Lobbying with Two Audiences: Public vs Private Certification *

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Abstract

This note compares public and private information certification in a simple class of communication games with one sender and two receivers. It also emphasizes the role of belief consistency conditions in a perfect Bayesian equilibrium of such games.

Keywords: Certifiable information, cheap talk, consistency of beliefs.

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1 Introduction

In this note we study a particular setting of lobbying activities with a single lobby and several decisionmakers. We refer to lobbying activities as meetings between the lobbyist (an informed interested party) and the decisionmakers in which the former try to influence the latter's choices by transmitting payoff-relevant information. In the simple setting introduced by Farrell and Gibbons [1989]—with two states of nature, two decisionmakers, and two actions each—we characterize the information revealed at equilibrium depending on whether meetings take place publicly or privately, and whether the information held by the lobbyist is certifiable or not.

Mutual discipline refers to a situation where information is revealed to neither decisionmaker when communication is private, but a fully revealing equilibrium exists when communication takes place publicly. The opposite situation, called *mutual subversion*, refers to a situation where there is a fully revealing equilibrium with each decisionmaker when communication takes place privately, but information is not revealed when communication is public. In Farrell and Gibbons's [1989] binary model, mutual discipline is possible in the cheap talk (non-certifiable) communication case, but there cannot be mutual subversion. We show that the opposite holds in the case of communication with certifiable information: there cannot be mutual discipline but mutual subversion is possible. From a theoretical point of view, our study emphasizes the role of belief consistency conditions that were irrelevant in previous work on strategic information revelation (e.g., Okuno-Fujiwara et al., 1990).

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2 Silent Game

We consider one sender, S (the lobbyist), and two receivers, Q and R (the decisionmakers). The sender observes the state $k \in \{k_1, k_2\}$, but the receivers do not. The prior probabilities of the states are $\Pr(k_1) = p \in (0,1)$ and $\Pr(k_2) = 1-p$. As in Farrell and Gibbons [1989], Q has two actions, q_1 and q_2, and R has two actions, r_1 and r_2. The receivers' payoff, represented in the second coordinate of Table 1, only depends on the state and their own action. The sender's payoff is the sum of the payoff he gets from Q's action and the payoff he gets from R's action. It is represented in the first coordinate of Table 1. For example, when Q chooses action q_1 and R chooses action r_2, the sender's payoff is $v_1 + 0$ if $k = k_1$, and $0 + w_2$ if $k = k_2$. The parameters x_1, x_2, y_1 and y_2 are assumed strictly positive.

	Q		R	
	q_1	q_2	r_1	r_2
k_1	v_1, x_1	$0, 0$	w_1, y_1	$0, 0$
k_2	$0, 0$	v_2, x_2	$0, 0$	w_2, y_2

Table 1: Silent game between the lobbyist and the two decisionmakers.

Without communication, the optimal actions of the decisionmakers are

$$q(p) = \begin{cases} q_1 & \text{if } p \geq \overline{x} \equiv \frac{x_2}{x_1+x_2}, \\ q_2 & \text{if } p \leq \overline{x}, \end{cases} \quad \text{and} \quad r(p) = \begin{cases} r_1 & \text{if } p \geq \overline{y} \equiv \frac{y_2}{y_1+y_2}, \\ r_2 & \text{if } p \leq \overline{y}. \end{cases}$$

3 Cheap Talk

We consider the direct cheap talk extension of the silent game in which before actions are taken, but after the state is revealed to the lobbyist, the latter can, whatever his type, send a costless message in $M = \{m_1, m_2\}$ to the decisionmakers. We say that communication is *public* when both decisionmakers observe the same message from the lobbyist. On the contrary, when the lobbyist can send a private, possibly different message to each decisionmaker, then communication is said *private*.[1] A perfect Bayesian equilibrium is defined as usual. In the cheap talk games considered in this section, the set of perfect Bayesian equilibrium outcomes coincide with the set of Nash equilibrium outcomes. These sets may differ in the case of certifiable information (see the next section).

When communication is public, the receivers' beliefs about the state should be the same since the receivers start exactly with the same prior beliefs, and the lobbyist's message is common knowledge. This is indeed true along any Nash equilibrium path (by Bayes' rule given the lobbyist's strategy), but also off the equilibrium path at a perfect Bayesian equilibrium

[1] The fact that a decisionmaker knows whether the lobbyist sends a message to the other decisionmaker is irrelevant because there is no payoff interaction between the two decisionmakers. In more general settings, private communication may differ from *secret* communication.

(see, e.g., Fudenberg and Tirole, 1991, condition B(iv) page 332).[2] Since Nash and perfect Bayesian equilibrium outcomes coincide in cheap talk games, this last requirement is irrelevant here but will matter in the associated information certification games.

We assume without loss of generality that $\overline{x} < \overline{y}$. Then, the decisionmakers' optimal actions given a (common) belief μ about state k_1 are

$$(q(\mu), r(\mu)) = \begin{cases} (q_1, r_1) & \text{if } \mu \geq \overline{y}, \\ (q_1, r_2) & \text{if } \overline{x} \leq \mu \leq \overline{y}, \\ (q_2, r_2) & \text{if } \mu \leq \overline{x}. \end{cases} \tag{1}$$

A *fully revealing equilibrium* is a (Nash or perfect Bayesian) equilibrium in which the lobbyist reveals all his information, for example by sending message m_1 when the state is k_1 and m_2 when the state is k_2. We immediately get the following proposition (Farrell and Gibbons, 1989).

Proposition 1 *Consider the cheap talk game. There exists a fully revealing equilibrium when the lobbyist communicates* privately *with the decisionmaker Q (R, respectively) if and only if $v_1 \geq 0$ and $v_2 \geq 0$ ($w_1 \geq 0$ and $w_2 \geq 0$, respectively). There exists a fully revealing equilibrium when the lobbyist communicates* publicly *with the two decisionmakers if and only if $v_1 + w_1 \geq 0$ and $v_2 + w_2 \geq 0$.*

Therefore, whenever there is a fully revealing equilibrium in each private cheap talk game, then there is also one in the public cheap talk game. In Farrell and Gibbons's [1989] terms, mutual subversion (full revelation with both receivers in private but not in public) is not possible in this setting. However, the unique equilibrium outcome may be non-revealing with both receivers in private, but a fully revealing equilibrium may exist in public (take, e.g., $v_1 = w_2 = 3$ and $v_2 = w_1 = -1$);[3] this situation is called mutual discipline.[4]

4 Information Certification

When the set of messages available to the lobbyist depends on his type, information is certifiable as in, e.g., Green and Laffont [1986], Okuno-Fujiwara et al. [1990], Seidmann and Winter [1997] or Forges and Koessler [2005]. To simplify the exposition, we assume that each state is certifiable, so the set of messages available to the lobbyist is $M(k_1) = \{m_1, \overline{m}\}$ when the state is k_1 and $M(k_2) = \{m_2, \overline{m}\}$ when the state is k_2. Of course, in this setting the conditions for a fully revealing equilibrium to exist are weaker than in the cheap talk case

[2]It is easy to see that a common belief for the receivers off the equilibrium path is a requirement of Kreps and Wilson's [1982] sequential equilibrium and Selten's [1975] perfect equilibrium because in the perturbed games receivers use the same trembling strategies of the sender to update their beliefs.

[3]Partially revealing equilibria, in mixed strategies, may also exist for generic parameters.

[4]A recent application of this effect includes, e.g., Levy and Razin [2004], in a binary model of conflict resolution between two countries in which (cheap talk) communication concerns the cost-benefit ratio from making concessions.

since now a lobbyist's type is not necessarily able to imitate the other type's message. As the following proposition shows, the relationship between these conditions in the public and private communication situations is also different, and depends on the adopted equilibrium concept.

Proposition 2 *Consider the information certification game. There exists a fully revealing Nash or perfect Bayesian equilibrium when the lobbyist communicates* privately *with the decisionmaker Q (R, respectively) if and only if* $v_1 \geq 0$ *or* $v_2 \geq 0$ *(*$w_1 \geq 0$ *or* $w_2 \geq 0$*, respectively). There exists a fully revealing* Nash equilibrium *when the lobbyist communicates* publicly *with the two decisionmakers if and only if condition (i), (ii), (iii) or (iv) below holds. There exists a fully revealing* perfect Bayesian equilibrium *when the lobbyist communicates* publicly *with the two decisionmakers if and only if condition (i), (ii) or (iii) below holds.*

(i) $v_1 + w_1 \geq 0$	*(ii)* $v_2 + w_2 \geq 0$
(iii) $w_1 \geq 0$ *and* $v_2 \geq 0$	*(iv)* $v_1 \geq 0$ *and* $w_2 \geq 0$.

Proof. The fully revealing Nash equilibrium in the private communication game with Q is supported by the fully revealing strategy $(m_1 \mid k_1, m_2 \mid k_2)$ and the off the equilibrium path action $q_1 \mid \overline{m}$ if $v_2 \geq 0$, $q_2 \mid \overline{m}$ if $v_1 \geq 0$, and any of the two if v_1, $v_2 \geq 0$. If v_1, $v_2 < 0$ then, whatever the mixed action (in $\Delta(\{q_1, q_2\})$ of Q after message $\overline{m}$, the sender has a strict incentive to deviate from full information revelation. To see that the fully revealing Nash equilibrium is also a perfect Bayesian equilibrium it suffices to remark that action $q_1 \mid \overline{m}$ can be made sequentially rational with a belief $\mu(k_1 \mid \overline{m})$ that puts, e.g., probability one to k_1 after message $\overline{m}$, and action $q_2 \mid \overline{m}$ with $\mu(k_1 \mid \overline{m}) = 0$. The same proof applies for the private communication game with R.

Next, consider the Nash equilibria of the public communication game. Full information revelation is supported by the following actions of the receivers in the different situations: (i) (q_2, r_2), (ii) (q_1, r_1), (iii) (q_1, r_2) and (iv) (q_2, r_1). We must also show that there is no fully revealing Nash equilibrium when neither of the conditions above are satisfied. In that case, four situations are possible: (a) $v_2 + w_2 < 0$ and $v_1, w_1 < 0$, (b) $v_1 + w_1 < 0$, $v_2 + w_2 < 0$ and $w_1, w_2 < 0$, (c) $v_1 + w_1 < 0$, $v_2 + w_2 < 0$ and $v_1, v_2 < 0$, and (d) $v_1 + w_1 < 0$ and $v_2, w_2 < 0$. In situation (a), since $v_1, w_1 < 0$, type k_1 does not deviate only if the receivers play $(q_1, r_1) \mid \overline{m}$. But then, since $v_2 + w_2 < 0$, type k_2 deviates and sends message $\overline{m}$ instead of m_2. Situation (d) is symmetric. Next, consider situation (b). If $v_1 < 0$ or $v_2 < 0$ we are also in situation (a) or (d), so assume that $v_1, v_2 \geq 0$. The sender does not deviate from full information revelation if and only if there is a mixed actions profile of the receivers,[5] (α_1, α_2), (β_1, β_2), where α_i is Q's probability of playing action q_i after $\overline{m}$, and β_i is R's probability of

[5]It can be checked that the result also holds when the receivers can use correlated strategies.

playing action r_i after $\overline{m}$, such that

$$
\begin{aligned}
v_1 + w_1 &\geq \alpha_1\beta_1(v_1 + w_1) + \alpha_1\beta_2 v_1 + \alpha_2\beta_1 w_1, \\
v_2 + w_2 &\geq \alpha_1\beta_2 w_2 + \alpha_2\beta_1 v_2 + \alpha_2\beta_2(v_2 + w_2).
\end{aligned}
$$

If $v_1 = 0$ or $v_2 = 0$, then these conditions cannot be satisfied simultaneously, so let $v_1, v_2 > 0$. The previous inequalities become equivalent to (recall that $v_1 > 0$ and $w_2 < 0$)

$$
\begin{aligned}
\alpha_1\beta_2 &\leq (1 - \alpha_1\beta_1) + (w_1/v_1)(1 - \alpha_1\beta_1 - \alpha_2\beta_1), \\
\alpha_1\beta_2 &\geq (1 - \alpha_2\beta_2) + (v_2/w_2)(1 - \alpha_2\beta_2 - \alpha_2\beta_1).
\end{aligned}
$$

Since $v_2 + w_2 < 0$, $w_2 < 0$ and $v_2 > 0$ imply $v_2/w_2 > -1$, and $v_1 + w_1 < 0$ and $v_1 > 0$ imply $w_1/v_1 < -1$, for a fully revealing Nash equilibrium to exist we must have $(1 - \alpha_2\beta_2) + (-1)(1 - \alpha_2\beta_2 - \alpha_2\beta_1) \leq (1 - \alpha_1\beta_1) + (-1)(1 - \alpha_1\beta_1 - \alpha_2\beta_1)$. This inequality is strict if $(1 - \alpha_2\beta_2 - \alpha_2\beta_1)$ or $(1 - \alpha_1\beta_1 - \alpha_2\beta_1)$ equals zero. If the first (second, respectively) term is zero, then type k_2 (k_1, respectively) deviates. But strict inequality implies $\alpha_2\beta_1 < \alpha_2\beta_1$, a contradiction. Therefore, there is no fully revealing Nash equilibrium. Situation (c) is symmetric.

Finally, consider the perfect Bayesian equilibria of the public communication game. The actions profiles $(q_2, r_2) \mid \overline{m}$, $(q_1, r_1) \mid \overline{m}$ and $(q_1, r_2) \mid \overline{m}$, respectively, can be made sequentially rational with the common off the equilibrium path beliefs $\mu(k_1 \mid \overline{m}) \leq \overline{x}$, $\mu(k_1 \mid \overline{m}) \geq \overline{y}$ and $\mu(k_1 \mid \overline{m}) \in [\overline{x}, \overline{y}]$, respectively. Hence, we get a fully revealing perfect Bayesian equilibrium in cases (i) to (iii). It remains to show that for any sequentially rational mixed actions profile of the receivers after message $\overline{m}$ (with a common belief), the sender deviates in case (iv) if neither (i), (ii) nor (iii) are satisfied. This situation implies $v_1 + w_1 < 0$, $v_2 + w_2 < 0$ and $v_1, v_2 \geq 0$. Since a sequentially rational mixed actions profile with common beliefs puts zero probability on (q_2, r_1) (see the best responses in Equation (1)), it is clear that in this situation both types of the sender want to deviate by sending message $\overline{m}$. ∎

Compared to the cheap talk situation, mutual discipline is impossible (whatever the equilibrium concept). Indeed, there is no fully revealing equilibrium in the private meetings if and only if v_1, v_2, w_1 and w_2 are strictly negative, which implies that conditions (i) to (iv) are not satisfied. On the contrary, mutual subversion becomes possible with the perfect Bayesian equilibrium concept (but not with the Nash equilibrium concept). For example, when $v_1 = w_2 = x_1 = x_2 = y_1 = 1$, $v_2 = w_1 = -2$ and $y_2 = 2$ there is a fully revealing perfect Bayesian equilibrium in both private meetings but there is none in the public meeting.

Yet, relaxing the definition of perfect Bayesian equilibrium by allowing *any* belief off the equilibrium path changes this last conclusion. Indeed, the actions profile $(q_2, r_1) \mid \overline{m}$ can be made sequentially rational with heterogeneous beliefs $\mu_Q(k_1 \mid \overline{m}) \leq \overline{x}$ for receiver Q and $\mu_R(k_1 \mid \overline{m}) \geq \overline{y} > \overline{x}$ for receiver R. Hence, under this weaker but less standard definition, perfect Bayesian equilibrium and Nash equilibrium outcomes coincide.

5 Conclusion

In this note we have studied strategic information certification in a simple model of communication with heterogeneous audiences. In general, contrary to the cheap talk case, public communication with certifiable information interfere with information revelation comparing to the case of private communication, at least under the standard assumption of perfect Bayesian equilibrium behavior. How this conclusion holds and depends on belief consistency requirements in more general settings is left for further research.

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