_H - N_{H1}$. The second-order condition for an interior minimum is that the following matrix of second-order partial derivatives be positive definite:

$$\begin{bmatrix} \frac{\partial^2 TSC}{\partial N_{L1}^2} & \frac{\partial^2 TSC}{\partial N_{L1}\partial N_{L2}} \\ \frac{\partial^2 TSC}{\partial N_{L2}\partial N_{L1}} & \frac{\partial^2 TSC}{\partial N_{L2}^2} \end{bmatrix} = \begin{bmatrix} 2c_{L\bullet}^{L} & c_{H\bullet}^{L} + c_{L\bullet}^{H} \\ c_{H\bullet}^{L} + c_{L\bullet}^{H} & 2c_{H\bullet}^{H} \end{bmatrix}.$$

The matrix is positive definite if $c_{L\bullet}^{L} > 0$, $c_{H\bullet}^{H} > 0$ (both conditions guaranteed) and

$4c_{L\bullet}^{L}c_{H\bullet}^{H} > \left(c_{H\bullet}^{L} + c_{L\bullet}^{H}\right)^2$ or

$$c_{L\bullet}^{L} c_{H\bullet}^{H} > c_{H\bullet}^{L} c_{L\bullet}^{H} + \frac{1}{4}\left(c_{H\bullet}^{L} - c_{L\bullet}^{H}\right)^{2}. \tag{A2}$$

Since the $c_{h\bullet}^{g}$ coefficients are constants (i.e. independent of N_{L1} and N_{H1}) *TSC* is a globally convex function of N_{L1} and N_{H1} if Condition (A2) holds.

7.3 Appendix C: Proof of Proposition 2

The equal-route-cost conditions for an integrated UE given in eqn. (2) are

$$\begin{bmatrix} c_{L\bullet}^{L} & c_{H\bullet}^{L} \\ c_{L\bullet}^{H} & c_{H\bullet}^{H} \end{bmatrix} \begin{bmatrix} N_{L1}^{e} \\ N_{H1}^{e} \end{bmatrix} = \begin{bmatrix} c_{L2}^{L} N_L + c_{H2}^{L} N_H + F_2^L - F_1^L \\ c_{L2}^{H} N_L + c_{H2}^{H} N_H + F_2^H - F_1^H \end{bmatrix}, \tag{A3}$$

where the superscript "e" denotes the UE. The first-order conditions for an integrated SO given in eqn. (11) are

$$\begin{bmatrix} 2c_{L\bullet}^{L} & c_{H\bullet}^{L} + c_{L\bullet}^{H} \\ c_{H\bullet}^{L} + c_{L\bullet}^{H} & 2c_{H\bullet}^{H} \end{bmatrix} \begin{bmatrix} N_{L1}^{o} \\ N_{H1}^{o} \end{bmatrix} = \begin{bmatrix} 2c_{L2}^{L} N_L + \left(c_{H2}^{L} + c_{L2}^{H}\right) N_H + F_2^L - F_1^L + e_{L2} - e_{L1} \\ \left(c_{H2}^{L} + c_{L2}^{H}\right) N_L + 2c_{H2}^{H} N_H + F_2^H - F_1^H + e_{H2} - e_{H1} \end{bmatrix}, \tag{A4}$$

where the superscript "o" denotes the SO. By Assumptions 1 and 2 of Proposition 2, the fixed cost and externality cost terms drop out of eqns. (A3) and (A4). Given Assumption 3 of Proposition 2, $c_{h2}^{g} = \rho c_{h\bullet}^{g}$, $g = L,H$, $h = L,H$ for some constant $\rho \in (0,1)$. Eqns. (A3) and (A4) can therefore be rewritten

$$\underbrace{\begin{bmatrix} c_{L\bullet}^{L} & c_{H\bullet}^{L} \\ c_{L\bullet}^{H} & c_{H\bullet}^{H} \end{bmatrix}}_{A} \begin{bmatrix} N_{L1}^{e} \\ N_{H1}^{e} \end{bmatrix} = \rho \underbrace{\begin{bmatrix} c_{L\bullet}^{L} & c_{H\bullet}^{L} \\ c_{L\bullet}^{H} & c_{H\bullet}^{H} \end{bmatrix}}_{A} \begin{bmatrix} N_L \\ N_H \end{bmatrix}. \tag{A5}$$

$$\underbrace{\begin{bmatrix} 2c_{L\bullet}^{L} & c_{H\bullet}^{L} + c_{L\bullet}^{H} \\ c_{H\bullet}^{L} + c_{L\bullet}^{H} & 2c_{H\bullet}^{H} \end{bmatrix}}_{B} \begin{bmatrix} N_{L1}^{o} \\ N_{H1}^{o} \end{bmatrix} = \rho \underbrace{\begin{bmatrix} 2c_{L\bullet}^{L} & c_{H\bullet}^{L} + c_{L\bullet}^{H} \\ c_{H\bullet}^{L} + c_{L\bullet}^{H} & 2c_{H\bullet}^{H} \end{bmatrix}}_{B} \begin{bmatrix} N_L \\ N_H \end{bmatrix}. \tag{A6}$$

By the stability condition (5), Matrix *A* in (A5) is nonsingular, and by the second-order condition (9), Matrix *B* in (A6) is nonsingular. The UE and SO therefore have a common solution with $N_{L1}^{e} = N_{L1}^{o} = \rho N_L$ and $N_{H1}^{e} = N_{H1}^{o} = \rho N_H$.

7.4 Appendix D: Proof of Proposition 3

The social optimum can be integrated, partially separated or segregated. The three cases are considered in turn.

(1) Integrated optimum

The equal-route-cost conditions for an integrated UE are given as in eqn. (A3) with toll differentials added to the right-hand side:

$$\begin{bmatrix} c_{L\bullet}^{L} & c_{H\bullet}^{L} \\ c_{L\bullet}^{H} & c_{H\bullet}^{H} \end{bmatrix} \begin{bmatrix} N_{L1}^{e} \\ N_{H1}^{e} \end{bmatrix} = \begin{bmatrix} c_{L2}^{L} N_L + c_{H2}^{L} N_H + F_2^L - F_1^L + \tau_2^L - \tau_1^L \\ c_{L2}^{H} N_L + c_{H2}^{H} N_H + F_2^H - F_1^H + \tau_2^H - \tau_1^H \end{bmatrix}. \tag{A7}$$

Toll differentials that support the SO can be solved simply by replacing N_{L1}^{e} and N_{H1}^{e} in (A7) by N_{L1}^{o} and N_{H1}^{o}, and rearranging the equations:

$$\begin{bmatrix} \tau_2^L - \tau_1^L \\ \tau_2^H - \tau_1^H \end{bmatrix} = \begin{bmatrix} F_1^L - F_2^L - c_{L2}^{L} N_L - c_{H2}^{L} N_H \\ F_1^H - F_2^H - c_{L2}^{H} N_L - c_{H2}^{H} N_H \end{bmatrix} + \begin{bmatrix} c_{L\bullet}^{L} & c_{H\bullet}^{L} \\ c_{L\bullet}^{H} & c_{H\bullet}^{H} \end{bmatrix} \begin{bmatrix} N_{L1}^{o} \\ N_{H1}^{o} \end{bmatrix}.$$

Since the SO is integrated, the stability condition is satisfied, and hence the equilibrium with tolls is stable.

(2) Partially separated optimum

Without loss of generality suppose *Lights* use both routes while *Heavies* are confined to Route 2. The number of *Lights* taking Route 1 in equilibrium is derived by substituting eqn. (1a) into (3a), setting $N_{H1} = 0$, and using the relationships $N_{L2} = N_L - N_{L1}$ and $N_{H2} = N_H - N_{H1} = N_H$:

$$N_{L1}^{e} = \left(c_{L\bullet}^{L}\right)^{-1} \left(c_{L2}^{L} N_L + c_{H2}^{L} N_H + F_2^L - F_1^L + \tau_2^L - \tau_1^L\right). \tag{A8}$$

The SO is derived in the same way except with eqn. (11) for *Lights*, i.e. $MSC_1^L = MSC_2^L$, in place of eqn. (3a):

$$N_{L1}^{o} = \left(c_{L\bullet}^{L}\right)^{-1} \left(c_{L2}^{L} N_L + \frac{c_{H2}^{L} + c_{L2}^{H}}{2} N_H + \frac{F_2^L - F_1^L}{2} + \frac{e_{L2} - e_{L1}}{2} \right). \tag{A9}$$

The toll differential for which $N_{L1}^{e} = N_{L1}^{o}$ is

$$\tau_2^L - \tau_1^L = \frac{F_1^L - F_2^L}{2} + \frac{e_{L2} - e_{L1}}{2} + \frac{c_{L2}^{H} - c_{H2}^{L}}{2} N_H. \tag{A10}$$

To deter *Heavies* from taking Route 1 the toll differential for *Heavies* must be such that $C_1^H \geq C_2^H$. After straightforward calculation one obtains

$$\begin{aligned} \tau_1^H - \tau_2^H \geq F_2^H - F_1^H + \frac{F_2^L - F_1^L}{2} + \frac{e_{L2} - e_{L1}}{2} + \left[c_{L2}^{H} - c_{L\bullet}^{H} \left(c_{L\bullet}^{L}\right)^{-1} c_{L2}^{L} \right] N_L \\ + \left[c_{H2}^{H} - c_{L\bullet}^{H} \left(c_{L\bullet}^{L}\right)^{-1} \frac{c_{H2}^{L} + c_{L2}^{H}}{2} \right] N_H. \end{aligned} \tag{A11}$$

The pair of toll differentials given in (A10) and the equality version of (A11) supports the SO if the stability condition is satisfied. But if the stability condition fails the solution is unstable. Figure A1 illustrates an unstable case in which the reaction curve for *Lights*, labeled *L*, is flatter than the reaction curve for *Heavies*, labeled *H*. Point *A* where the reaction curves intersect is an unstable equilibrium because the direction of adjustment in the north-west sector is away from *A*. Point *B* is a stable equilibrium. To support *A* as a stable equilibrium the toll differential for *Heavies*, $\tau_1^H - \tau_2^H$, must be increased until the *Heavies* reaction curve is shifted to H′ where it intersects the *L* reaction curve where $N_{L1} = 0$.

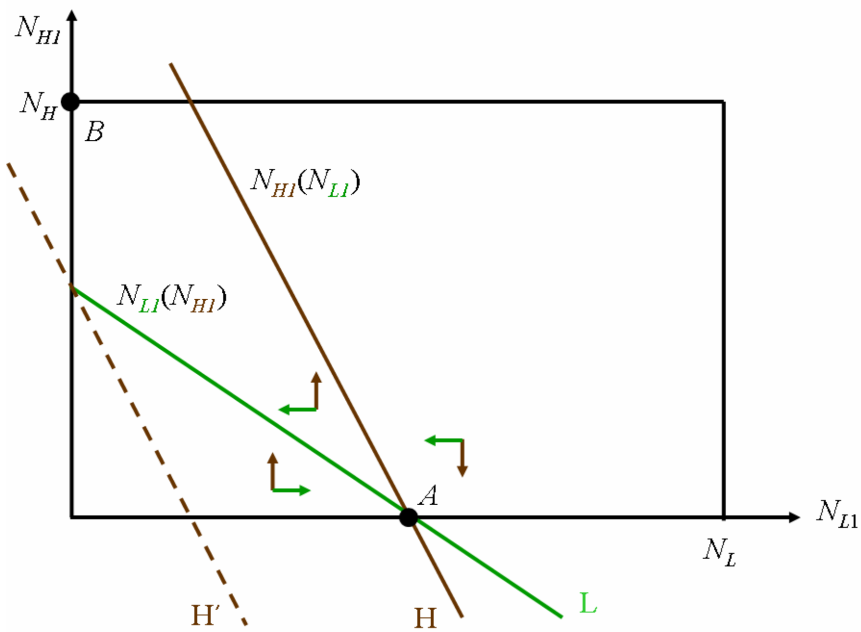


Figure A1: Unstable equilibrium with partial separation

(3) Segregated optimum

Suppose that in the SO *Lights* are relegated to Route 1 and *Heavies* to Route 2. The toll differentials that deter each type from unilaterally switching to the other route are

$$\tau_2^L - \tau_1^L \geq F_1^L - F_2^L + c_{L1}^L N_L - c_{H2}^L N_H \,, \tag{A12}$$

$$\tau_1^H - \tau_2^H \geq F_2^H - F_1^H + c_{H2}^H N_H - c_{L1}^H N_L \,. \tag{A13}$$

The equality variants of these differentials support the system optimum if the stability condition is met. But if it is not satisfied the differentials have to be raised. As Figure A2 illustrates, this can be done in three ways: (1) by shifting the *Lights* reaction curve right to *L*′, (2) by shifting the *Heavies* reaction curve down to *H*′, or (3) by shifting both curves; e.g. to *L*′′ and *H*′′.

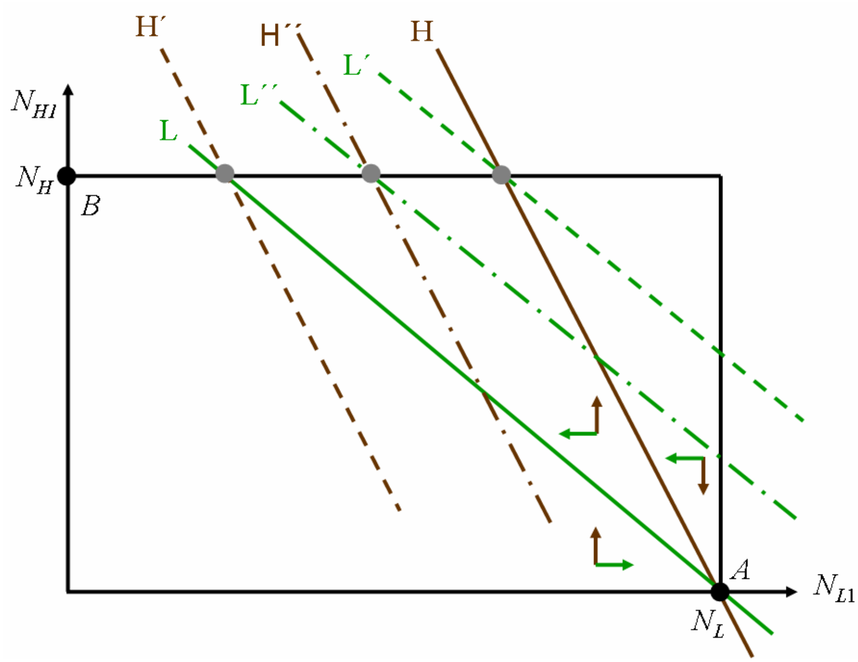


Figure A2: Unstable equilibrium with segregation

7.5 Appendix E: Explanation for Proposition 4

Proposition 4 can be proved by calculating *TSC* in the integrated UE, *TSC* in any of the four possible partially separated UE, and evaluating the difference in costs. To understand Proposition 4, suppose – following the text – that in the restricted UE, *Heavies* are confined to Route 2. The number of *Lights* taking Route 1 without tolls is given by eqn. (A8), and the number taking Route 1 in the SO is given by (A9). Subtracting (A9) from (A8) gives

$$N_{L1}^e - N_{L1}^o = \frac{1}{2}\left(c_{L\bullet}^L\right)^{-1}\left(F_1^L - F_2^L + e_{L1} - e_{L2} + \left(c_{H2}^L - c_{L2}^H\right)N_H\right). \qquad \text{(A14)}$$

Under Assumptions 1-3 of Proposition 2, (A14) simplifies to

$$N_{L1}^e - N_{L1}^o = \frac{1}{2}\left(c_{L\bullet}^L\right)^{-1}\left(c_{H2}^L - c_{L2}^H\right)N_H = \frac{\rho}{2}\frac{c_{H\bullet}^L - c_{L\bullet}^H}{c_{L\bullet}^L}N_H\,,$$

where ρ is defined as in Appendix C by the equation $c_{h2}^g = \rho c_{h\bullet}^g,\ g = L,H,\ h = L,H$.

Routes			
Characteristic		Route 1	Route 2
Capacity		4,000 PCE /hour	2,000 PCE /hour
Speed limit		65 mph	65 mph
Length		32.5 miles	32.5 miles
Demand			
$N_L + N_H$		40,000 trips per day	
Proportion of *Heavies*		Range 0-100%	
Volume-independent user costs			
Component	Symbol in model	*Light* vehicles	*Heavy* vehicles
Operating & maint. (incl. fuel tax)		$0.131/mile[b]	
Variable component of vehicle capital cost		$0.063/mile[b]	
Sum		$0.194/mile[b]	$0.42/mile[c]
Values of time	v^g	$12/hour[d]	$50/hour[d]
Congestion cost coefficients			
Component	Symbol in model	*Light* vehicles	*Heavy* vehicles
Light-Light coeff.	$cong_{Lr}^{L}$	See text[d]	
PCE, *Heavies*	PCE_{cong}		2[a]
Relative impedance of *Lights* by *Heavies*	λ_H^L	1[e]	
Accident cost coefficients			
Component	Symbol in model	*Light* vehicles	*Heavy* vehicles
Light-Light coeff.	acc_{Lr}^{L}	See text[d]	
PCE, *Heavies*	PCE_{acc}		0.75[a]
Relative cost of accident for *Heavies*	μ^H		1[d]
Hazard factor	ϕ_H^L	1[e]	
External costs			
Road damage		$0.0000/mile[a]	$0.0740/mile[a]
Local pollution		$0.0133/mile[a]	$0.0857/mile[a]
Global pollution		$0.0080/mile[a]	$0.0286/mile[a]
Noise		$0.0010/mile[a]	$0.0270/mile[a]
Sum	e_{gr}	$0.0223/mile[a]	$0.2153/mile[a]

Table 1: Base-case parameter values for numerical example

Sources: [a] Parry (2006, Table 1), [b] Small & Verhoef (2006, Table 3.3), [c] Samuel, Poole and Holguin-Veras (2002, Table 4-3), [d] Authors' judgment [e] Authors' assumption

Variant	Parameters	Stability condition satisfied?	Second-order condition satisfied?	Toll differential: $\tau_2^\bullet - \tau_1^\bullet$ Lights Min	Lights Max	Heavies Min	Heavies Max	Maximum welfare gain	Frac. of Heavies in traffic mix for which segregation beneficial (steps of 0.01)
Base case		Yes	Yes	\$0.00	\$0.00	\$0.00	\$0.00	\$0	None
(a) Value of time for heavy vehicles									
1	v^H = \$15/h	Yes	No	-\$0.81	\$0.92	-\$1.32	\$1.50	\$931	[0.23,0.27], [0.55,0.60]
2	v^H = \$75/h	Yes	No	-\$2.54	\$1.86	-\$6.01	\$4.40	\$7,128	[0.11,0.18], [0.33,0.48]
(b) Congestion-cost parameters									
3	PCE_{cong}=1.5	Yes	No	-\$2.53	\$1.98	-\$4.18	\$3.28	\$7,275	[0.14,0.24], [0.39,0.56]
4	PCE_{cong}=3	Yes	Yes	\$0.00	\$0.00	\$0.00	\$0.00	\$0	None
5	$\lambda^L{}_H$=2	No	No	-\$2.81	\$2.27	-\$4.19	\$5.98	\$66	0.13, 0.38
6	$\lambda^L{}_H$=2, v^H = \$25/h	No	No	-\$1.60	\$1.60	-\$8.46	\$11.37	\$6,982	[0.22,0.32], [0.52,0.65]
(c) Accident-cost parameters									
7	PCE_{acc}=1.5	Yes	No	-\$1.48	\$1.12	-\$3.29	\$2.48	\$2,747	[0.14,0.19], [0.39,0.48]
8	μ^H = 2	Yes	No	-\$1.94	\$1.46	-\$4.12	\$3.11	\$4,782	[0.13,0.19], [0.37,0.49]
9	$\varphi^L{}_H$=2	Yes	No	-\$1.65	\$1.24	-\$3.29	\$2.48	\$4,634	[0.14,0.20], [0.38,0.50]
10	$\varphi^L{}_H$=4	No	No	-\$2.00	\$1.54	-\$2.39	\$1.85	\$3,002	[0.13,0.18], [0.38,0.47]
(d) Route characteristics									
11	Rte 2 length 30 miles	Yes	No	\$0.00	\$1.43	-\$0.08	\$3.01	\$2,233	[0.15,0.21]
12	Rte 2 length 35 miles	Yes	No	-\$1.77	\$0.00	-\$3.88	\$0.08	\$2,480	[0.14,0.16], [0.41,0.50]
13	Rte capacs 3,000/h. v^H = \$75/h	Yes	No	-\$2.16	\$2.13	-\$5.11	\$5.04	\$5,569	[0.20,0.31]

Table 2: Sensitivity analysis

Source: Authors' construction

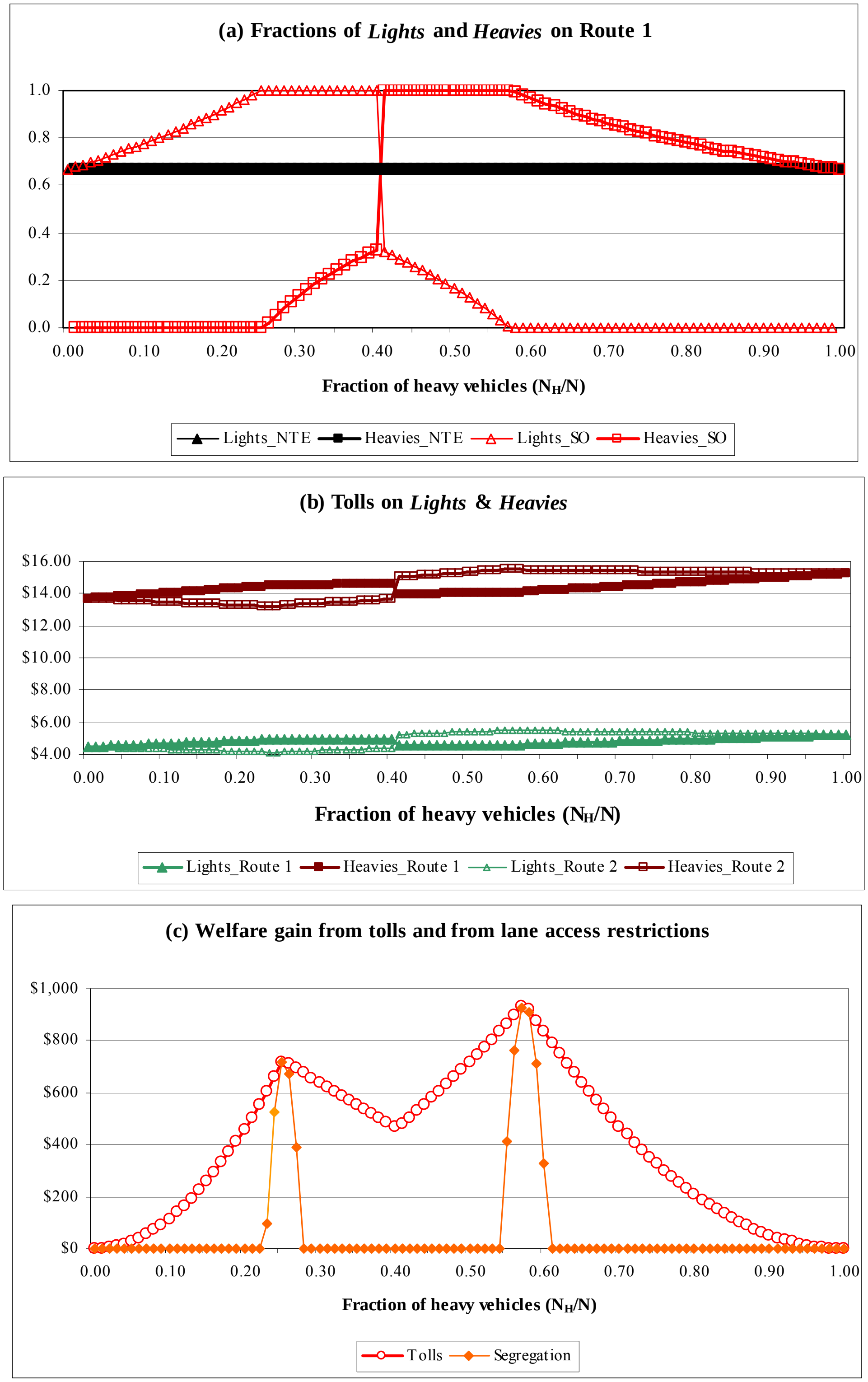


Figure 1: Results for Variant 1: $v^H = \$15/h$

(a) Fraction of *Lights* and *Heavies* on Route 1, (b) Tolls on *Lights* and *Heavies*, (c) Welfare gain from tolls and from lane access restrictions

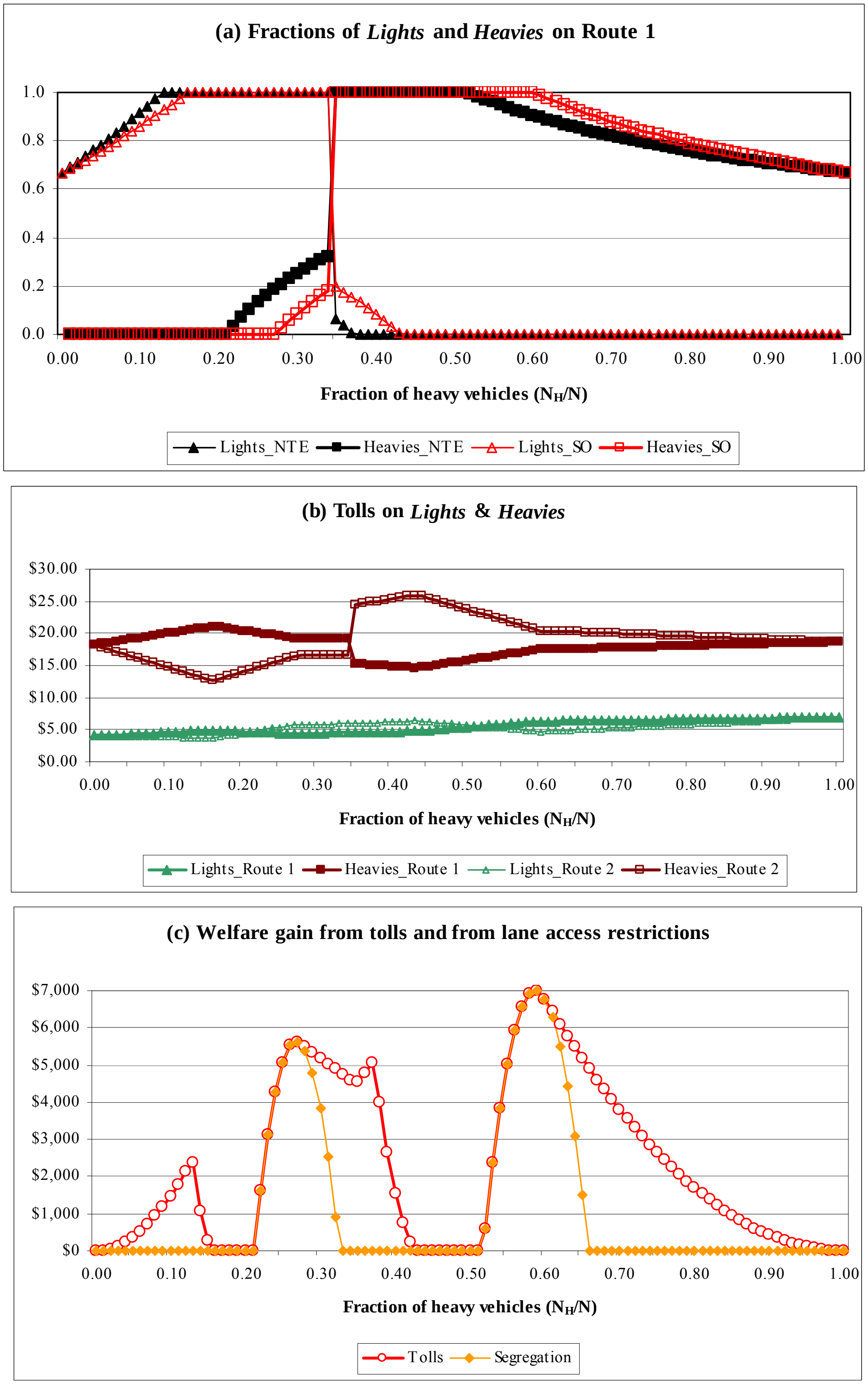


Figure 2: Results for Variant 6: $\lambda_H^L = 2$, $v^H = \$25/h$

(a) Fraction of *Lights* and *Heavies* on Route 1, (b) Tolls on *Lights* and *Heavies*, (c) Welfare gain from tolls and from lane access restrictions

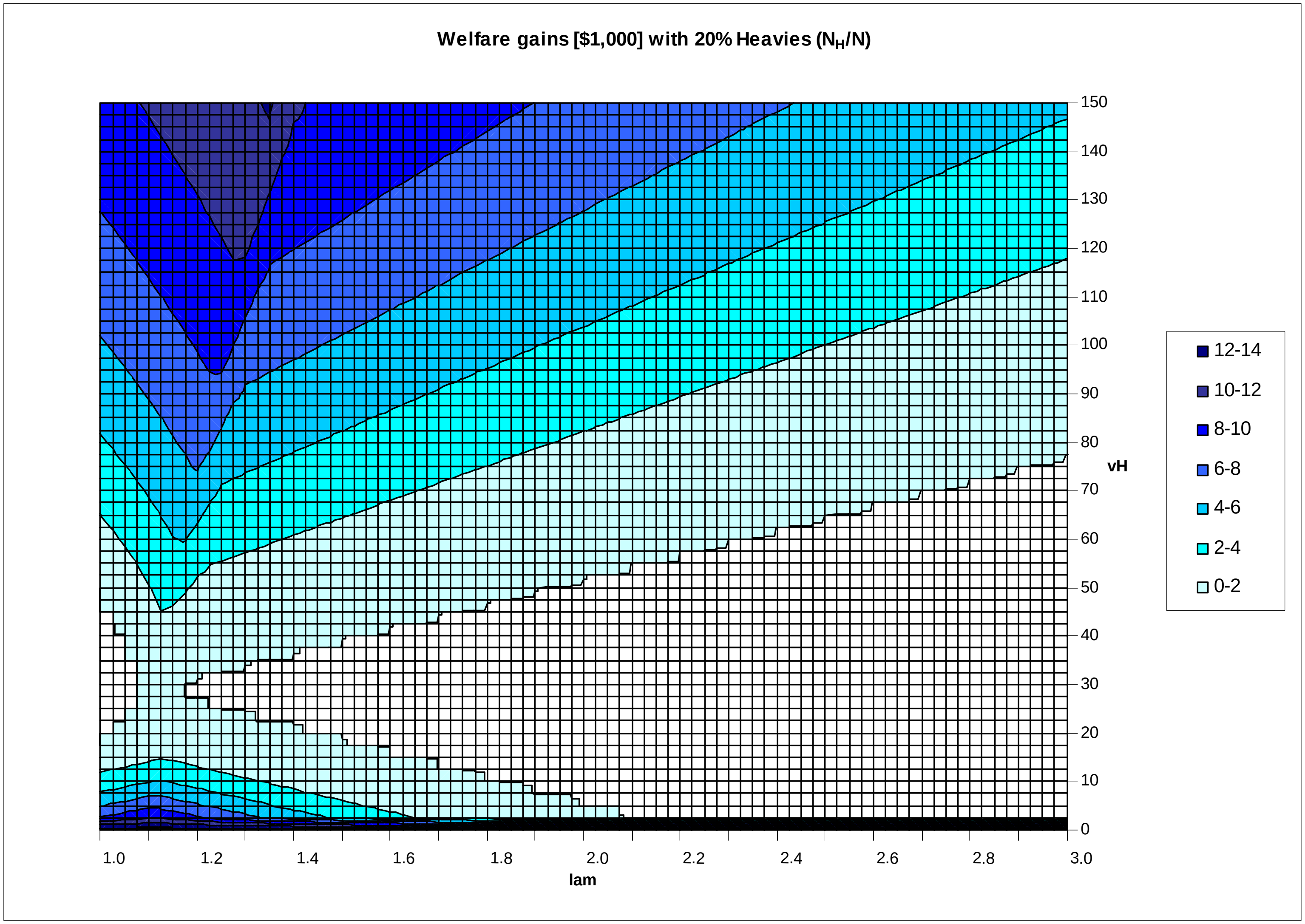


Figure 3: Welfare gains from tolling as a function of v^H and λ_H^L with 20% *Heavies*

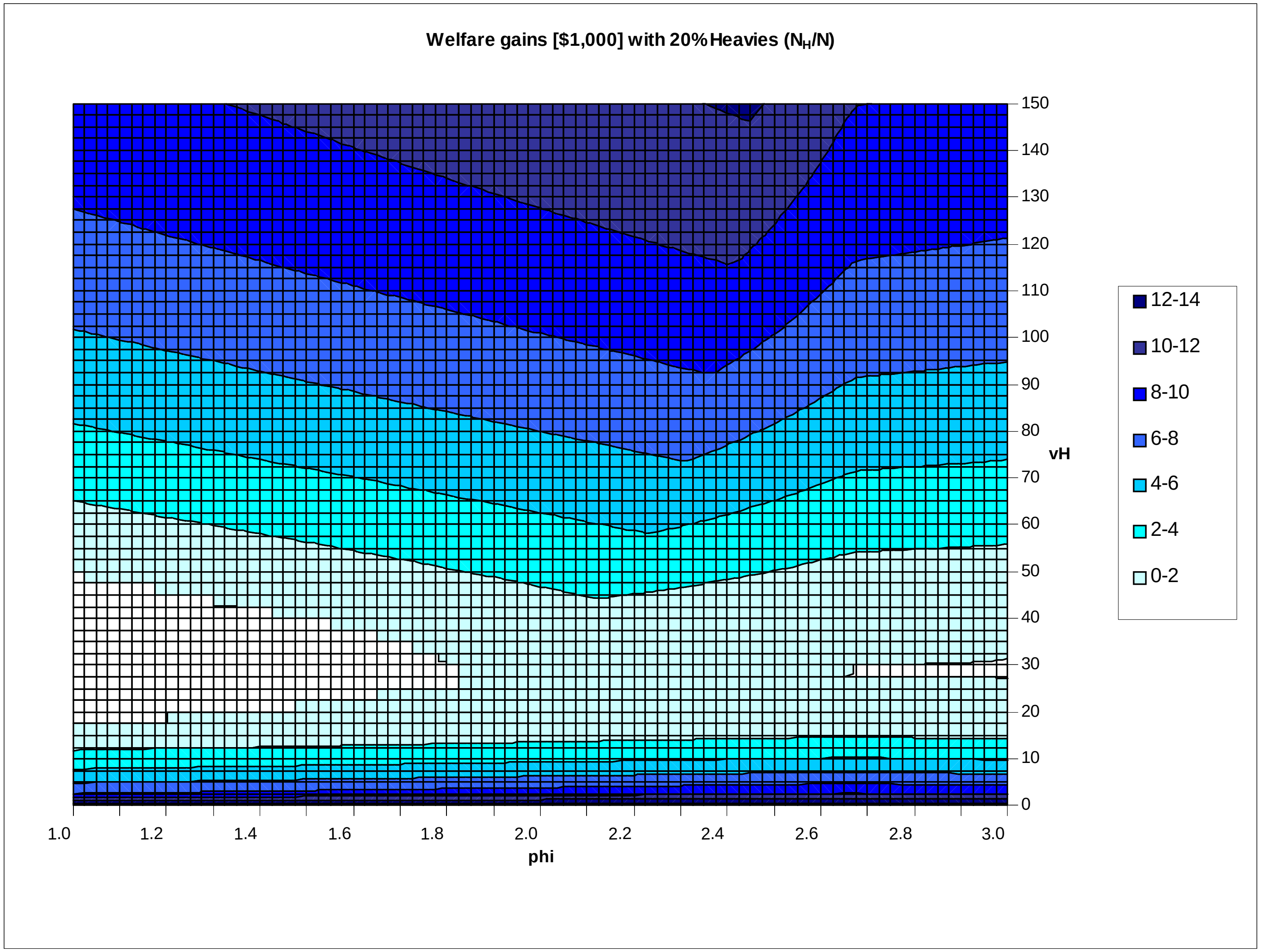
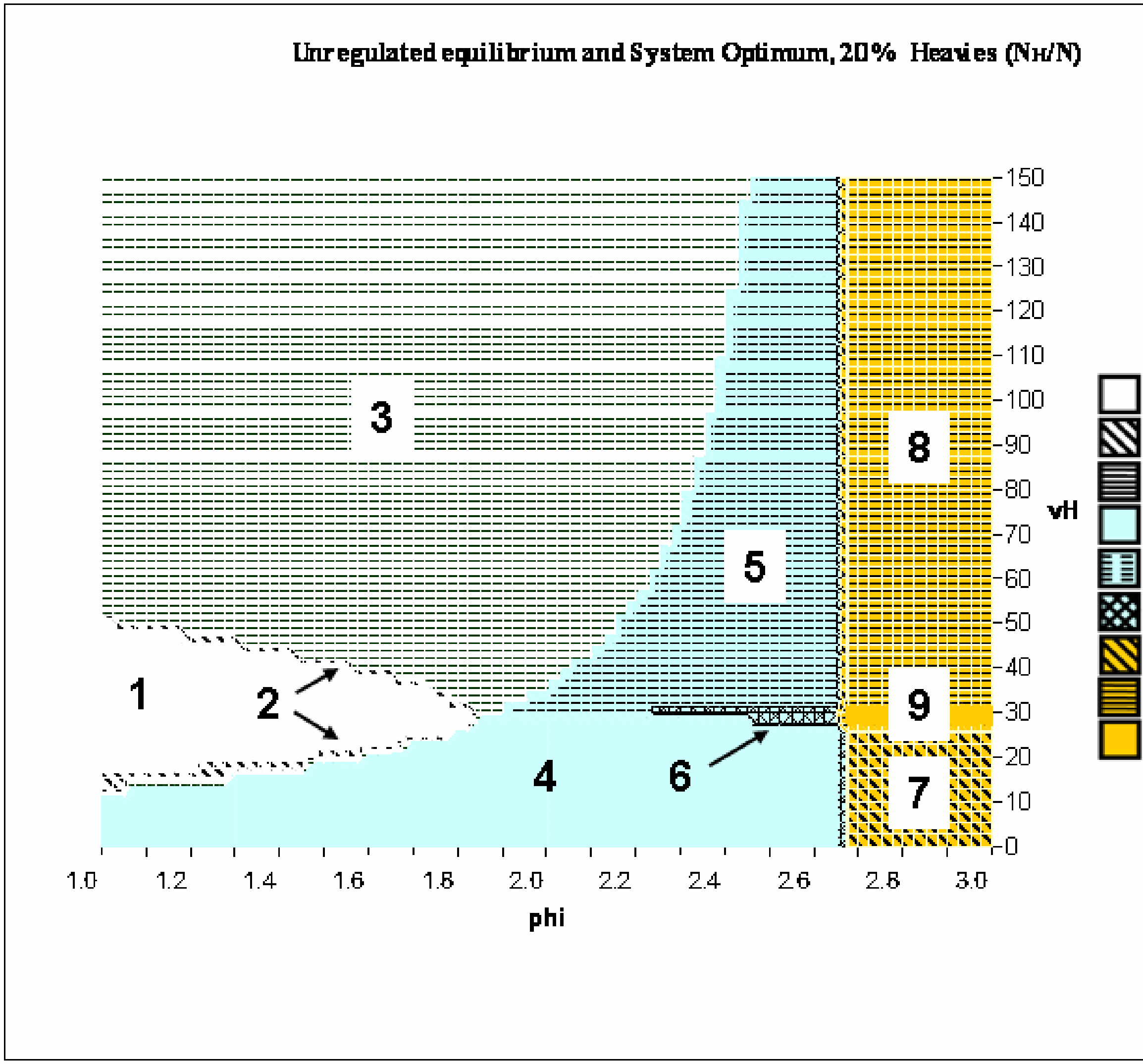


Figure 4: Welfare gains from tolling as a function of v^H and ϕ_H^L with 20% *Heavies*



Region	Unregulated equilibrium Rte 1 | Rte 2	System Optimum Rte 1 | Rte 2
1	LH | LH	LH | LH
2	LH | LH	L | LH
3	LH | LH	LH | H
4	L | LH	L | LH
5	L | LH	LH | H
6	L | LH	L | H
7	L | H	L | LH
8	L | H	LH | H
9	L | H	L | H

Figure 5: Allocation patterns in unregulated equilibrium and social optimum as a function of v^H and ϕ_H^L with 20% *Heavies*