On linking microsimulation and applied GE by exact aggregation of heterogeneous discrete-choice making agents

Riccardo Magnani and Jean Mercenier*†

THEMA - U. de Cergy-Pontoise, France

February 22, 2007

Abstract

Our paper contributes to bridge the gap between the microsimulation's approach and applied GE models, by making use of exact aggregation results from the discrete choice literature: heterogeneous individuals choosing (possibly continuous amounts) within a set of discrete alternatives may be aggregated into a representative agent with CES/CET preferences/technologies. These results therefore provide a natural link between the two policy evaluation approaches. We illustrate the usefulness of these results by evaluating potential effects of population ageing on the dynamics of income distribution and inequalities, using a simple OLG model when individuals have to make leisure/work decisions, and choose a profession among a discrete set of alternatives.

JEL classification: C63; C68; C81; D31; D58; E17; J10; J22.

Keywords: Microsimulation; Applied OLG models; Exact aggregation; Discrete choice; Population ageing; Income inequality.

*Riccardo.Magnani@u-cergy.fr and Jean.Mercenier@u-cergy.fr (corresponding author)

†We wish to thank Nathalie Picard for very helpful discussions, as well as André De Palma for comments.

1 Introduction

During the last twenty years, applied GE models have become standard tools of quantitative policy assessment. Their appeal has built on their rigorous grounding in economic theory: individual agents' decision-making behavior is derived from explicit optimization under strictly specified technological or budget constraints, given market signals that ensure global consistency. These theoretical foundations have made applied GE models appear particularly useful for ex-ante evaluations of policy reforms. Convincing as this argument may be, it can only be sustained if ex-post performance evaluations are made, and sources of prediction errors identified and taken care of.[1] Many reasons could of course contribute to explain why most applied GE models would probably fail to a serious ex-post prediction test. The theoretical mechanism hypothesized in the model may not be appropriate: Kehoe (2003), for example, suggests that "no plausible parameter changes can get the models of NAFTA built on the Dixit-Stiglitz specification to match what actually has happened". Another reason, often (over- ?) stressed by statistically oriented econometricians, is that applied GE modelers tend to excessively rely on guesstimated rather than on rigorously estimated parameter values; more generally, that applied GE modelers pay too little attention on the data-set they use (see *e.g.* Mercenier and Yeldan, 1999). Yet another – and potentially more serious – reason is that the whole apparatus relies on the concept of "representative agent" despite unclear aggregation procedures to link these aggregate optimizing decision-makers to the numerous individual agents whose behavior they are meant to capture.

During the same period, microsimulation models have also become increasingly popular tools for policy analysis precisely because they avoid any reliance on *typical agents* by fully taking into account the heterogeneity of individuals as they are observed in micro-

[1] Amazingly, the methodology has rarely been submitted to such tests: notable exceptions are Kehoe *et al.* (1995) and Kehoe (2003). Though the former's conclusion – based on a single-country perfectly-competitive model of the 70s' – sounded rather positive and optimistic, the latter's assessment – built on three of the most prominent applied GE models constructed to predict the impact of NAFTA – is quite devastating: "Theses models drastically underestimated the impact of NAFTA on North American trade. Furthermore, the models failed to capture much of the relative impacts on different sectors." (Kehoe, 2003, p0).

data sets. See Bourguignon and Spadaro (2006) for an excellent survey and an extensive list of references. Indeed, working with myriads of actual economic agents rather than with a few hypothetical ones makes it possible to precisely identify the winners and the losers of a reform – obviously a major concern to policy-makers – yet, making it possible by simple addition to accurately measure this impact on aggregate variables. The increasing availability of large and detailed data-sets on individual agents makes this quite appealing. The drawback of this approach is of course that it is partial equilibrium in essence: individuals' labor supply adjustments to some new tax incentive scheme, for instance, may be accurately captured for given wages and other policy parameters, but market equilibrium and government budget constraints can be expected to have a feedback influence that is typically neglected. One could of course imagine iterations between the microsimulation and the applied GE models, and indeed, a few efforts have successfully been done in this direction: see for instance Savard (2003) and Arntz *et al.* (2006). Though this iterative strategy might indeed be satisfactory for some problems – in particular when dynamics are thought unimportant – it is likely to be unfeasible for those requiring more sophisticated apparatus such as OLG models. Analyzing policy issues in a context of a demographic change, for instance, would obviously require a different approach. It is the object of this paper to suggest one such approach.

Our paper contributes to bridge the gap between the two approaches by making use of some simple yet powerful exact aggregation results due to Anderson, de Palma and Thisse (1992) (here after: AdPT). They show that, under reasonably mild conditions, heterogeneous individuals that have to choose (possibly continuous amounts) within a set of (possibly subsets of) discrete alternatives may be aggregated into a representative agent with (possibly multiple-level) CES/CET preferences/technologies. These results therefore provide a natural and appealing link between the standard applied GE apparatus and the microsimulations approach. It also potentially makes available to applied GE modelers a growing body of empirical results drawn from panel-data econometrics. There is no free lunch, unfortunately: some details captured by the microsim approach could be lost in the aggregation, a cost that one should balance against the benefits of accounting for the GE feedbacks.

We illustrate the usefulness of these results in the context of a simple OLG model.

Simulations will be done in vitro – i.e., using a computer generated data-set – to explore the potential consequences of population ageing on the dynamics of income distribution and inequalities, when individuals have to make leisure/work decisions, and choose a profession among a discrete set of alternatives.

The paper is organized as follows: in Section 2, we provide a refresher on probabilistic discrete choice models, and show how the basic aggregation results emerge from assuming multinomial logit heterogeneity in preferences. We then apply these results in Section 3 to modeling nested choices between leisure/work - professions and imbed this decision problem into an OLG model that is sketched in Section 4. We then submit in Section 5 the economy to an ageing shock, and plug into individual decision problems the computed equilibrium prices to evaluate the effect of population ageing on the dynamic path of income inequality indicators. The paper closes with a brief conclusion.

2 Discrete-choice models: a refresher

Assume a population of individuals $h = 1, ..., N$ has to choose among a set $i, j = 1, ..., n$ of discrete alternatives with associated utility levels:

$$\widetilde{u}_j^h = u_j + \epsilon_j^h \qquad j = 1, ..., n$$

where u_j is a deterministic component (for now, assumed common to all individuals) and ϵ_j^h is a random term. Each h is therefore characterized by a draw $\epsilon = (\epsilon_1^h, ..., \epsilon_n^h)$ in a probability distribution with cumulative density function $F(\epsilon)$. Assume that individuals in this population are not only statistically identical but also statistically independent. Then, the distribution of choices is multinomial with mean $\overline{X}_j = NP_j$, $j = 1, ..., n$, where P_j denotes the probability that alternative j be chosen by h. $\overline{X}_j$ is the mathematical expectation of demand for alternative j; for N large enough, $\overline{X}_j$ is a close approximation of aggregate demand for j in this population. In other words, aggregate demands for each alternative may be readily determined from the choice probabilities from the individual discrete decision problem.

The probability that h will choose alternative j is:

$$\begin{aligned} P_j &= prob\left[\widetilde{u}_j^h \geq \widetilde{u}_i^h, \forall i = 1, ..., n\right] \\ &= prob\left[u_j + \epsilon_j^h \geq u_i + \epsilon_i^h, \forall i = 1, ..., n\right] \\ &= prob\left[\epsilon_i^h - \epsilon_j^h \leq u_j - u_i, \forall i = 1, ..., n\right] \end{aligned}$$

The determination of the choice probabilities using $F(\epsilon)$ is in principle always possible but in general extremely difficult, in particular if ϵ is assumed normally distributed, as would seem natural. Fortunately, a theorem due to McFadden[2] identifies a class of distribution functions $F(\epsilon)$ – of which the multinomial logit is a special case – for which these probabilities may be easily determined indirectly. Consider the generalized extreme value distribution function

$$F(\epsilon_1, ..., \epsilon_n) = \exp\left[-H(e^{-\epsilon_1}, ..., e^{-\epsilon_n})\right]$$

with H a nonnegative function defined over R_+^N satisfying the following properties: (i) H is homogeneous of degree $1/\mu$; (ii) $lim_{x_i \to \infty} H(x_1, ..., x_n) = \infty \ \forall i = 1, ..., n$; (iii) the mixed partial derivatives of H with respect to k different variables exist and are continuous, non-negative if k is odd, non-positive if k is even, $k = 1, ..., n$. (These technical conditions are needed to ensure that $F(\epsilon)$ is indeed a cumulative probability distribution function.) Then, the choice probabilities P_j may be determined as:

$$P_j = \mu \frac{\partial \ln H(e^{u_1}, ..., e^{u_n})}{\partial u_j}$$

It can easily be checked that the following particularization of H,

$$H(\epsilon_1, ..., \epsilon_n) = \sum_{j=1}^{n} \epsilon_j^{1/\mu}$$

satisfies the previous properties. The cumulative distribution function becomes:

$$F(\epsilon_1, ..., \epsilon_n) = \exp\left[-\sum_{j}^{n} e^{-\epsilon_j/\mu}\right] = \prod_{j}^{n} \exp\left[-e^{-\epsilon_j/\mu}\right]$$

that is, the product of n i.i.d. double exponential distributions characterizes the stochastic behavior of utilities $\widetilde{u}_j$, and it follows from the theorem that

$$P_j = \mu \frac{\partial \ln \sum_i^n e^{u_i/\mu}}{\partial u_j} = \frac{e^{u_j/\mu}}{\sum_i^n e^{u_i/\mu}} \tag{1}$$

[2] See McFadden 1978, p.80; 1981, p.227.

which are the choice probabilities derived from a multinomial-logit population with dispersion parameter μ. This of course makes the MNL quite appealing. It turns out that, in addition, it provides a good approximation to the normal distribution.[3] Observe that, from (1),

$$\frac{\partial P_j}{\partial u_i} = -\frac{P_j P_i}{\mu} \qquad i, j = 1, ...n, \ i \neq j$$

so that the cross-elasticities

$$Elas(P_j, u_i) = -\frac{P_i u_i}{\mu} \qquad i, j = 1, ...n, \ i \neq j$$

are independent of j. Any change in the deterministic utility level associated with alternative i will therefore affect symmetrically the choice probabilities of all other alternatives: relative aggregate demands between two alternatives are unaffected by variations in the utility level of a third alternative. This over-restrictive property, known as the *independence of irrelevant alternatives*, can be bypassed by nesting multinomial logit systems, as we shall now illustrate.

Assume that the set A of alternatives $j = 1, ..., n$ can be partitioned into m subsets $\{A_l; l = 1, ..., m\}$ of close alternatives. We particularize the $H(\epsilon_1, ..., \epsilon_n)$ function as follows:

$$H_A(\epsilon_1, ..., \epsilon_n) = \sum_{l=1}^{m} \left[\sum_{i \in A_l} \epsilon_i^{1/\mu_2} \right]^{\mu_2/\mu_1} \tag{2}$$

This function is homogeneous of degree $1/\mu_1$; McFadden has shown that if $\mu_1 \geq \mu_2$, this function satisfies all the properties required to apply the extreme value theorem. It follows that

$$F(\epsilon_1, ..., \epsilon_n) = \exp \left\{ -\sum_{l}^{m} \left[\sum_{i \in A_l} e^{-\epsilon_i/\mu_2} \right]^{\mu_2/\mu_1} \right\}$$

and

$$\begin{aligned} P_j &= \mu_1 \frac{\partial \ln \sum_l^m \left[\sum_{i \in A_l} e^{u_i/\mu_2} \right]^{\mu_2/\mu_1}}{\partial u_j} \\ &= \frac{\left[\sum_{i \in A_l} e^{u_i/\mu_2} \right]^{\mu_2/\mu_1}}{\sum_l^m \left[\sum_{i \in A_l} e^{u_i/\mu_2} \right]^{\mu_2/\mu_1}} \cdot \frac{e^{u_j/\mu_2}}{\sum_{i \in A_l} e^{u_i/\mu_2}} \qquad j \in A_l \end{aligned} \tag{3}$$

[3] Ben Akiva and Lerman (1985, p128) write: "there is still no evidence to suggest in which situations the greater generality of the multinomial probit is worth the additional computational problems resulting from its use." We are not aware that such evidence has been reported in the literature since then.

This expression has a structure that makes it straightforward to understand. The second term is the probability that, within the subset A_l of alternatives, j be chosen. The first term represents the probability that among all subsets of A, A_l be chosen.

The expression can be given an alternative welfare interpretation. To see this, consider a subset A_l of alternatives, and define

$$\begin{aligned} H_{A_l} &= H_{A_l}(\epsilon_i, i \in A_l) = \sum_{i \in A_l} \epsilon_i^{1/\mu_2} && (4) \\ G_{A_l} &= G_{A_l}(u_i, i \in A_l) = \mu_2 \ln \sum_{i \in A_l} e^{u_i/\mu_2} && (5) \end{aligned}$$

It can be shown (see e.g. AdPT, p60) that G_{A_l} is the *expected value of the maximum of utilities from the alternatives in subset A_l*, which can therefore be interpreted as a measure of the attractiveness of the subset A_l. Dividing G_{A_l} by μ_1 and using an exponential transform yields:

$$e^{G_{A_l}/\mu_1} = \left[\sum_{i \in A_l} e^{u_i/\mu_2} \right]^{\mu_2/\mu_1}$$

Upon substitution of H_{A_l} into (2), we get:

$$H_A = H_A(H_{A_l}, l = 1, ..., m) = \sum_{l}^{m} [H_{A_l}]^{\mu_2/\mu_1}$$

Note the similarity of this expression with (4). We can write the expected value of the maximum of utilities from choosing between the different subsets of alternatives as:

$$G_A = \mu_1 \ln \sum_{l}^{m} e^{G_{A_l}/\mu_1}$$

which can be transformed to yield:

$$e^{G_A/\mu_1} = \sum_{l}^{m} e^{G_{A_l}/\mu_1} = \sum_{l}^{m} \left[\sum_{i \in A_l} e^{u_i/\mu_2} \right]^{\mu_2/\mu_1}$$

Hence, making use of those expressions into (3), the probability P_j takes an intuitive structure:

$$P_j = \frac{e^{G_{A_l}/\mu_1}}{\sum_l^m e^{G_{A_l}/\mu_1}} \cdot \frac{e^{u_j/\mu_2}}{\sum_{i \in A_l} e^{u_i/\mu_2}} \qquad (6)$$

Comparing (6) with (3), we see that the first term is a logit choice probability between $l = 1, ..., m$ alternatives, each alternative being priced by the expected maximum utilities from alternatives belonging to subset A_l. The nested discrete choice problem can therefore quite simply be solved sequentially, one level after the other, up the decision tree. It is immediate to generalize this to any number q of nested discrete choices, provided that $\mu_1 \geq \mu_2 \geq ... \geq \mu_q$ where q is the lowest level in the decision tree, i.e. where individual heterogeneity is lowest.

Note that the non idiosyncratic part of the utilities will in general depend on some exogenous characteristics of both the option (such as a market price) and of the decision-maker (such as age, sex etc). For illustrative purpose, assume u_j depends on the market price p_j associated with option j, on the individual's income y^h and on some exogenous characteristics z_j common to a subset of individuals within the population:

$$\begin{aligned}\widetilde{u}_j^h &= const^h + \alpha \ln y^h - \ln p_j + \gamma_j z_j + \epsilon_j^h \\ &= const^h + \alpha \ln y^h - \ln (p_j / f(z_j)) + \epsilon_j^h \\ &= const^h + \alpha \ln y^h - \ln \widetilde{p}_j + \epsilon_j^h\end{aligned}$$

where $f(z_j) = \exp(\gamma_j z_j)$ and $\widetilde{p}_j = p_j / f(z_j)$. We see that the presence of the decider's characteristics into the utilities only affects the valuation of the option by all individuals in the population who share the same characteristics. Assuming the random terms $\epsilon_1, ..., \epsilon_n$ are i.i.d. double exponentials, the choice probabilities are given by:

$$\begin{aligned}P_j &= \frac{\exp\left\{\left(const^h + \alpha \ln y^h - \ln \widetilde{p}_j\right)/\mu\right\}}{\sum_i^n \exp\left\{\left(const^h + \alpha \ln y^h - \ln \widetilde{p}_i\right)/\mu\right\}} \qquad j = 1, ..., n \\ &= \frac{\exp\left\{-\ln \widetilde{p}_j / \mu\right\}}{\sum_i^n \exp\left\{-\ln \widetilde{p}_i / \mu\right\}} \qquad j = 1, ..., n \\ &= \frac{\widetilde{p}_j^{\,-1/\mu}}{\sum_i^n \widetilde{p}_i^{\,-1/\mu}} \qquad j = 1, ..., n \end{aligned} \tag{7}$$

Aggregate demands for each option j from the population subset with common characteristics z_j are then closely approximated by multiplying this individual choice probability by the (large enough) number of individuals N in that subset:

$$\begin{aligned}\overline{X}_j &= P_j \cdot N \\ &= \frac{\widetilde{p}_j^{\,-1/\mu}}{\sum_i^n \widetilde{p}_i^{\,-1/\mu}} \cdot N \qquad j = 1, ..., n\end{aligned}$$

We are now equipped to represent individual nested discrete choice between leisure and work in different professions, and to derive a representative agent formulation that replicates the aggregation of individuals' decisions.

3 Modeling leisure/work decisions and the choice of a profession

3.1 The discrete choice formulation

The population is partitioned into k=*1,...,*K cells according to as many characteristics as made possible by the available data, such as sex, age-class etc. In what follows, we model the decision problems of individuals belonging to one such cell, and neglect the subscript of the cell to ease notation. In the applied GE model there will be one representative agent for each cell.

Consider one individual h belonging to a cell, therefore belonging to a sub-population *with the same socioeconomic characteristics*. This individual has to decide whether to work or not, and if he does, in which profession. We model this as a two-level discrete choice problem; we take advantage of the nested structure to solve the problem sequentially starting with the choice of profession.

3.1.1 Choosing between professions

There are I possible professions indexed i, j. We write the utility as a log-linear function:

$$\widetilde{v}_i^h = \ln \theta_i + \ln w_i + \epsilon_i^h \qquad i = 1, ..., I$$

The first term captures the (common to all options) disutility of working as well as the welfare costs/benefits of various characteristics specific to profession i, and w_i is the market wage (adjusted for characteristic-specific efficiency) expressed in terms of the consumption good. Note that these two terms are common to all h within the considered population cell. We therefore assume here that, upon making their optimal decisions, individuals ignore possible within-cell idiosyncratic productivity differences, that will ex-post be responsible for the observed distribution of wages in the data.[4] We refer to this within-cell average

[4] The additional information contained in the within-cell distribution of individual wages w_i^h will be

wage w_i as the Mincer wage for that cell as opposed to the individual wage w_i^h. In the ex-post microsimulations we will of course evaluate how distortive will be the substitution of w_i for w_i^h in individual decisions. Intra-cell individual heterogeneity in preferences is then captured by the i.i.d. double exponential stochastic term ϵ_i^h with dispersion parameter μ. From the previous section, we know that the probability h will choose profession i is:

$$\begin{aligned} P_i &= \frac{\exp\left(\frac{\ln\theta_i+\ln w_i}{\mu}\right)}{\sum_j \exp\left(\frac{\ln\theta_j+\ln w_j}{\mu}\right)} \\ &= \frac{\theta_i^{1/\mu}\cdot w_i^{1/\mu}}{\sum_j \theta_j^{1/\mu}\cdot w_j^{1/\mu}} \end{aligned}$$

3.1.2 Choosing whether to work or to leisure

Let the utility h enjoys from not working be:

$$\widetilde{V}_0^h = \ln\Theta_0 + \varepsilon_0^h$$

where ε_0^h is a random term which captures individual heterogeneity in the valuation of leisure (the disutility of working). The alternative is for the individual to work, taking into account that if he does so, he will be able to choose the best profession. The valuation of the alternative *work* that is consistent with the second stage decision problem is, from (5):

$$\widetilde{V}_1^h = V_1 + \varepsilon_1^h$$

where:

$$\begin{aligned} V_1 &= \mu\ln\sum_i \exp\left(\frac{\ln\theta_i+\ln w_i}{\mu}\right) \\ &= \mu\ln\sum_i \theta_i^{1/\mu}\cdot w_i^{1/\mu} \end{aligned}$$

We assume that $\varepsilon_0^h, \varepsilon_1^h$ are double exponential i.i.d. random terms with dispersion

used in the econometric estimation of the parameters of the discrete-choice preferences, in the calibration of the general equilibrium model, and in the ex-post microsimulations.

parameter $\upsilon > \mu$. The probability that h will choose to *farniente* is therefore:

$$\begin{aligned} P_0 &= \frac{\Theta_0^{1/\upsilon}}{\Theta_0^{1/\upsilon} + \exp(V_1/\upsilon)} \\ &= \frac{\Theta_0^{1/\upsilon}}{\Theta_0^{1/\upsilon} + \left[\sum_i \theta_i^{1/\mu} \cdot w_i^{1/\mu}\right]^{\mu/\upsilon}} \end{aligned}$$

3.1.3 Aggregation of individual choices

Let there be a large enough set N of statistically identical and independent individuals in this population cell; each individual has one unit of time. The within-cell aggregate labor supply resulting from individual discrete choices is then closely approximated by the mathematical expectations for option "work":

$$\begin{aligned} L &= (1 - P_0) \cdot N \\ &= \frac{\left[\sum_i \theta_i^{1/\mu} \cdot w_i^{1/\mu}\right]^{\mu/\upsilon}}{\Theta_0^{1/\upsilon} + \left[\sum_i \theta_i^{1/\mu} \cdot w_i^{1/\mu}\right]^{\mu/\upsilon}} \cdot N \end{aligned}$$

from which the cell's labor-supply by professions follows immediately:

$$\begin{aligned} L_i &= P_i L \\ &= \frac{\theta_i^{1/\mu} \cdot w_i^{1/\mu}}{\sum_j \theta_j^{1/\mu} \cdot w_j^{1/\mu}} \cdot \frac{\left[\sum_j \theta_j^{1/\mu} \cdot w_j^{1/\mu}\right]^{\mu/\upsilon}}{\Theta_0^{1/\upsilon} + \left[\sum_j \theta_j^{1/\mu} \cdot w_j^{1/\mu}\right]^{\mu/\upsilon}} \cdot N \qquad i = 1, \cdots, I \end{aligned} \tag{8}$$

3.2 The representative agent formulation

Our next task is to write an optimization problem for a representative agent[5] seeking to split his total time N between leisure and professional activities, such that the optimal allocation coincides with the one generated from aggregation of individual discrete choices (8). We proceed in two steps.

We first determine the optimal share of total time N between leisure and work. Let $S_{\mathcal{L}}$ and S_L denote some measure of time devoted respectively to leisuring ($\mathcal{L}$) and working (L), and λ be the household's relative valuation of leisure: the index λ is of course inversely related to market wages, in a way that will be established later, but is here assumed given.

[5] One for each population-cell, but here again we neglect the cell index k to ease notation.

The representative agent chooses $S_{\mathcal{L}}$ and S_L so as to maximize $\lambda S_{\mathcal{L}} + S_L$ subject to a constant elasticity of transformation (CET) constraint:

$$\left(\alpha_{\mathcal{L}}\left[S_{\mathcal{L}}\right]^{\frac{\tau+1}{\tau}} + \alpha_L\left[S_L\right]^{\frac{\tau+1}{\tau}}\right)^{\frac{\tau}{\tau+1}} = 1 \qquad \tau > 0$$

that captures the fact that moving in and out of the job market is not costless. It immediately follows from the FOC that the optimal ratio of time spent on the two activities is:

$$\frac{S_{\mathcal{L}}}{S_L} = \left[\frac{\alpha_{\mathcal{L}}}{\alpha_L}\right]^{-\tau} \cdot \lambda^{\tau} \tag{9}$$

Making use of (9) jointly with the resource constraint $\mathcal{L}+L = N$ yields the household's optimal labor supply:

$$L = \frac{\alpha_L^{-\tau}\lambda^{-\tau}}{\alpha_{\mathcal{L}}^{-\tau} + \alpha_L^{-\tau}\lambda^{-\tau}} \cdot N \tag{10}$$

The second step of the decision problem consists in allocating this work time between professions taking into account relative market wages and the fact that switching profession is not costless. Formally, the representative agent problem is to choose s_i so as to maximize $\sum_i w_i s_i$ subject to a constant elasticity of transformation (CET) constraint:

$$\left(\sum_i \alpha_i \cdot \left[s_i\right]^{\frac{\sigma+1}{\sigma}}\right)^{\frac{\sigma}{\sigma+1}} = 1 \qquad \sigma > 0$$

This yields the optimal ratios:

$$\frac{s_i}{s_j} = \left[\frac{\alpha_i}{\alpha_j}\right]^{-\sigma} \cdot \left[\frac{w_i}{w_j}\right]^{\sigma} \qquad i \neq j$$

which, jointly with the resource constraint $\sum_i L_i = L$ determines the amount of time devoted to working in each profession:

$$L_i = \frac{\alpha_i^{-\sigma} \cdot w_i^{\sigma}}{\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}} \cdot L$$

Making use of (10), we can substitute out L and get:

$$L_i = \frac{\alpha_i^{-\sigma} \cdot w_i^{\sigma}}{\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}} \cdot \frac{\alpha_L^{-\tau} \cdot \lambda^{-\tau}}{\alpha_{\mathcal{L}}^{-\tau} + \alpha_L^{-\tau} \cdot \lambda^{-\tau}} \cdot N \tag{11}$$

Let the household's relative leisure valuation index λ be inversely related to market wages by the following function:

$$\lambda = \alpha_L^{-1}\left[\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}\right]^{-\frac{1}{\sigma}}$$

so that (11) can be rewritten as:

$$L_i = \frac{\alpha_i^{-\sigma} \cdot w_i^{\sigma}}{\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}} \cdot \frac{\left[\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}\right]^{\frac{\tau}{\sigma}}}{\alpha_{\mathcal{L}}^{-\tau} + \left[\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}\right]^{\frac{\tau}{\sigma}}} \cdot N \tag{12}$$

Comparing this expression with (8), we see that, though the interpretation of the parameters differs considerably, the two expressions are identical provided that we set:

$$\begin{cases} \sigma = 1/\mu \\ \tau = 1/\upsilon \\ \alpha_i = 1/\theta_i \\ \alpha_{\mathcal{L}} = 1/\Theta_0 \end{cases} \tag{13}$$

For each population cell $k = 1, ..., K$, there are N^k individuals facing a specific Mincer wage vector w_i^k, $i = 1, ..., n$, and having preference characteristics θ_i^k, $\mu^k, \Theta_0^k, \upsilon^k$. These parameters can, in principle, be estimated using discrete choice econometric techniques and the aggregate labor-supply systems (12) plugged into the general equilibrium model. In this paper, however, we computer-generate the micro data-set and assume arbitrary though reasonable values for the parameters.

3.3 The OLG set-up

We now have, for each population cell, a different labor-supply system generated from aggregation of individual discrete choices, that is, there are as many representative labor-supplying agents as there are socioeconomic characteristics of interest in the micro data-base. This could suggest that, without restrictions on the number of these characteristics, we would rapidly run into the "curse of dimensionality" in the general equilibrium set-up, which would of course drastically limit the appeal of the current approach. Fortunately, this is not the case. Indeed, if we adopt identical and standard homothetic intertemporal preferences, we can aggregate further these representative labor-supplying agents into a single (per-generation) representative consumer that optimally allocates its human wealth to lifetime consumption.[6]

[6] The OLG structure we use is fairly standard; see e.g. Mercenier *et al.* (2005) for an illustrative use in the context of population ageing. To avoid excessive lengthening of the paper, we only sketch it here. A complete list of equations is available upon request.

We distinguish between G generations that coexist at each time period t. At the end of each period, the oldest group $g(G)$ disappears and a new generation $g(1)$ enters the active population according to the following rule:

$$N_{g(1),t+1} = \eta_t \cdot N_{g(1),t} \tag{14}$$

where $N_{g(1),t}$ denotes the number of young people at time t and η_t is an exogenous gross reproduction rate. Each agent maximizes its intertemporal utility subject to its wealth constraint. Doing so, he chooses: *(a)* the intertemporal profile of consumption (and therefore of assets accumulation); *(b)* how much to work, and in which profession (for those generations that are active, retirement is exogenously fixed at some late age). Formally, lifetime utility for the generation that becomes active at time t is:

$$U_t = \sum_{k=1}^{G} \mathfrak{R}^{k-1} \cdot \ln c_{g(k),t+k-1} \tag{15}$$

where $\mathfrak{R}$ is an exogenous discount factor and c is consumption. U is maximized subject to:

$$\sum_{k=1}^{G} R_{t+k-1} \cdot \left(m_{g(k),t+k-1} - c_{g(k),t+k-1}\right) = 0 \tag{16}$$

where R_t is the market determined discount factor: $R_{t+k-1} = \prod_{s=t+1}^{t+k-1} \left(\frac{1}{1+r_s}\right)$ and $m_{g(k),t}$ is labor income (net of social security contributions at rate τ_{sc}) and pension benefits:

$$\begin{aligned} m_{g(k),t} \quad = \quad & \sum_{i=1}^{I} \sum_{sex} (1-\tau_{sc}) \cdot A_{i,g(k),sex,t} \cdot w_{i,t} \cdot s_{g(k),sex,t} \cdot l_{i,g(k),sex,t} \\ & +pens_{g(k),t} \end{aligned}$$

where $w_{i,t}$ is the per unit of effective labor wage in profession i, $s_{g(k),sex,t}$ the proportion of males and females in the population by class age, $l_{i,g(k),sex,t}$ the proportion of professions by class age and sex ($l_{i,g(k),sex,t} = \frac{L_{i,g(k),sex,t}}{N_{g(k),t} \cdot s_{g(k),sex,t}}$ from (12)), and labor productivity $A_{i,g(k),sex,t}$ depends on characteristics such as age and sex:

$$\ln A_{i,g(k),sex,t} = \varphi_{1,i} k + \varphi_{2,i} k^2 + \varphi_{3,i} sex \tag{17}$$

The economy produces one good in amount X using physical capital K and effective labor of different professions $\mathfrak{L}_i$ with a constant returns to scale Cobb-Douglas technology:

$$X_t = \prod_{i=1}^{I} \mathfrak{L}_{i,t}^{\alpha_i} \cdot K_t^{\beta}$$

A pension system is Pay-As-You-Go with fixed social security rate τ_{sc}, the replacement ratio γ being endogenously determined to ensure balanced social security budget at each t:

$$pens_{g(k),t} = \gamma_t \cdot \sum_{i=1}^{I} \sum_{sex} A_{i,g(k),sex,t} \cdot w_{i,t} \cdot s_{g(k),sex,t} \cdot l_{i,g(k),sex,t} \tag{18}$$

The capital stock accumulation depends on investments and on capital depreciation:

$$K_{t+1} = K_t \cdot (1 - \delta) + Inv_t \tag{19}$$

The price system ($w_{i,t}$, r_t) is determined so that markets balance at each time period:

$$X_t = \sum_k N_{g(k),t} \cdot c_{g(k),t} + Inv_t \tag{20}$$

$$\mathfrak{L}_{i,t} = \sum_k \sum_{sex} N_{g(k),t} \cdot A_{i,g(k),sex,t} \cdot s_{g(k),sex,t} \cdot l_{i,g(k),sex,t} \tag{21}$$

4 The dynamics of income distribution in an ageing population: an illustrative example

In this section, we wish to illustrate the usefulness of the aggregation results, and test their robustness. We evaluate potential effects of population ageing on the dynamics of income distribution and inequalities, using the OLG model particularized to the case where individuals have to make leisure/work decisions, and choose one of two possible professions (indicated by Prof-0 and Prof-1). Addressing such issues requires a consistent use of both the microsimulation set-up – to keep track of individuals – and the general equilibrium. For this, we shall use a plausible artificial computer-generated micro data-set of 30.000 individuals, and link this to an applied OLG model calibrated on a fictitious macro data-set that can be thought of as representative of some archetype OECD economy. Assuming the dynamic economy is initially in a stationary steady state, we then submit it to a quite drastic demographic slowdown.

4.1 The micro data-set

In this stationary population, we distinguish individuals by gender and age groups of ten years each, starting at age 15. Only those belonging to the first five age classes have discrete choices to make: to work or not to work, and in which profession. Those from the last three generations are exogenously retired from the labor force. There are 30.000 such decision-making individuals, each belonging to one specific cell of characteristics, in proportions conveyed by **Table 1.**

Males	Number	***Females***	Number
g(1) : 15-24	3000	*g(1)* : 15-24	3000
g(2) : 25-34	3000	*g(2)* : 25-34	3000
g(3) : 35-44	3000	*g(3)* : 35-44	3000
g(4) : 45-54	3000	*g(4)* : 45-54	3000
g(5) : 55-64	3000	*g(5)* : 55-64	3000
total	15000	total	15000

Table 1: Number of individual decision-makers by age and sex

Mincer wages by professions are generated using the following equation:

$$\ln w_i = const_i + \alpha_{1i} \cdot age + \alpha_{2i} \cdot age^2 + \alpha_{3i} \cdot sex$$

The parameters adopted for this equation are reported in **Table 2**. The quadratic term is of course meant to capture the hump-shape of labor productivity with respect to age.

	Prof-0	Prof-1
constant	5	6
age	0.3	0.45
age x age	-0.005	-0.003
sex	-0.4	-0.35

Table 2: The parameters of the Mincer equations

Idiosyncratic productivity differences and wages w_i^h are generated using a normal distribution with average levels w_i and standard deviations as reported in **Table 3a** and **Table 3b**. Observe that the latter are chosen sufficiently large for the accuracy test to be meaningful.

	Males		*Females*	
	Mean	*Stand dev*	*Mean*	*Stand dev*
g(1) : 15-24	170.669	41.891	114.008	28.463
g(2) : 25-34	288.831	55.985	220.420	37.137
g(3) : 35-44	401.843	77.223	298.008	49.980
g(4) : 45-54	512.599	95.286	388.631	65.002
g(5) : 55-64	661.020	127.120	489.150	83.912

Table 3a: Parameters of the distributions of individual wages by age and sex for Prof-0

	Males		*Females*	
	Mean	*Stand dev*	*Mean*	*Stand dev*
g(1) : 15-24	554.909	137.339	348.526	88.735
g(2) : 25-34	875.275	210.080	606.427	139.897
g(3) : 35-44	1399.315	316.811	922.660	213.110
g(4) : 45-54	2089.774	508.444	1397.954	320.523
g(5) : 55-64	3175.437	767.594	2045.990	487.340

Table 3b: Parameters of the distributions of individual wages by age and sex for Prof-1

The preference parameters are chosen so as to generate reasonable shares of leisure and work, as well as contrasted activity shares by professions: see **Table 4**.

	Leisure / Total	Work / Total	Prof-0 / Work	Prof-1 / Work
Males				
g(1) : 15-24	17.47%	82.53%	61.79%	38.21%
g(2) : 25-34	15.33%	84.67%	74.61%	25.39%
g(3) : 35-44	14.60%	85.40%	51.80%	48.21%
g(4) : 45-54	12.67%	87.33%	58.59%	41.41%
g(5) : 55-64	25.47%	74.53%	51.03%	48.97%
Females				
g(1) : 15-24	18.47%	81.53%	62.18%	37.82%
g(2) : 25-34	31.43%	68.57%	57.07%	42.93%
g(3) : 35-44	15.87%	84.13%	65.17%	34.83%
g(4) : 45-54	15.43%	84.57%	62.83%	37.17%
g(5) : 55-64	22.50%	77.50%	57.59%	42.41%

Table 4: Leisure/work rates, and activity rates by profession

Finally, intra-cell individual heterogeneity in preferences is then generated using i.i.d. double exponential stochastic terms with dispersion parameters μ and υ that are the inverse of the transformation elasticities between professions σ and between leisure and work τ of (13) which values are reported in **Table 5**.

	Leisure / Work	Prof-0 / Prof-1
Males		
g(1) : 15-24	0.752	1.336
g(2) : 25-34	0.753	0.177
g(3) : 35-44	0.739	1.527
g(4) : 45-54	0.749	0.449
g(5) : 55-64	0.727	0.824
Females		
g(1) : 15-24	0.759	1.397
g(2) : 25-34	0.772	1.226
g(3) : 35-44	0.789	0.827
g(4) : 45-54	0.742	0.961
g(5) : 55-64	0.755	0.417

Table 5: Transformation elasticities of the aggregate supply systems

4.2 The macro data-set and the ageing shock

In this illustrative simulation exercise, we assume the economy initially in a steady-state that is stationary. Because all individuals are assumed to exit at the same age of 95, the dependency ratio is rather high in this economy, at 60%. (We could have taken care of this by introducing mortality rates at each age but with little additional insight given illustrative-only ambition of the exercise.) The main parameters and data of the macro model are summarized in **Table 6**.

Consumption / GDP	80%
Investments / GDP	20%
Gross capital remuneration / GDP	33.3%
Remuneration of *Prof-0* / GDP	15.3%
Remuneration of *Prof-1* / GDP	51.4%
Social security contributions	20%
Gross interest rate	8.3%
Depreciation rate	5.0%
Intertemporal substitution elasticity	1

Table 6: The main parameter values used in the illustrative OLG model

The ageing shock is implemented by a temporary drop of the parameter η_t (see (14)) with resulting population time-path displayed in **Figure 1**, and old-age dependency ratio (the ratio of retired to active population) as displayed in **Figure 2**. This is indeed a quite drastic ageing shock. The reason for choosing an admittedly excessive demographic change is that we want to ensure significant factor-price changes and hence, induce significant switches in individual discrete decisions: only then can we gain true confidence in our methodology.

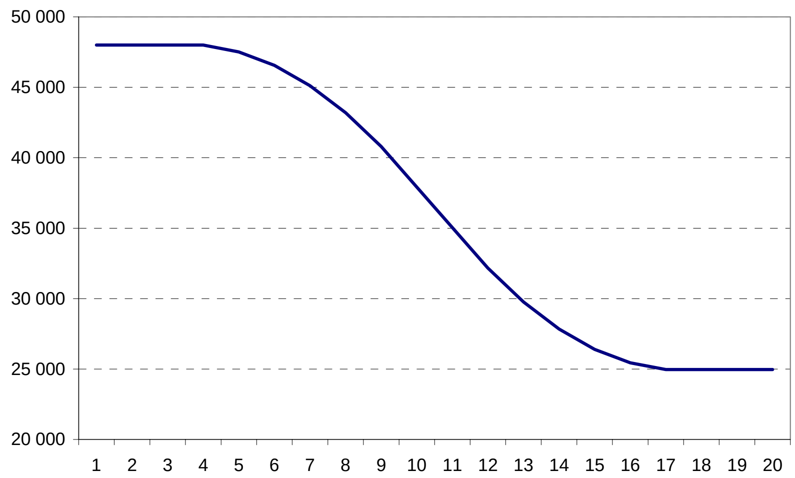

Figure 1: The demographic shock: total population

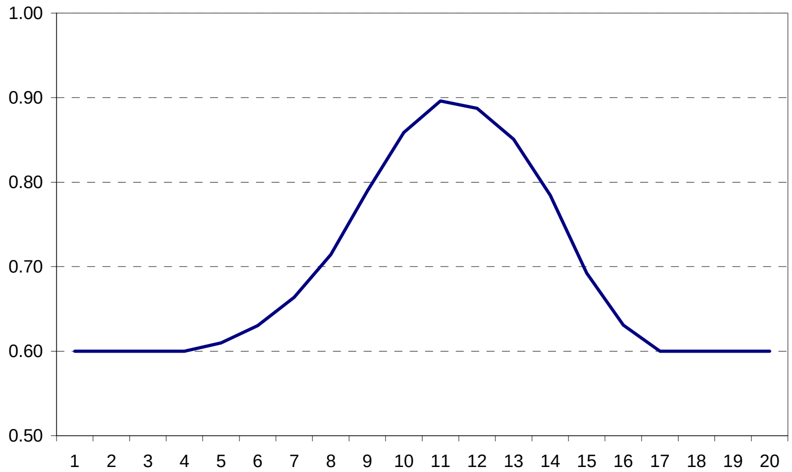

Figure 2: The old-age dependency ratio resulting from the demographic shock

The resulting solution time-path of factor prices is displayed in **Figure 3**, and is as one expects.[7]

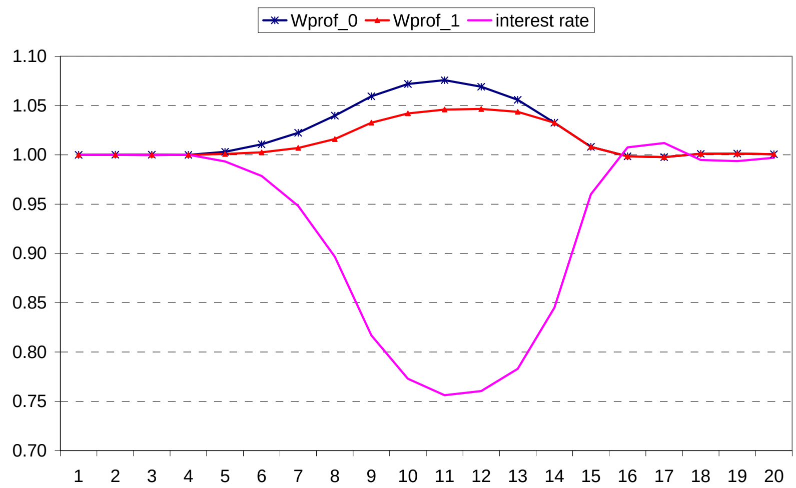

Figure 3: The dynamics of factor prices induced by the demographic shock

4.3 Accuracy

Having computed the equilibrium path of wages, we now plug back these factor prices into the microsimulation model, and compute the new optimal discrete choices for each of the 30.000 individuals, aggregate these per population cells and compare with those generated from the representative agent formulation in the OLG model. Why could these predictions differ, given that we use exact aggregation results? The reader will remember that, within each population cell, we assumed that the labor supply decision results from considering – both in the micro and in the macro approach – the Mincer wage w_i rather than the true individual wage w_i^h, which is w_i adjusted for within-cell idiosyncratic productivity differences. The ex-post microsimulation evaluation uses this individual information that has been lost in the aggregation process. Checking for these errors is therefore indeed meaningful.

[7] We only report the first 20 periods though the model is solved over a horizon of 40 periods of ten years each.

Table 7 and **Table 8** provide a sample of accuracy results, measured as % discrepancies between the two predicted labor supplies. Observe that the first time period discrepancies are all of the order of 1.E-10 which only reflects the quality of the calibration: indeed, the demographic shock only affects the economy at later periods. Looking at the time path of errors, we see that the largest is roughly equal to half a per cent, a very small number given the severity of the demographic shock: clearly, a discrepancy that is unlikely to affect the equilibrium wages and is therefore without GE implication.

	g(1)	*g(2)*	*g(3)*	*g(4)*	*g(5)*
1	-1.19904E-12	5.55112E-13	2.22045E-12	2.5091E-12	3.9968E-12
2	-2.16404E-08	-3.09893E-09	-1.64776E-08	-3.75063E-09	-8.0231E-09
3	-4.9683E-08	-7.1157E-09	-3.78319E-08	-8.61163E-09	-1.8421E-08
4	1.97866E-08	2.83493E-09	1.5069E-08	3.43074E-09	7.33644E-09
5	-0.000564295	-7.45465E-05	-0.000395638	-9.02095E-05	-0.000192817
6	-0.00203157	-0.000268202	-0.00144886	-0.000331816	-0.000707877
7	-0.003751061	-0.000493583	-0.002565971	-0.000606362	-0.001291068
8	-0.005325312	-0.000763898	-0.004172046	-0.000938141	-0.001960737
9	-0.005843257	-0.000846965	-0.004418017	-0.001069888	-0.001866271
10	-0.006287636	-0.000973801	-0.004982881	-0.00124621	-0.00188436
11	-0.006212755	-0.000921774	-0.00530581	-0.001172894	-0.001807266
12	-0.004738384	-0.000700114	-0.003978269	-0.000890913	-0.001526259
13	-0.002638207	-0.000387584	-0.002217207	-0.000460682	-0.000641863
14	-2.63511E-05	-3.8454E-06	-2.21728E-05	-4.57406E-06	-5.1224E-06
15	-6.75913E-06	-9.86695E-07	-5.68882E-06	-1.17413E-06	-1.3115E-06
16	1.2707E-06	1.85523E-07	1.0696E-06	2.208E-07	2.46377E-07
17	2.02767E-06	2.96044E-07	1.70678E-06	3.52341E-07	3.93118E-07
18	-8.86393E-07	-1.29408E-07	-7.46087E-07	-1.54008E-07	-1.71897E-07
19	-1.07553E-06	-1.57021E-07	-9.05284E-07	-1.86869E-07	-2.0858E-07
20	-4.98291E-07	-7.27476E-08	-4.19418E-07	-8.65777E-08	-9.66293E-08

Table 7: Total labor supply, males, % differences between the micro and the macro predictions

	g(1)	g(2)	g(3)	g(4)	g(5)
1	-8.24896E-12	8.88178E-13	2.9976E-12	-3.475E-12	-2.88658E-12
2	-2.1646E-08	-3.09953E-09	-1.64797E-08	-3.75189E-09	-8.02445E-09
3	-4.96885E-08	-7.11652E-09	-3.78343E-08	-8.61299E-09	-1.84221E-08
4	1.97811E-08	2.8342E-09	1.50672E-08	3.42919E-09	7.33544E-09
5	-0.000564295	-7.45465E-05	-0.000395638	-9.02095E-05	-0.000192817
6	-0.00203157	-0.000268202	-0.00144886	-0.000331816	-0.000707877
7	-0.003751061	-0.000493583	-0.002565971	-0.000606362	-0.001291068
8	-0.005325312	-0.000763898	-0.004172046	-0.000938141	-0.001960737
9	-0.005843257	-0.000846965	-0.004418017	-0.001069888	-0.001866271
10	-0.006287636	-0.000973801	-0.004982881	-0.00124621	-0.00188436
11	-0.006212755	-0.000921774	-0.00530581	-0.001172894	-0.001807266
12	-0.004738384	-0.000700114	-0.003978269	-0.000890913	-0.001526259
13	-0.002638207	-0.000387584	-0.002217207	-0.000460682	-0.000641863
14	-2.63511E-05	-3.8454E-06	-2.21728E-05	-4.57406E-06	-5.1224E-06
15	-6.75914E-06	-9.86696E-07	-5.68882E-06	-1.17413E-06	-1.3115E-06
16	1.2707E-06	1.85523E-07	1.0696E-06	2.20799E-07	2.46376E-07
17	2.02766E-06	2.96043E-07	1.70678E-06	3.5234E-07	3.93117E-07
18	-8.86398E-07	-1.29409E-07	-7.46089E-07	-1.54009E-07	-1.71898E-07
19	-1.07554E-06	-1.57021E-07	-9.05287E-07	-1.8687E-07	-2.08581E-07
20	-4.98296E-07	-7.27481E-08	-4.1942E-07	-8.65788E-08	-9.66304E-08

Table 8: Labor supply, males in Prof-1, % diff. between the micro and the macro predictions

4.4 Income inequalities induced by population ageing

We now report how the ongoing ageing of our economies may affect income inequalities, an issue that can be rigorously addressed thanks to the microsimulations model. Among the various inequality indices, we choose two without apologies: our results are purely illustrative and do not require thorough dwelling.

We first report in **Figure 4** the median, tenth percentile, and ninetieth percentile of the (net of social security contributions) total income distribution for the entire active population (that is, excluding the retired cohorts). The dynamics of the median and ninetieth percentile are easy to understand from the time path of wages (see **Figure 3**): the former individual is a young lower-skilled – i.e., working in the profession where wages are lowest – who benefits from increasing wages in Profession 0 and is largely unaffected by the drastic reduction in capital returns; the latter individual is an older – and hence with more accumulated assets – qualified worker (i.e., working in Profession 1) whose rising wage more than compensates depressed returns on capital during the first half of the time horizon, and whose recovering capital income offsets later the contracting labor earnings. Not surprisingly, the time profile of the tenth percentile of income is more erratic reflecting undiversified factor ownership in the lower tail of the income distribution: up to time period 8, the pivotal individual is a (possibly up to then unemployed) low-skilled worker who benefits from rising wages and is immune to fluctuations in capital returns, whereas for the next ten years, the pivotal individual, because unemployed, is strongly affected by low interest rates.

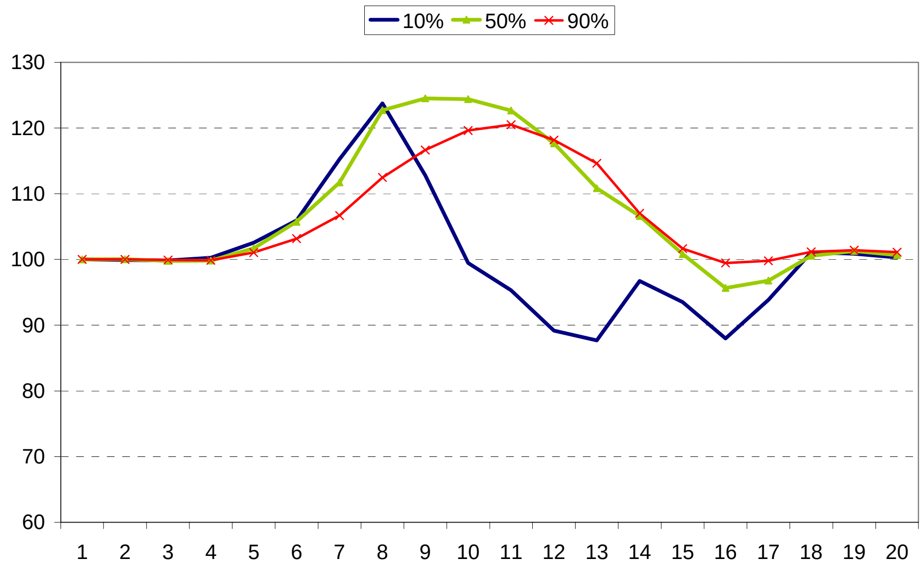

Figure 4: The dynamics of income inequality, the 10^{th}, 50^{th}, and 90^{th} percentiles

We end this section by reporting in **Figure 5** and **Figure 6** the contrasted time path of the Gini coefficients for age-groups 45-54 and 55-64 which of course are what we expect from what we know on the cohorts (from **Table 4**) and from the factor-price movements.

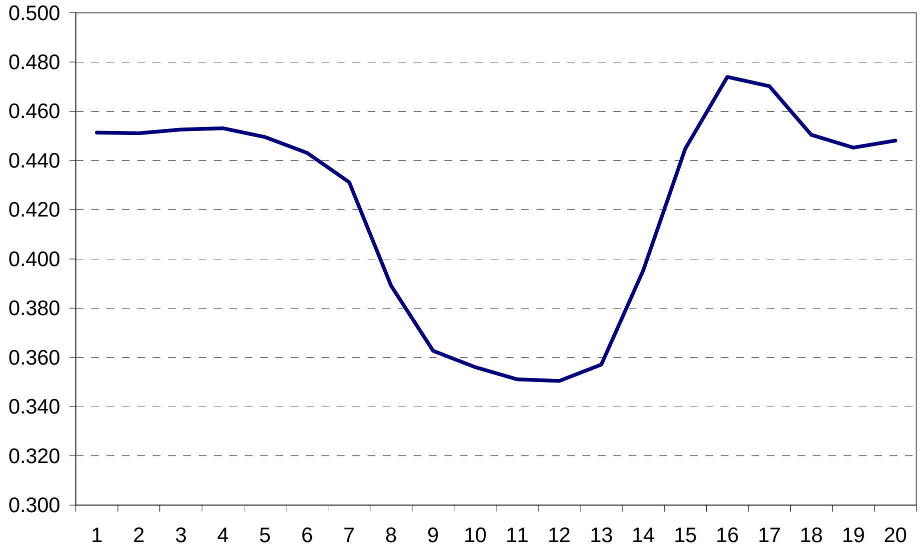

Figure 5: The dynamics of income inequality, the Gini index for the age group 45-54

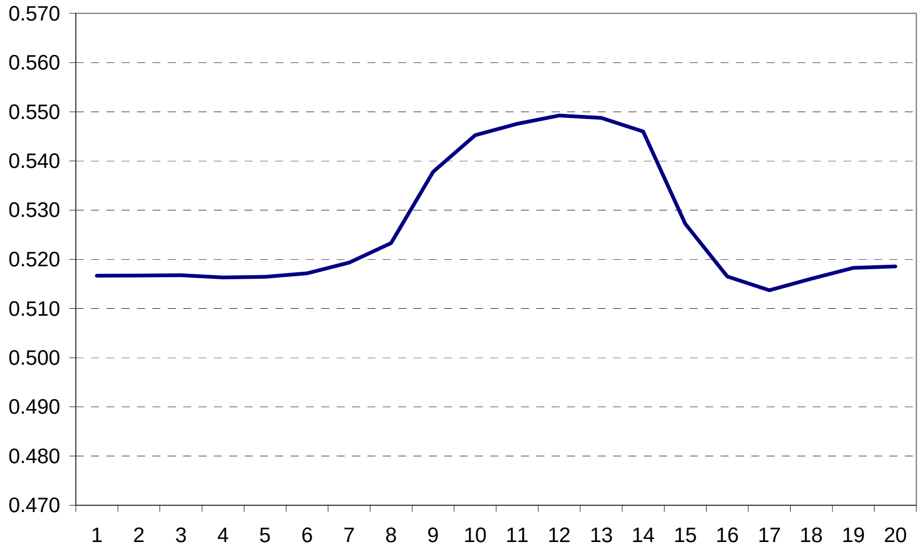

Figure 6: The dynamics of income inequality, the Gini index for the age group 55-64

5 Conclusion

Applied GE models have become indispensable tools of qualitative policy assessment. By essence, they rely on some form of representative agents' simplification of the economy so as to make explicit and manageable the consistency imposed on individual decisions by technological and resource constraints. As huge micro data-sets have increasingly been made available in recent years, the microsimulation approach has developed that apprehends the full heterogeneity of individual behavioral adjustments to policy reforms at the expense of global consistency. In these models, individual decision-making often is of the discrete-choice type. In this paper, we suggested a bridge between these two approaches by making use of exact aggregation results due to Anderson, de Palma and Thisse (1992). We have argued that this provides an extremely useful interface between the two approaches: it makes possible to counter the major weakness of each of the two approaches making them consistently complementary. We have illustrated this in a dynamic setting, by linking a microsimulation model built from a computer-generated micro data-set to an OLG-GE representation of an economy submitted to demographic ageing.

References

[1] Anderson S., A. de Palma and J-F. Thisse (1992) *Discrete Choice Theory of Product Differentiation*. Cambridge: MIT Press.

[2] Arntz M., S. Boeters, N. Gurtzgen and S. Schubert (2006) "Analysing Welfare Reform in a Microsimulation-AGE Model: The Value of Disaggregation". Mimeo.

[3] Ben-Akiva M. and S. Lerman (1985) *Discrete Choice Analysis: Theory and Application to Predict Travel Demand*. Cambridge: MIT Press.

[4] Bourguignon F. and A. Spadaro (2006) "Microsimulation as a Tool for Evaluating Redistribution Policies". *Journal of Economic Inequality*, 4 (1), 77-106.

[5] Kehoe T.J. (2005) "An Evaluation of the Performance of Applied General Equilibrium Models of the Impact of NAFTA". In T.J. Kehoe, T.N. Srinivasan and J. Whalley (eds), Frontiers in Applied General Equilibrium Modeling: Essays in Honor of Herbert Scarf. Cambridge Univ. Press, 341-77.

[6] Kehoe T.J., C. Polo and F. Sancho (1992) "An Evaluation of the Performance of an Applied General Equilibrium Model of the Spanish Economy". *Economic Theory*, 6, 115-141.

[7] McFadden D. (1978) "Modeling the Choice of Residential Location". In A. Karlqvist, L. Lundqvist, F. Snickars and J. Weibull (eds), Spatial Interaction Theory and Planning Models. Amsterdam: North-Holland.

[8] McFadden D. (1981) "Econometric Models of Probabilistic Choice". In C. Manski and D. McFadden (eds), Structural Analysis of Discrete Data with Econometric Applications. Cambridge: MIT Press.

[9] Mercenier J., M. Mérette et M. Fougère, 2005, "A Sectoral and Occupational Analysis of Population Ageing in Canada Using a Dynamic CGE Overlapping Generations Model", *Economic Modelling*, forthcoming.

[10] Mercenier J. and E. Yeldan (1999) "A Plea For Greater Attention To Data In Policy Analysis". *Journal of Policy Modeling*, 21 (7), 851-873.

[11] Savard L. (2003) "Poverty and Income Distribution in a CGE-Household Microsimulation Model: Top-Down/Bottom-Up Approach". CIRPEE WP 03-43.