What Discount Rate should Bankruptcy Judges Use? Estimates from Canadian Reorganization Data

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Abstract

Using data from financial reorganization plans filed by insolvent Canadian firms, we estimate the discount rate implicit in the unsecured creditors' reorganization decision. Using (HARA) utility functions, we find the implicit monthly discount rate of creditors to be 4.9%, which corresponds to an annual discount rate of 77%. This is 7 to 10 times higher than discount rates used in previous empirical studies of reorganization. The discount rate estimates are robust to varying degrees of risk aversion and market to book value ratio of assets.

1 Introduction

Bankruptcy systems such as the U.S. Bankruptcy Code and the Canadian Bankruptcy and Insolvency Act can be modeled as a multi-stage game. At stage one, the manager of a financially distressed firm decides between liquidation and reorganization. At stage two, if reorganization is chosen, creditors vote on the plan and, if certain conditions are met, the bankruptcy judge confirms it. At stage three, conditional upon the court's confirmation of the plan, the firm must execute the terms of the plan in order to be discharged from bankruptcy.

From the perspective of corporate finance, stage two of the bankruptcy game represents a capital budgeting decision in which creditors choose between two alternatives: rejecting the plan and liquidating the firm's assets, or accepting the plan and the specified structure of repayments of the outstanding debt. Accordingly, creditors will accept the plan if their 'participation constraint' is satisfied, i.e., if their expected gain from acceptance (reorganization) is larger than their expected gain from rejection (liquidation). A plan that satisfies the creditors' participation constraint is, in judicial terms, said to pass the 'best-interests test'.

Court-supervised reorganization procedures typically require judges to consider the best-interests test. According to §1129(a)(7)(A) of the U.S. Bankruptcy Code, a Chapter 11 plan cannot be confirmed unless creditors receive as much under the plan as under liquidation. Under §59(2) of the Canadian Bankruptcy and Insolvency Act, the bankruptcy court may reject reorganization proposals that are "not calculated to benefit the general body of creditors." Determining whether a plan is in the creditors' interests involves comparing a stream of payments under reorganization to the payment under liquidation. As Klee (1995: 567–568) points out, the correct way for the court to do this is to compare the present value of payments under reorganization to the present value of payments under liquidation. Given the present value methodology, a vital issue is then: what discount rate should the court use? Empirical studies evaluating the best interests test have simply assumed a value for the discount rate. In a study of bankrupt Japanese firms, Eisenberg and Tagashira (1994) assume a 7% annual discount rate on the basis of the long-term interest rate prevailing over their sample period. Taking into account a higher rate of inflation, Fisher and Martel (1999) assume a 10% annual interest rate in a study of bankrupt Canadian firms. Given the assumed discount rates, both studies conclude there is substantial compliance with the best interests test in Japan and Canada.

It is clearly desirable to have a sounder basis than assumption for choosing the discount rate a bankruptcy court should use. In this paper, we use data on reorganization and liquidation payments to estimate the discount rate implicit in creditors' reorganization plan decisions. In deciding how to vote on a reorganization plan, we hypothesize that rational creditors compare the expected present value of the payment under reorganization to the expected present value of the payment under liquidation. Everything else equal, the firm will choose the stream of reorganization payments such that the median creditor is indifferent between liquidation and reorganization, i.e., such that the present value of payments in liquidation and reorganization are equal. Thus, given data on the stream of payments in reorganization and liquidation, we can solve from the equality of present values for the discount rate of the median creditor.

Our methodology, which involves using a real-world decision posing tradeoffs between the near future and a more distant future, has often been used to estimate discount rates for individual consumers. By comparing the tradeoff between price and long-term running costs, Hausman (1979) finds that individuals use a discount rate of about 20% in the purchase decision for air conditioners. Using a similar methodology, Gately (1980) reports implicit discount rates of 45–300% for the purchase of refrigerators, while Ruderman, Levine, and McMahon (1987) estimate a discount rate of 243% for gas water heaters. Other studies estimate discount rates implicit in wage-risk tradeoffs in the range 1–14% [Moore and Viscusi (1990)] and in auto-safety decisions in the range 11–17 percent [Dreyfus and Viscusi (1995)] Warner and Pleeter (2001) estimate discount rates of 0–30% from the decisions of U.S. military personnel involved in a downsizing program.

For risk neutral creditors, we find that the mean implicit monthly discount rate is equal to 4.9%, corresponding to an annual discount rate of 77%. This is seven to ten times higher than discount rates typically used in empirical studies of the best-interests test. We examine the impact of risk aversion on the implicit discount rate using (HARA) utility functions. Generally speaking, an increase in risk aversion leads to a reduction in creditors' discount rates into the range of 1.1–4.8% per month. We also investigate the impact on the estimated discount rates of different assumptions about the market value of assets. We find that a reduction in the market value of assets reduces discount rates, and this result holds for different specification of utility functions.

The paper is outlined as follows. Section 2 defines the participation constraint of creditors voting on a financial reorganization plan and describes the method used to estimate the implicit discount rate for each observation in the data. Section 3 describes the data and Section 4 reports the estimated discount rates under a varying degrees of risk aversion among creditors' and on different assumptions about the market to book value of assets. Section

5 concludes the article.

2 The Creditors' Participation Constraint

Under Canadian insolvency law, ordinary (unsecured) creditors face two options when a debtor files for reorganization: they can reject the proposal and get the proceeds resulting from the liquidation of the firm's assets, or they can accept the multi-period repayment structure of the proposal.[1] The value of the payment in liquidation depends on five factors: (i) the market value of liquidated assets, (ii) the value of secured claims (SC), (iii) the value of preferred claims (PC), (iv) administration costs ($ADMIN$), and (v) expected time to liquidate the assets (T). We assume the market value of assets is equal to the book value of assets (A), taken from the firm's balance sheet, times an adjustment factor δ. In accordance with the priority rule under Canadian bankruptcy law, ordinary creditors receive the proceeds of liquidation after the payment of administration costs, secured claims, and preferred claims. Thus, the value of the payment in liquidation is:

$$L = \delta A - SC - PC - ADMIN \tag{1}$$

The value of the payment in reorganization depends on the payment structure specified in the proposal. Let M_t be the proposed cash payment specified in reorganization at month t and N be the maximum number of

[1] By and large, Canadian and U.S. bankruptcy laws are very similar, though there are some differences in terminology and procedure. The Canadian counterpart to a Chapter 11 plan is a reorganization 'proposal'. Only 'ordinary' unsecured creditors (as distinct from 'preferred' unsecured creditors such as employees) vote on the proposal. Canadian law allows incumbent management to retain control of the firm while mandating the presence of a trustee, who deals with payments to creditors, during reorganization.

months specified in the proposal. We express the payment structure as a vector $M = (M_1, \ldots, M_N)$. To capture the inter-temporal choice offered to ordinary creditors and various risk preferences for ordinary creditors, we adopt the discounted utility approach wherein the creditors's inter-temporal utility function can be written as:

$$U(M_1, \ldots, M_N) = \sum_{t=0}^{N} \frac{U(M_t)}{(1+r)^t}$$

An additional element that needs to be considered is the uncertainty surrounding payment in reorganization. Payment to creditors is contingent upon 'success' of the debtor, meaning the debtor meets all the terms of the proposal and the trustee is discharged by the bankruptcy court. In the case of liquidation, there is no uncertainty as we assume that the creditors will get the liquidation proceeds with a probability equal to one at period T. Let $p(s)$ denote the probability that a proposal is successful.

Formally, the median creditors' 'participation constraint' can be written:

$$\frac{U(\delta A - SC - PC - ADMIN)}{(1+r)^T} = \left[\sum_{t=0}^{N} \frac{U(M_t)}{(1+r)^t} \right] \times p(s) \tag{2}$$

which states simply that the median ordinary creditor will be indifferent between reorganization and liquidation, and hence just indifferent between 'participating' in reorganization, if the present value of utility in liquidation is equal to the present value of expected utility in reorganization.

The implicit discount rate on a project, r^*, is the rate such that the net present value NPV of the project is equal to zero, that is:

$$NPV(r^*) = PVL(r^*) - PVR(r^*) = 0 \tag{3}$$

where PVL is the present value of utility in liquidation (i.e., the left-hand side of (2)) and PVR is the present value of utility in reorganization (i.e., the right-hand side of (2)). In view of (2), it is clear that r^* is a function of δ, A, SC, PC, $ADMIN$, M and p. It is also clear from (3) that the implicit discount rate is simply the internal rate of return (IRR) from the participation constraint.

To capture creditors' attitudes towards risk, we assume that the utility functions belong to the class of hyperbolic absolute risk aversion (HARA) functions of general from:

$$U(W) = \frac{1-\gamma}{\gamma}\left(\frac{aW}{1-\gamma}+b\right)^{\gamma}$$

with $a > 0$, $\gamma \neq 1$, and $W > 0$. Table 1 reports the different utility functions depending on the value of parameters. Note that we can retrieve the special case of risk neutrality from the power utility function if $\gamma = 1$, or from the negative exponential utility function if a approaches zero.

Each observation in our sample corresponds to a single equation with one unknown variable: the discount rate. Given the complexity of the equation, an explicit analytical solution cannot be found, so we need to search numerically for a solution. The IRR for each reorganization proposal is found where the NPV function intersects the horizontal axis. A grid search method estimates the shape of the $PVL(r)$ and $PVR(r)$ functions in order to find the zero of the $NPV(r)$ function.[2]

Before we proceed to the results one point is worth noting. Whenever the (undiscounted) expected utility of reorganization payments to be received up to and including period T is greater than the utility of the liquidation payment to be received at period T, it follows that $PVR(r) > PVL(r)$ for any value of r. In other words, reorganization proposals with the property

[2]Barthélémy, Fisher & Martel (2004) contains a detailed description of the estimation method as well as results from an alternative Newton algorithm.

that

$$U(L) < \left[\sum_{t=0}^{T} U(M_t) \right] \times p(s) \tag{4}$$

will satisfy the participation constraint whatever discount rate creditors are using. For our purposes, proposals that satisfy (4) do not yield any information about the discount rate used by creditors. In such cases, bankruptcy judges do not need to be concerned with using the 'correct' discount rate, because it is immaterial to the creditors reorganization decision. A common special case of (4) is when reorganizing firms offer their creditors a zero return in liquidation, i.e., $L = 0$. In these cases, a non-zero return in reorganization, however small, is preferable for creditors and the value of the discount rate irrelevant. A corollary is that only proposals that *do not* satisfy (4) yield information about the creditors' discount rate. But this is as it should be, since only in these cases does a bankruptcy judge need to worry about the correct value of the discount rate creditors are using.

3 Data

The data in the present study are collected directly from documents filed in bankruptcy courts by 305 firms in Montreal and Toronto during the period 1977–88. All of the variables in (2) are observed directly in the data with the exception of T (expected time to liquidate assets). We approximate T by the average time in liquidation, which is 27 months, from a related sample of liquidated firms [Martel (1995]. We eliminated 110 files with a liquidation payment equal to zero and 112 files for which the expected utility of reorganization payments within 27 months is greater than the utility of the liquidation payment, because as mentioned above, given the assumption of 27 months in liquidation, these files do not contain any information about the discount rate. Thus, the final sample contains 83 observations: 222 observations do not yield any information on the discount rate. This suggests that the discount rate is an important consideration in about 1 proposal in 4 (83/305).

Table 2 gives a snapshot of the characteristics of firms and proposals in the sample. The average value of assets for the firms is \$2.2 million while the average value of liabilities is \$2.4 million. Most claims are ordinary unsecured claims, followed by secured (typically bank) claims. Firms in reorganization offer a mean payoff rate on ordinary unsecured claims of 43.6 cents on the dollar. On average, the cash payment is equal to 2.2 cents, the rest being paid by installments over a maximum period of 60 months. The maximum liquidation payment is equal to 100% of the value of unsecured claims.

As previously mentioned, the expected payment in reorganization depends not only on the structure of payments but also on $p(s)$, the estimated probability of success of the proposal. When creditors vote on the proposal they must form expectations about $p(s)$ and, following Martel (2004), we model this process. Given the dichotomous nature of the outcome in reorganization (i.e., success or failure) and using information from the data on success or failure, a logit model is used to estimate the probability of success.[3] As indicated by (2), $p(s)$ is assumed to be constant for the time period covered by the proposal. For example, a proposal making payments to creditors at 3, 6 and 9 months is assumed to have an estimated probability of success that is the same at 3, 6 or 9 months. Put another way, the hazard rate for proposals (i.e., the probability that a proposal will fail in the next time interval) is assumed to be constant for the life of the proposal. This approach results from the lack of information on the exact time of default

[3]The logit estimates are available from the authors upon request.

for failed proposals. Anecdotal evidence suggests that when default occurs, it typically happens very soon after the creditors vote.

4 Results

To estimate the implicit discount rate for each proposal we start with a base case against which we can investigate the impact of creditors' attitude towards risk and other factors. The base case scenario is defined as follows:

1. market value equals the book value of assets ($\delta = 1$);
2. liquidation time of assets is 27 months ($T = 27$);
3. creditors are risk neutral.

Figure 1 illustrates how the grid search process works for a representative observation. The net present value function is plotted against various values for the discount rate. The solution to (2) occurs where the function intersects the horizontal axis, which in this case is at 2.37%. Given that (2) is a high order polynomial, we could expect to find multiple solutions for many observations. In fact, only one proposal exhibits multiple solutions (two solutions: 8.4% and 24.2%) and we assume the lower discount rate is the relevant solution. The other 82 solutions to (2) are unique. Results from the grid search are reported in the column headed "Linear" in Table 3.[4] The mean monthly discount rate is 4.86%, the median is 3.85%; the lowest is 0.079% and the highest value is 16.10%. Using the mean value, the annual discount rate for creditors in financial reorganization is equal to 76.7%. These estimates

[4]Individual solutions for all 83 observations in the sample are available from the authors on request.

imply discount rates an order of magnitude greater than the literature has previously considered.

Risk aversion

Risk aversion introduces curvature into the utility function of creditors. Because the liquidation payment is larger than the reorganization payments, risk aversion will have a larger relative impact on the utility of the liquidation payment than on the utility of the reorganization payments. Thus, we expect that the discount rate that solves (2) will be lower in the presence of risk aversion. The remaining columns of Table 3 report discount rate estimates for the natural logarithmic, negative exponential, and power utility functions. The negative exponential function was estimated for different values of a, with risk aversion decreasing as a approaches zero. The power function was also estimated for four different values of γ, with risk aversion decreasing as γ approaches one. Table 3 clearly shows risk aversion lowers the estimated discount rates. The natural log and all the negative exponential and power utility discount rates are lower than the linear (risk neutral) rates. For the negative exponential utility function estimated discount rates fall as a decreases, reflecting the expected effect of a greater degree of risk aversion on the discount rates. Similarly, the discount rates fall as γ decreases for the power utility function.

Table 3 also shows that introducing risk aversion reduces the number of observations. This is due to the sample selection process discussed in section 3. In the presence of risk aversion, we eliminate all files for which the expected utility within 27 months is greater than the utility in liquidation, because, given the assumption of 27 months in liquidation, these files satisfy the participation constraint for any positive discount rate. For a given utility function, the number of observations eliminated increases with risk aversion. The drop in observations is largest for the logarithmic function since it exhibits the highest value of absolute risk aversion (for reasonable values of the parameters).[5]

Ratio of market to book value of assets

Martel (1995) finds evidence consistent with deviations between the book and market value of assets in liquidation (δ), which could impact the estimated discount rate of ordinary creditors. To see how δ may impact the discount rate, simplify the participation constraint by assuming that the constant values SC, PC and $ADMIN$ are equal to zero and the success probability of the proposal is equal to one. Thus, the constraint simplifies to:

$$\frac{U(\delta A)}{(1+r)^{27}} = \sum_{t=0}^{N} \frac{U(M_t)}{(1+r)^t}$$

or, alternatively:

$$U(\delta A) = \sum_{t=0}^{N} U(M_t)(1+r)^{27-t}$$

Under this setting, there are three types of contracts:

Type 1: proposals where all payments to unsecured creditors are made before 27 months;

Type 2: proposals where all payments to unsecured creditors are made after 27 months;

Type 3: proposals with payments made before and after 27 months.

[5]The measures of $ARA(W) = -U''/U'$ for the natural logarithmic, negative exponential, and power function are, respectively, $1/(1+W)$, a and $-(\gamma-1)/W$. For the logarithmic, negative exponential, and power utility functions, ARA converges to the linear utility function, respectively, as $W \rightarrow \infty$, $a \rightarrow 0$, and $\gamma \rightarrow 1$.

It turns out there are 65 Type 1 observations, 17 Type 3 observations, and one Type 2 observation. The impact on the discount rate of a change in δ depends on the time structure of the contract, as follows:

Type 1: $\left.\frac{\partial r}{\partial \delta}\right|_{r^*} > 0$

Type 2: $\left.\frac{\partial r}{\partial \delta}\right|_{r^*} < 0$

Type 3: $\left.\frac{\partial r}{\partial \delta}\right|_{r^*}$ is undetermined

These follow from (2). Figure 2 gives an idea of what is going on for a Type 1 observation. As δ decreases the NPV function shifts to the left. For observations with low discount rates for $\delta = 1$, shifts to the left may result in no solution to the problem. Thus, a reduction in δ has two effects on Type 1 observations: the sample size shrinks as low discount rate observations are dropped from the sample while the remaining observations have lower discount rates.

To keep the analysis of δ tractable, we examine $0.95 \leq \delta \leq 0.50$ in 0.05 steps. For each observation there exists a value of δ for which there is no solution to the participation constraint. As δ decreases, the liquidation value of assets can be reduced to a point where the expected utility in reorganization will always be higher than the utility in liquidation for any reasonable value of the discount rate. This implies that the number of observations for which there exists a solution decreases as δ falls.

Table 4 shows the estimated discount rates for different values of δ for the various utility functions. The base case (linear utility) reports a mean monthly discount rate of 4.86%, the minimum mean value occurs for $\delta = 0.80$ (4.42%) and the maximum mean value occurs for $\delta = 0.55$ (6.63%). As expected, the number of observations for which there exists a reasonable solution decreases with δ—going from 83 when $\delta = 1$ to 10 when $\delta = 0.50$. As can be seen, the mean discount rate is quite stable across market to book value ratios. The important fact that emerges from Table 4 is that the mean discount rate is quite stable across market to book value ratios for the various utility functions.

5 Conclusion

Using data on 83 reorganization proposals filed by bankrupt Canadian firms, we compare the payout to creditors from liquidation to the stream of future payments specified in the reorganization proposals. Based on the payment data together with the creditors' participation constraint, we are able to determine the discount rate implicit in the creditors' reorganization decision. Using a base case scenario where investors are assumed to be risk neutral, we estimate the implicit monthly discount rate in reorganization to be equal to 4.86%, which corresponds to an annual discount rate of about 77 percent. We also show that the discount rate decreases as investors become more risk averse.

In practical terms, our results have two implications for court-administered reorganization procedures such as Chapter 11. The first implication is that in many reorganization cases, the creditors' discount rate is irrelevant. This considerably lightens the computational burden on bankruptcy courts. In cases where the liquidation return is zero, a bankruptcy judge need only check whether creditors are offered a non-zero return in reorganization in order to evaluate the best interests test. In cases where the sum of reorganization payments offered before a liquidation payment would be received, a bankruptcy judge need only check whether the sum the reorganization payments is greater than the liquidation payment. In our Canadian data, either scenario would cover three-quarters of the proposals. For the remaining files, however, discount rates are a vital consideration in the best interests test. Moreover, our results clearly demonstrate that annual rates of the order of 7–10% are an order of magnitude lower than the rates creditors are likely to be using. Annual discount rates of around 80% can clearly have a major impact of the relative benefits of reorganization and liquidation, potentially yielding drastically different results to the best interests test than previously considered discount rates.

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TABLE 1
HARA Utility Functions

γ	b	$U(W)$	
0	0	$\ln(1+W)$	natural log
$-\infty$	1	$-e^{-aW}$	negative exponential
< 1	0	$(1/\gamma)W^{\gamma}$	power

Note: We use $\ln(1+W)$ to avoid problems when monthly payments are equal to zero, for which the logarithmic function would not be defined.

TABLE 2
Descriptive Statistics of Firms in Reorganization

Variable	Mean	Std Dev	Minimum	Maximum
Financial Variables				
Assets	2,219.4	4,006.2	48.5	25,518.6
Liabilities	2,407.6	3,268.0	67.2	19,703.0
Ordinary claims	1,245.1	1,801.3	24.3	8,375.2
Secured claims	977.1	1,828.2	0.0	11,568.2
Preferred claims	98.3	154.5	0.0	689.1
Administration costs	58.4	125.5	2.0	1,008.3
Reorganization proposal variables				
Total payment	43.60	28.02	5.0	124.0
Cash payment	2.23	8.85	0.0	60.2
1-Month	8.85	16.59	0.0	100.0
3-Month	5.95	10.11	0.0	50.0
6-Month	4.28	9.54	0.0	65.0
9-Month	3.25	7.91	0.0	50.0
12-Month	4.60	10.53	0.0	71.0
15-Month	1.72	4.33	0.0	25.0
18-Month	2.44	6.29	0.0	40.0
21-Month	0.95	2.59	0.0	12.0
24-Month	1.89	4.85	0.0	25.0
36-Month	5.28	17.83	0.0	123.9
48-Month	1.34	5.23	0.0	35.0
60-Month	0.82	4.75	0.0	35.0
Estimated probability of success	0.770	0.152	0.426	0.988

Notes: $N = 83$ proposals. Dollar values are reported in thousands of December 1998 Canadian dollars, deflated by the GDP deflator (series D20556). The expected payoff in liquidation, the payoff in reorganization and the payments variables are reported in percent.

TABLE 3
Summary of Estimated Discount Rates (in %) for different Utility Functions

	Linear	Nat log	Negative exponential			Power		
			$a = 8$	$a = 5$	$a = 1$	$\gamma = 0.8$	$\gamma = 0.5$	$\gamma = 0.1$
Mean	4.86	1.06	4.85	2.88	0.78	3.76	2.16	1.09
Median	3.85	1.16	3.85	3.18	1.08	2.91	1.89	1.16
Std Dev	4.74	0.72	4.74	1.68	0.46	3.24	1.61	0.77
Minimum	0.08	0.16	0.08	0.04	0.04	0.10	0.03	0.16
Maximum	16.10	5.58	16.01	14.74	5.41	12.48	9.10	5.59
N	83	38	83	62	38	74	56	38
Percentile								
5	0.36	0.24	0.36	0.10	0.11	0.34	0.12	0.24
10	0.39	0.34	0.39	0.25	0.17	0.57	0.26	0.34
25	1.68	0.48	1.68	0.58	0.27	1.49	0.74	0.51
75	6.95	1.03	6.93	3.69	0.66	5.46	2.79	1.06
90	9.53	1.60	9.52	6.31	1.19	8.26	4.81	1.67
95	12.39	3.78	12.39	8.43	3.23	9.33	5.51	3.90

TABLE 4

Impact of δ on Estimated Discount Rates (in %) for different Utility Functions

		δ										
		1.00	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50
Linear	Mean	4.86	4.65	4.68	4.64	4.42	4.64	5.17	5.16	5.38	6.63	5.77
	N	83	75	62	55	45	35	26	22	18	12	10
Nat log	Mean	1.06	1.02	1.00	1.02	0.94	0.93	0.78	0.75	0.79	0.81	0.79
	N	38	37	36	33	33	29	26	24	19	14	12
Neg exp ($a = 5$)	Mean	2.88	2.80	2.74	2.58	2.46	3.08	3.40	3.60	3.05	3.95	4.70
	N	62	55	47	44	38	26	22	19	16	11	8
Power ($\gamma = 0.8$)	Mean	3.76	3.53	3.58	3.30	3.23	3.82	4.44	3.77	4.48	4.48	4.66
	N	74	68	57	50	42	29	21	19	13	12	10

Figure 1: Observation 1 (risk neutral utility function)

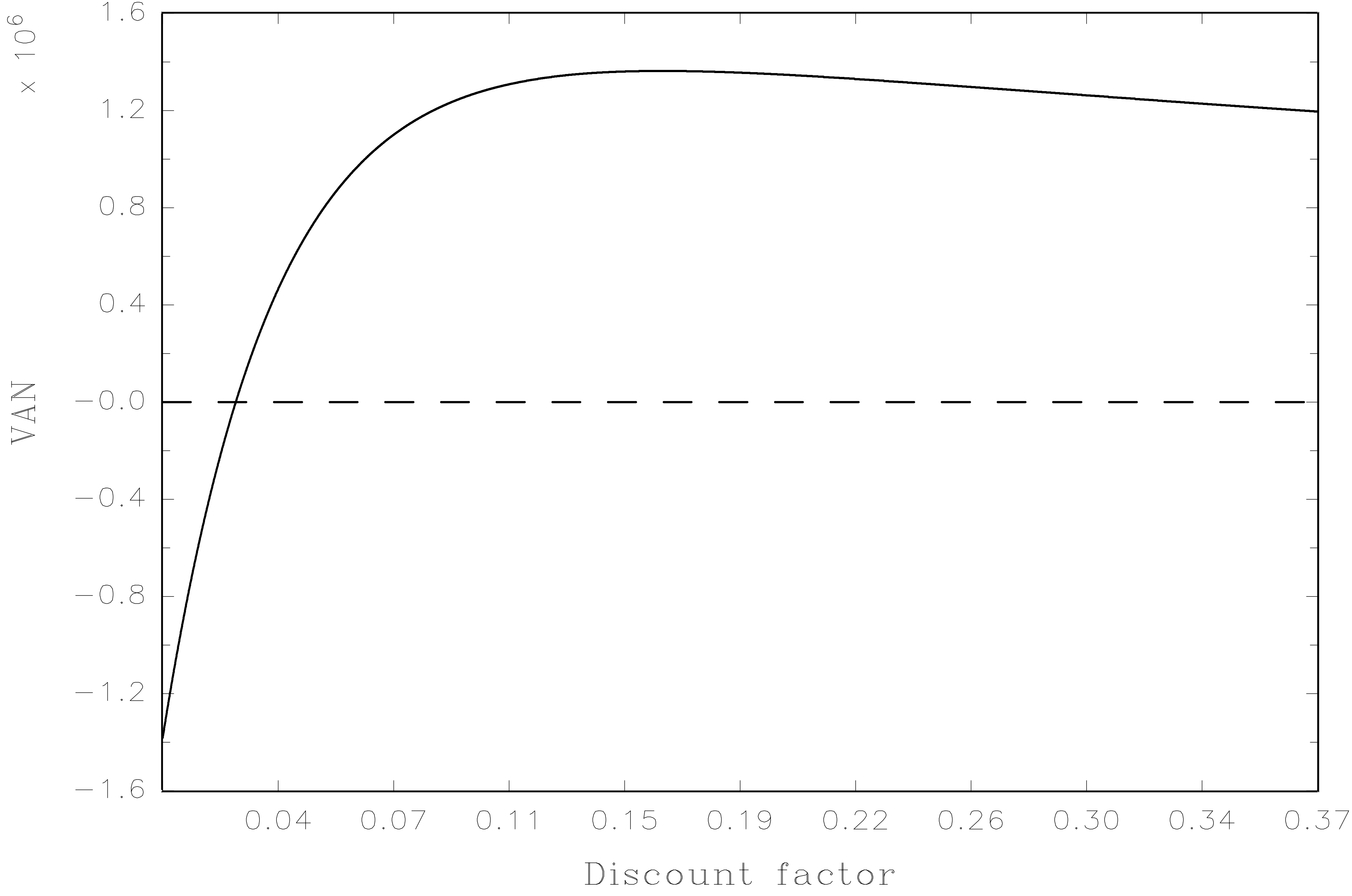

Figure 2: Observation 22 – solution as function of δ
Neutral Risk utility function